UCLID5's Elements: Formal Modeling, Verification, Synthesis, and Learning

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https://github.com/uclid-org/uclid

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Overview of this Tutorial

An **introduction to UCLID5**, a system for formal modeling, verification and synthesis of computational systems

- ✓ Motivation – Verification of Trusted Computing Platforms
- ✓ Multi-Modal Modeling with UCLID5
- Verification by Reduction to Synthesis
- Syntax-Guided Synthesis
- Formal Inductive Synthesis & Oracle-Guided Inductive Synthesis
- Satisfiability and Synthesis Modulo Oracles

Formal Synthesis

- Given:
  - Class of Artifacts C
  - Formal (mathematical) Specification ϕ
- Find $f \in C$ that satisfies ϕ
- Example 1:
  - C: all affine functions f of $x \in R$
  - ϕ: $\forall x.\ f(x) \geq x + 42$
- Example 2: SyGuS
- Example 3: Reactive synthesis (from LTL)

C → defined by grammar
ϕ → SMT formula

ϕ → LTL property
C → set of all FSMs

Induction vs. Deduction

- **Induction**: Inferring general rules (functions) from specific examples (observations)
  - Generalization
- **Deduction**: Applying general rules to derive conclusions about specific instances
  - (generally) Specialization
- **Synthesis** can be Inductive or Deductive or a combination of the two

Inductive Synthesis

- Given
  - Class of Artifacts C
  - Set of (labeled) Examples E (or source of E)
  - A stopping criterion Ψ
    - May or may not be formally described
- Find, using only E, an $f \in C$ that meets Ψ
- Example:
  - C: all affine functions f of $x \in R$
  - E = {(0,42), (1, 43), (2, 44)}
  - Ψ -- find consistent f

Inductive Synthesis for Formal Methods

- **Modeling / Specification**
  - Generating environment/component models
  - Inferring (likely) specifications/requirements
- **Verification**
  - Synthesizing verification/proof artifacts such as inductive invariants, abstractions, interpolants, environment assumptions, etc.
- **Synthesis** (of programs/designs/controllers, etc.)

Verification by Reduction to Synthesis

Artifacts Synthesized in Verification

- Inductive invariants
- Abstraction functions / abstract models
- Auxiliary specifications (e.g., pre/post-conditions, function summaries)
- Environment assumptions / Env model / interface specifications
- Interpolants, Frames in IC3/PDR
- Ranking functions (for proofs of termination)
- Intermediate lemmas for compositional proofs
- Simulation/Bisimulation Relations
- Theory lemma instances in SMT solving
- Patterns for Quantifier Instantiation in SMT solving
- …

One Reduction from Verification to Synthesis

NOTATION

Transition system $M = (I, \delta)$

Safety property $\Psi = G(\psi)$

VERIFICATION PROBLEM

Does M satisfy Ψ?

↓

SYNTHESIS PROBLEM

Synthesize ϕ s.t.

$$I \Rightarrow \phi \wedge \psi$$

$$\phi \wedge \psi \wedge \delta \Rightarrow \phi' \wedge \psi'$$

Two Reductions from Verification to Synthesis

NOTATION

Transition system $M = (I, \delta)$, S = set of states

Safety property $\Psi = G(\psi)$

VERIFICATION PROBLEM

Does M satisfy Ψ?

SYNTHESIS PROBLEM #1

Synthesize ϕ s.t.

$$I \Rightarrow \phi \wedge \psi$$

$$\phi \wedge \psi \wedge \delta \Rightarrow \phi' \wedge \psi'$$

SYNTHESIS PROBLEM #2

Synthesize $\alpha : S \rightarrow \hat{S}$ where

$\alpha(M) = (\hat{I}, \hat{\delta})$

s.t.

