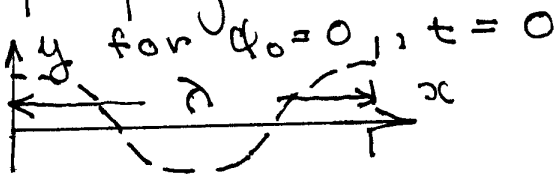
Time-varying ⇒ Coupling of Electric and (1)
Magnetic effects ⇒ waves. - Maxwell

Fields are vectors so waves are vectors
However the waves we will deal with mostly are transverse - The fields ($\bar{E}$ and $\bar{H}$) are $\perp$'r to the direction of propagation

Eq (1.17) of Ulaby



$$y(x,t) = A\cos\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0\right) \qquad (1.19)$$

why does this describe a wave? Ans. lets follow a given amplitude (value of $y = y_0$)
Then $\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0$ must be a constant $= K$
(in fact $y_0 = A\cos K$ or $K = \cos^{-1}(y_0/A)$)
Thus taking differentials

$$\frac{2\pi}{T}dt - \frac{2\pi}{\lambda}dx = 0 \text{ or!}$$

$$dx/dt = (\lambda/T)$$ - We must move at speed λ/T in the x-direction in order to ride with the wave! This is called the phase speed (or phase velocity)
Fig (1-12) shows this movement as time evolves.

In time T the waveform has moved

$$\frac{dx}{dt} T = \lambda/T \, T = \lambda$$

λ as we see is the length of one cycle of the wave or "wavelength"

If x is held fixed, say 0 then

$$y(0,t) = A \cos\left(\frac{2\pi}{T} t\right) \qquad (\phi_0 = 0 \text{ still})$$

In $t = T$ the wave has gone through 1 cycle. T is the "period" of the wave

Other parameters we will use

f = frequency = no of cycles / sec (for fixed x) $= \frac{1}{T}$

$\frac{\beta}{2\pi}$ = spatial frequency = no of cycles/m (for fixed t) $= \frac{1}{\lambda}$

$2\pi f = \omega$ = angular frequency
[ωt has units of radians]

$2\pi \frac{\beta}{2\pi} = \beta$ = "propagation vector"
[βx also has units of radians]

So $y(x,t) = A \cos(\omega t - \beta x + \phi_0)$ for the above wave form (1.32)

Lossy medium → waves decay exponentially

$$y(x,t)\Big|_{\text{Lossy}} = e^{-\alpha x}\, y(x,t)\Big|_{\text{Lossless}}$$

$$= e^{-\alpha x} A\cos(\omega t - \beta x + \phi_0)$$

Example 1.-2

$$E(x,t) = 150\, e^{-.03x} \cos(3\times10^{15}\, t - 10^7 x) \quad V/m$$

$$\text{speed} = \frac{3\times10^{15}}{10^7} = 3\times10^8 \ m/sec. = \omega/\beta = f\Big/\left(\frac{\beta}{2\pi}\right) = \frac{\lambda}{T}$$

Note: $f\Big/\left(\frac{\beta}{2\pi}\right) = f\Big/\frac{1}{\lambda} = f\lambda = \text{speed} = c = 3\times10^8 \frac{m}{s}$

At $x = 200\,m$, the electric field amplitude is

$$e^{-\alpha x} A = 150\, e^{-.03\times200} = .37 \ V/m.$$

<u>Note</u>: Differential Equation obeyed by $y(x,t)$ for the lossless case $y(x,t) = A\cos(\omega t - \beta x)$

① $\frac{\partial y}{\partial x} \overset{\text{means}}{=} \frac{dy}{dx}\Big|_{t\text{ - a constant}} = -A[\sin(\omega t - \beta x)][-\beta]$

② $\frac{\partial y}{\partial t} = -A[\sin(\omega t - \beta x)]\,\omega$

Thus $\frac{\text{Eq ①}}{\beta} + \frac{\text{Eq (2)}}{\omega} = 0$ or

$$\frac{1}{\beta}\frac{\partial y}{\partial x} + \frac{1}{\omega}\frac{\partial y}{\partial t} = 0 \quad \text{or} \quad (\text{multiplying by } \beta)$$

powerful because this has more general solutions than cos() (much more!!)

$$\boxed{\frac{\partial y}{\partial x} + \frac{1}{v_{ph}}\frac{\partial y}{\partial t} = 0}$$

← a wave equation where $v_{ph} = \omega/\beta$