Time-varying Coupling of Electric and Magnetic effects \( \Rightarrow \) waves. - Maxwell

Fields are vectors so waves are vectors.

However the waves we will deal with mostly are transverse - The fields \( (E, H) \) are \( \perp \) to the direction of propagation.

Eq (1.17) of Ulaby:

\[
y(x,t) = A \cos \left( \frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0 \right) \quad (1.19)
\]

Why does this describe a wave? Ans. Let's follow a given amplitude (value of \( y = y_0 \)).

Then \( \frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0 \) must be a constant = \( K \)

(in fact \( y_0 = A \cos K \) or \( K = \cos^{-1}(y_0/A) \))

Thus taking differentials:

\[
\frac{2\pi}{T} \frac{dt}{dt} - \frac{2\pi}{\lambda} \frac{dx}{dt} = 0 \quad \text{or} \quad \frac{dx}{dt} = \frac{(\lambda/T)}{2\pi}.
\]

- We must move at speed \( \lambda/T \) in the \( x \)-direction in order to ride with the wave! This is called the phase speed (or phase velocity).

Fig (1-12) shows this movement as time evolves.
In time $T$ the waveform has moved
\[
\frac{dx}{dt} = \frac{\lambda}{T} \quad T = \frac{\lambda}{v}
\]

$\lambda$ as we see is the length of one cycle of the wave or "wavelength".

If $x$ is held fixed, say $0$ then
\[
y(0,t) = A \cos \left( \frac{2\pi}{T} t \right) \quad (\phi_0 = 0 \text{ still})
\]

In $t = T$ the wave has gone through 1 cycle. $T$ is the "period" of the wave.

Other parameters we will use
\[
\nu = \text{frequency} = \text{no of cycles/sec} = \frac{1}{T} \quad \text{(for fixed $x$)}
\]
\[
\frac{\beta}{2\pi} = \text{spatial frequency} = \text{no of cycles/m} = \frac{1}{\lambda} \quad \text{(for fixed $t$)}
\]
\[
2\pi \nu = \omega = \text{angular frequency}
\]

[\omega has units of radians]
\[
2\pi \frac{\beta}{2\pi} = \beta = \text{"propagation vector"}
\]

[\beta also has units of radians]

So \[ y(x,t) = A \cos (\omega t - \beta x + \phi_0) \text{ for the above waveform} \] (1.32)
Lobby medium - waves decay exponentially

\[ y_f(x, t) = e^{-\alpha x} y(x, t) \]

\[ = e^{-\alpha x} A \cos(\omega t - \beta x + \phi_0) \]

Example 1.2

\[ E(x, t) = 150 e^{-0.03x} \cos(3 \times 10^5 t - 10^7 x) \text{ V/m} \]

Speed = \[ \frac{3 \times 10^5}{10^7} = 3 \times 10^8 \text{ m/sec} \]

\[ \frac{\beta}{\omega} = \frac{3 \times 10^8}{3 \times 10^{15}} = \frac{3}{10^7} \text{ m/s} \]

Note: \[ \frac{\beta}{\omega} = \frac{1}{\lambda} = \frac{\lambda}{c} = \text{speed} = c = 3 \times 10^8 \text{ m/s} \]

At \( x = 200 \text{ m} \), the electric field amplitude is

\[ e^{-\alpha x} A = 150 e^{-0.03 \times 200} \]

\[ = 0.37 \text{ V/m} \]

Note: Differential Equation obeyed by \( y(x, t) \)

for the lossless case \( y(x, t) = A \cos(\omega t - \beta x) \)

1. \[ \frac{\partial y}{\partial x} = \frac{dy}{dx} \]

\[ = -A [\sin(\omega t - \beta x)] [-\beta] \]

2. \[ \frac{\partial y}{\partial t} = -A \cos(\omega t - \beta x) \]

Thus

\[ \frac{\partial y}{\beta} + \frac{1}{\omega} \frac{\partial y}{\partial t} = 0 \]

(multiplying by \( \beta \))

\[ \frac{1}{\beta} \frac{\partial y}{\partial x} + \frac{1}{\omega} \frac{\partial y}{\partial t} = 0 \]

\[ \frac{\partial^2 y}{\partial x^2} + \frac{1}{\omega} \frac{\partial y}{\partial t} = 0 \]

\[ \frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial x^2} \]

\[ \frac{\partial y}{\partial t} = \frac{\partial y}{\partial t} \]

\[ \frac{1}{\omega} \frac{\partial y}{\partial t} = \frac{1}{\omega} \frac{\partial y}{\partial t} \]

\[ \frac{\partial^2 y}{\partial x^2} + \frac{1}{\omega} \frac{\partial y}{\partial t} = 0 \]

\[ \text{a wave equation} \]

where \( N_{\text{ph}} = \frac{\omega}{\beta} \)