Do your work on the exam.  
If you do need to use extra sheets  
attach these to the exam  
so that these are considered as well.  

Make your methods clear so that partial credits is possible.  

There are two problems  
THE PROBLEMS ARE EACH WORTH THE SAME
Problem Number One)
Consider a point charge \( Q \) located a distance \( h \) from the origin along the \( z \)-axis.

![Diagram of point charge Q at z = h](image)

a) What is the electric potential at point P in terms of \( r_1 \)?

\[
V(P) = \frac{Q}{4\pi \varepsilon_0 r_1}
\]

b) Express the electric potential of part a) in terms of \( r \) and \( \theta \) using the cosine law (\( r \) and \( \theta \) are the spherical coordinates)

\[
r_1 = \left( r^2 + h^2 - 2rh \cos \theta \right)^{\frac{1}{2}}
\]

\[
V(P) = \frac{Q}{4\pi \varepsilon_0 \left( h^2 + r^2 - 2rh \cos \theta \right)^{\frac{1}{2}}}
\]

A second charge \(-Q\) is now placed at \( z = -h\).

C) What is the total potential at P due to the two charges in terms of \( r_1 \) and \( r_2 \), the distance from the second charge to the point P

![Diagram of second charge Q at z = -h](image)

\[
V(P) = \frac{Q}{4\pi \varepsilon_0 \left( \frac{1}{r_1} - \frac{1}{r_2} \right)}
\]
Now the charges are made to oscillate by having a sinusoidal current flow between the two charges. \( I = \tilde{I} e^{j\omega t}/2 + \text{c.c.} \)

\[ \text{d)} \text{ If the charge is represented in phasor form } (q = \tilde{q} e^{j\omega t}/2 + \text{c.c.}) \text{ how is } \tilde{q} \text{ related to } \tilde{I}? \]

\[ \frac{\partial}{\partial t} \nabla \cdot \mathbf{D} + \varepsilon_0 \varepsilon \mathbf{E} = 0 \]

Integrate over small sphere

\[ \int_{s} \varepsilon_0 \varepsilon \mathbf{E} \cdot d\mathbf{A} = \int_{s} \mathbf{D} \cdot d\mathbf{A} = \frac{\tilde{q}}{2} \mathbf{e} \cdot \mathbf{w} \mathbf{t} + \text{c.c.} \]

\[ \frac{\partial}{\partial t} \int_{s} \varepsilon_0 \varepsilon \mathbf{E} \cdot d\mathbf{A} = \frac{\partial}{\partial t} \int_{s} \mathbf{D} \cdot \mathbf{w} \mathbf{t} + \text{c.c.} \]

Thus

\[ \nabla \mathbf{E} = \frac{j \omega \mu_0 \mathbf{H}}{2} \]

\[ \mathbf{H} = \nabla \times \mathbf{E} \]

\[ e^{-jkr_1} = e^{-jkr_1 (1 + \frac{k}{r} \cos \theta)} = e^{-jkr_1} (1 + j k h \cos \theta) \]

\[ e^{-jkr_2} = e^{-jkr_2} (1 - j k h \cos \theta) \]

\[ \nabla \mathbf{E}(r) = \frac{q}{\varepsilon_0 \varepsilon} \left( \frac{1}{r} e^{j\omega t - jkr} \left( \frac{1 + \frac{k}{r} \cos \theta}{r} \right) \right) \]

\[ 
\begin{align*}
\nabla \mathbf{E}(r) &= \frac{q}{\varepsilon_0 \varepsilon} \left( \frac{1}{r} e^{j\omega t - jkr} \left( \frac{(1 + \frac{k}{r} \cos \theta)}{r} \right) \right) \\
\n&= \frac{q}{\varepsilon_0 \varepsilon} \left( \frac{1}{r} e^{j\omega t - jkr} \right) \left( \frac{2 \frac{k}{r} \cos \theta + 2 j k h \cos \theta}{r} \right)
\end{align*}
\]

\[ e^{-jkr} \]

\[ \text{time delay from respective charge to point } P \]
Problem Number Two
Consider an infinite line current \( I_1 = I_2 \pi e^{i\omega t - kx} \). Take \( \omega = 0 \) and \( k = 0 \) for parts a), b), c), and d).