$\alpha(M)$ satisfies Ψ

iff

M satisfies Ψ

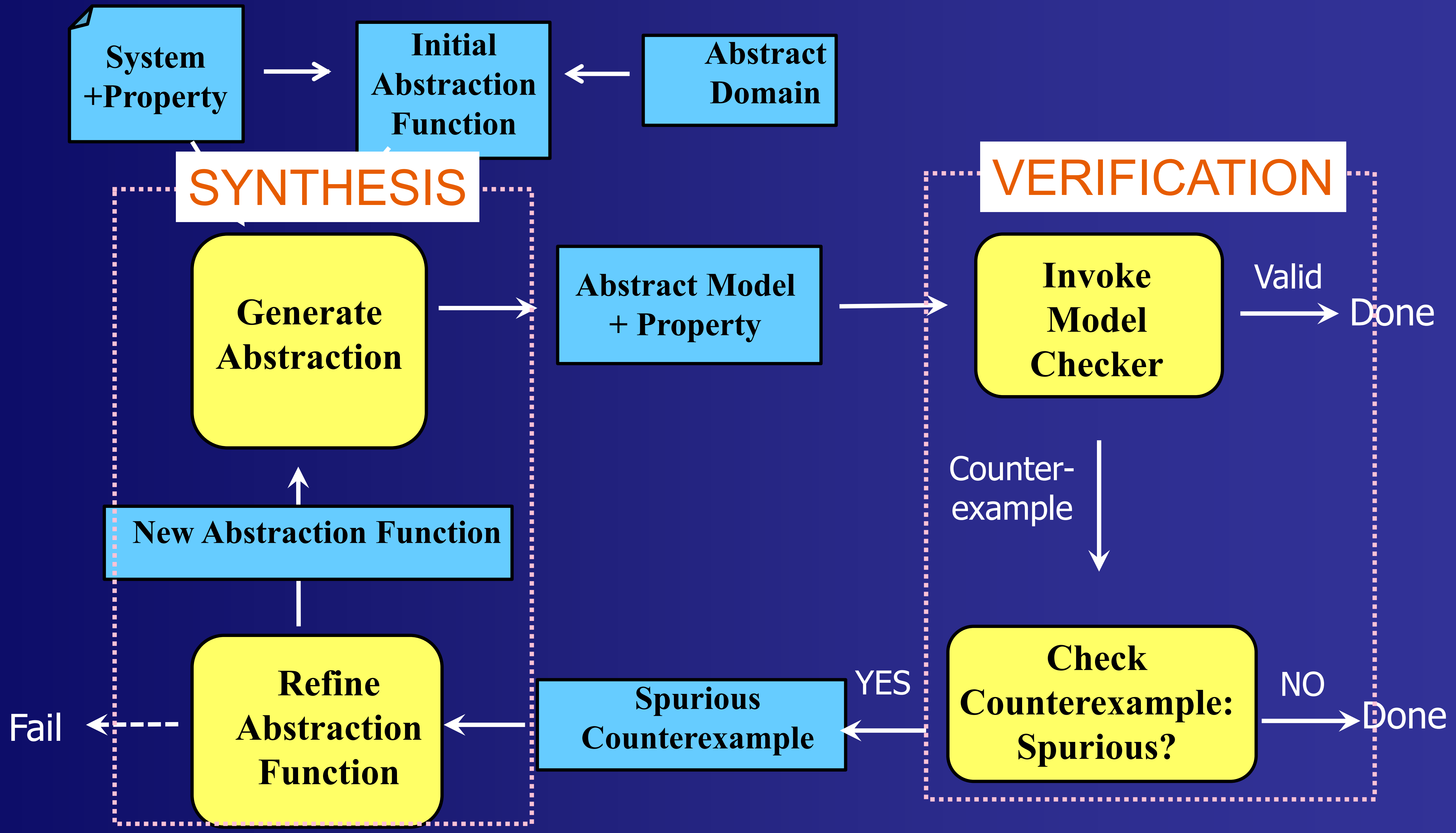
Common Approach for both: Inductive Synthesis

