Antenna Parameters:

$D =$ Directivity $= \left(\dfrac{\underline{\text{Peak Intensity}}}{\text{Avg Intensity}} \right) = \dfrac{P_{peak}}{\left(\dfrac{\int \text{Intensity}\, d\Omega}{4\pi} \right)}$

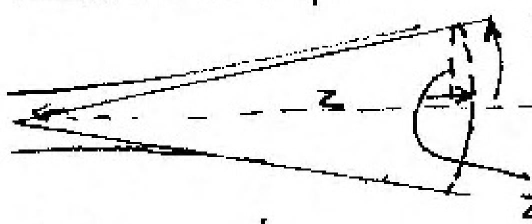
Solid · Angle: $\Omega\, P_{peak} =$ Total Power

$= \int \text{Intensity}\, d\Omega$

Thus $\boxed{D = \dfrac{4\pi}{\Omega}}$

Basic Relationship. - Gaussian Difracting Beam

$\dfrac{w(z)}{z} \rightarrow \dfrac{w_0}{z_0} = \dfrac{\lambda}{\pi w_0} = \theta_0$



solid angle.

$$\frac{\int_0^{\theta_0} 2\pi z \sin\theta \, z\, d\theta}{z^2} = 2\pi (1 - \cos\theta_0) \simeq \pi \theta_0^2$$

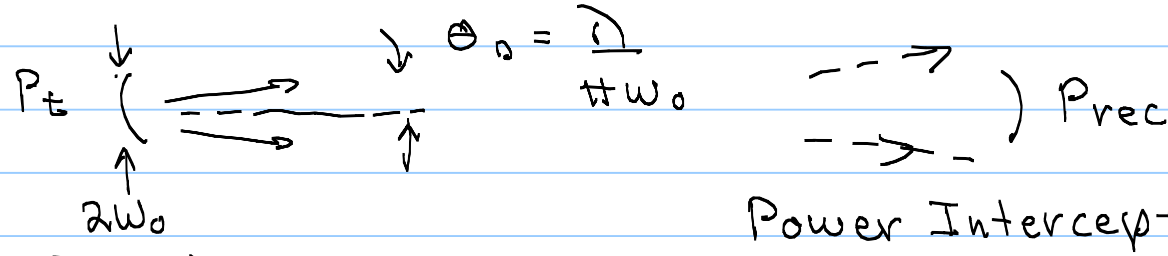
Consequently $D = \dfrac{4\pi}{\pi \theta_0^2} = \dfrac{4}{\lambda^2} \pi^2 w_0^2$

$= \dfrac{4\pi \text{ Area}}{\lambda^2}$

In particular if $D = 1$ Area $= \dfrac{\lambda^2}{4\pi}$

Minimum area is roughly $\lambda \times \lambda$

The Friis Equation



Power Intercepted

(Gaussian Assumed)

$(\text{Area} = A = \pi w_0^2)$

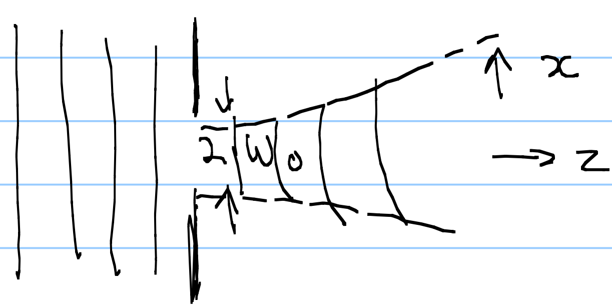
$$P_{rec} = \underbrace{\boxed{\frac{P_t D}{4\pi r^2}}}_{\text{Power / Area}} \times A_{rec}$$

$$P_{rec} = \frac{P_t}{\cancel{4\pi}} \left(\frac{\cancel{4\pi}}{\Omega}\right) \frac{A_{rec}}{r^2} = \frac{P_t (\pi w_0^2) A_{rec}}{\lambda^2 r^2}$$

Since $\Omega = \dfrac{\lambda^2}{\pi w_0^2}$ $\qquad \Omega = 2\pi (1 - \cos\theta_D)$

$$\simeq \pi \theta_D^2 \simeq \pi \frac{\lambda^2}{\pi^2 w_0^2}$$

The basic argument for Diffraction



Uncertainty in x due to aperture

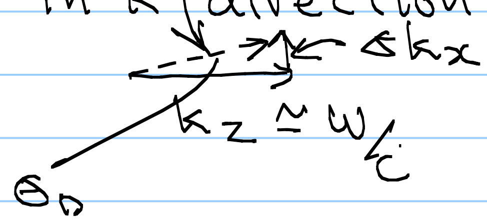
$$\Delta k_x \Delta x \simeq 1$$

$$\Delta x = 2w_0$$

Aperture

uncertainty in k direction due to

i.e. $\Delta k_x \simeq \dfrac{1}{2w_0}$



Thus $\theta_D \simeq \dfrac{\Delta k_x}{k} = \dfrac{1/2w_0}{2\pi/\lambda}$

$= \lambda / \pi w_0$