a) Using Amperes law, what is the magnetic field as a function of \( r_1 \), the radial distance from the line current, \( I_1 \)?

\[
\mathbf{B} = \mu \mathbf{H} = \mu \frac{I_1}{2\pi r_1} \hat{\phi}
\]

b) A second parallel line current \( I_2 = -I_1 \pi e^{i\omega t + kx} \) is located a distance \( d \) from the first. What is the total magnetic field as a function of \( x \) in the plane of the two wires for \( \omega = 0 \) and \(-d < x < d\)?

\[
\mathbf{B} = \mu \mathbf{H} = \left( \frac{I_1}{2\pi r_1} - \frac{I_2}{2\pi r_2} \right) \hat{\phi}
\]

\[
r_1 = \frac{d}{2} + x \quad \text{and} \quad r_2 = \frac{d}{2} - x
\]
c) Calculate the magnetic flux between the wires for a differential distance \( \Delta z \) and \( -L < x < +L \), where \( L < d \).

\[
\Phi_{\text{flux}} = \int \mathbf{B} \cdot d\mathbf{s} = \int B \, dz \, dx = \frac{\mu_0}{2\pi} \left( \ln \left( \frac{r_1}{r_2} \right) \right) \left[ z \right]_{-\frac{L}{2}}^{\frac{L}{2}} = \frac{\mu_0}{2\pi} \left( \ln \left( \frac{\frac{d}{2} + \frac{L}{2}}{\frac{d}{2} - \frac{L}{2}} \right) - \ln \left( \frac{\frac{d}{2} - \frac{L}{2}}{\frac{d}{2} + \frac{L}{2}} \right) \right) = \frac{\mu_0}{\pi} \ln \left( \frac{d+L}{d-L} \right) \Delta z
\]

\[d) \text{ Using c), obtain an expression for the inductance per unit length}
\]

\[
\Phi_{\text{flux}} = \int B \, dz = \frac{\mu_0}{\pi} \ln \left( \frac{d+L}{d-L} \right)
\]

e) Does the result in d) change when \( \omega \neq 0 \). State a reason for your answer.

"Does not change. Two wire system supports TEM wave so transverse fields given by static solutions. Thus \( L \) remains the same."

f) Is there an electric field when \( \omega \neq 0 \)? If so give its value (and direction) along \( x \) for \( y = 0 \) and \( -L < x < +L \).

"Continuity \( \nabla \cdot \mathbf{E} + \frac{\partial \mathbf{D}}{\partial t} = 0 \) demands a line charge \( \mathbf{E} = \frac{k}{r} \mathbf{r} \) where \( k = \frac{\omega \mu_0 \varepsilon_0}{2} \).

Then by Gauss's Law,

\[
\mathbf{E} = \frac{\kappa}{4\pi \varepsilon_0} \left( \frac{1}{r_1^2} - \frac{1}{r_2^2} \right) \mathbf{e}_x \begin{align*}
\omega t + \mathbf{k} \mathbf{z}
\end{align*}
\]

we see that \( \mathbf{E} \) is

\[
\mathbf{E} = \frac{\mathbf{E}}{\mu_0 \varepsilon_0} = \sqrt{\frac{\mu_0 \varepsilon_0 \omega}{c}} = \sqrt{\frac{E}{c}}
\]
Problem Number Three
A thin wire carrying a current $I$ is located parallel to a conducting plane and a distance $d$ from it. Determine the direction and force per meter on the wire. [Hint: use the method of images]

Induced surface current

Perfectly conducting surface current $= H_t$

Image current $-I$, which makes $H_t$ continuous

Equivalent to that resulting from an

Force is thus $\left(\frac{\mu_0}{2\pi} \frac{I}{d}\right)$ Newton repulsive