Problem Number one ) Magnetometer
a) How many turns are required for a circular loop of 100-mm radius to develop a peak emf of 10mV if the loop rotates 30r/s in the earth’s magnetic field? Take $B = 60 \mu T$
b) How must the axis be oriented to reduce the voltage to zero and thus determine the direction of the field?
c) Consider the same rotating loop, with the modification that $B$ varies with time given by $B = B_0 \sin \omega t$. Leave the answer in terms of $\omega$. Note at $t=0$, $B = 0$ and the plane of the coil is perpendicular to the field.

Problem Number two ) A current loop
a) A loop of radius $R$ carries a current. Find the magnetic field
   1) At the center of the loop.
   2) As a function of distance along the axis of the loop (z-axis).
   3) At large distances from the loop.
b) Show that for case 3) the vector potential is given by

\[
\mathbf{A} = \frac{\mu_0}{4\pi} \mathbf{m} \times \frac{\mathbf{r}}{r^2}
\]

where $\mathbf{m} = \pi R^2 I \hat{z}$

where $\hat{z}$ is the unit vector in the z-direction.

Problem Number three ) Two line Transmission Line)

a) Establish the expression in table 2-1 for the inductance per unit length of a two line transmission line.
b) Show that the the phasor electric field and magnetic field of a coaxial line:

\[
\mathbf{H} = H_\phi e^{i(\omega t - kz)} \hat{\phi} \\
\mathbf{E} = E_r e^{i(\omega t - kz)} \hat{r}
\]

$H_\phi \hat{\phi}$ and $E_r \hat{r}$ are determined by

\[
\nabla \times (E_r \hat{r}) = 0
\]
\[ \nabla \times (H_\phi \hat{\phi}) = j \hat{z} \]

and \( \nabla = \nabla_t + \frac{\partial}{\partial z} = \nabla_t \) in this case since \( H_\phi \) and \( E_r \) are independent of \( z \). Hence \( H_\phi \) and \( E_r \) are determined by the static solutions.

Problem No. 4) Trans-Line review (Qucs design of a two section \( \lambda/16 \) matching line)

Matching of loads to transmission lines can be accomplished by using multiple sections with different characteristic impedances. This basic idea is used extensively for designing anti-reflective coatings for optical components. This is in actuality filter design and other parameters such as ripple in the passband, and steepness of the edges of the passband are important parameters. Or one might also want a "stop" band of frequencies. This problem deals with a two section \( \lambda/16 \) matching section (shorter than a single \( \lambda/4 \).

a) Assume a 400 \( \Omega \) load, and a 100 \( \Omega \) line. Using Qucs (or simulation software of your choosing) between these two insert two transmission line sections each 65 mm long (electrical length) (Qucs defines this as physical length times index - not phase!). Let the one nearest the load have \( Z_o = 100 \Omega \), and the other 400\( \Omega \). Now attach the 100 \( \Omega \) line. I put in a 100 mm length. Finally place a power source having \( Z = 100 \Omega \). Perform an \( S_{11} \) parameter simulation between 200 and 400 MHz Just do a cartesian plot. You should see a match around 300 MHz.

b) Trace out, on a Smith chart, the trajectory from the load to the source at the matching frequency (center of the passband).

c) Estimate the bandwidth of the match defined as VSWR < 1.2 (from the simulation)

d) Show that the 400 \( \Omega \) section can be replaced with a lumped series inductor with \( \omega L = 173 \) and the 100 \( \Omega \) section with a shunt capacitor with \( \omega C = .00433 \)

e) Look at the filter synthesis tools in Qucs. Pick one and give one parameter optimized by your choice!