Synthesis of:-

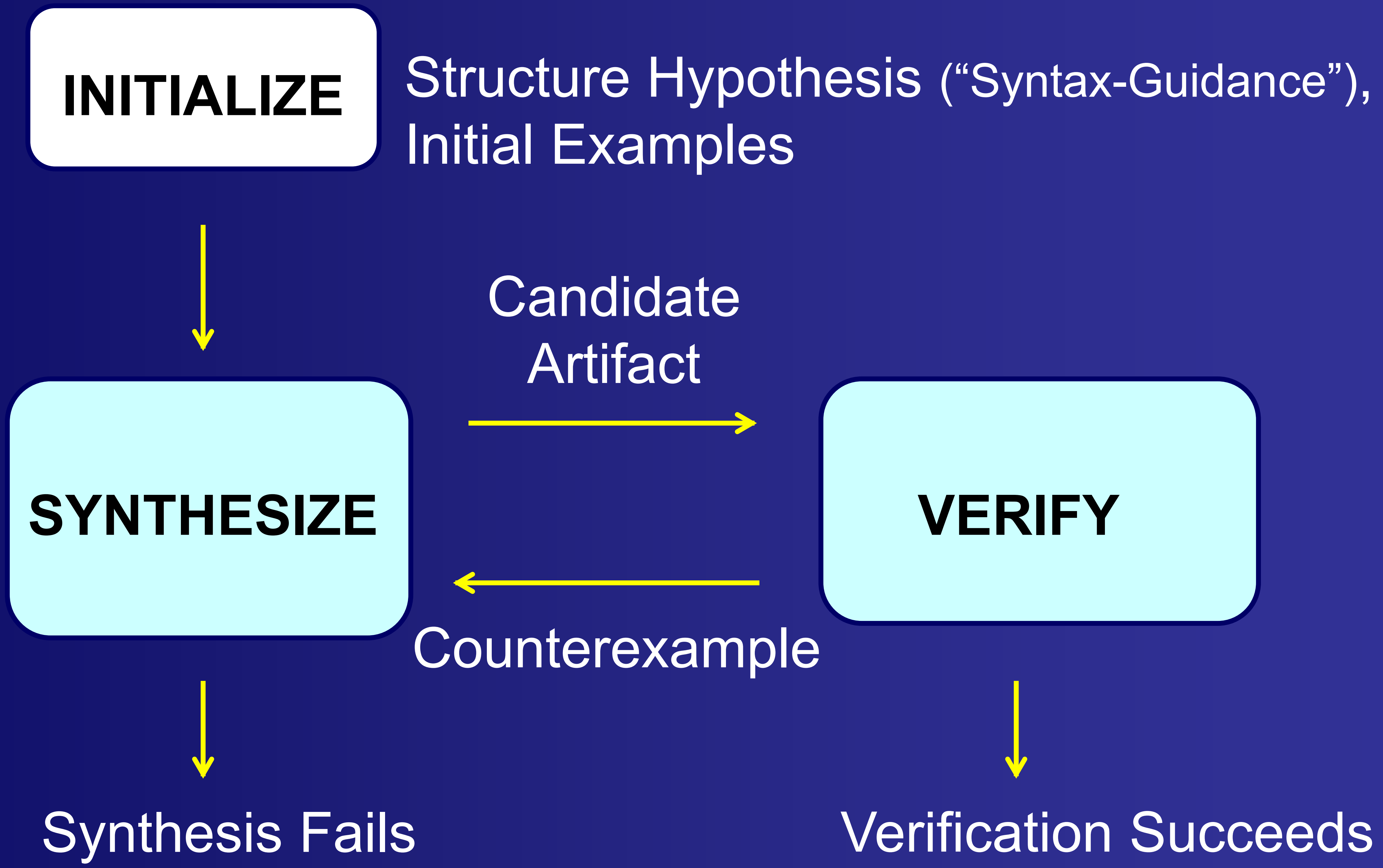
- Inductive Invariants
  - Choose templates for invariants
  - Infer likely invariants from tests (examples)
  - Check if any are true inductive invariants, possibly iterate
- Abstraction Functions
  - Choose an abstract domain
  - Use Counter-Example Guided Abstraction Refinement (CEGAR)

Counterexample-Guided Abstraction Refinement (CEGAR) is Inductive Synthesis/Learning

[Anubhav Gupta, '06]



CEGAR = Counterexample-Guided Inductive Synthesis (of Abstractions)



Syntax-Guided Synthesis

Definition

(FMCAD 2013)

Given:

1. Set of functions $f_1, f_2, \ldots, f_k$ to be synthesized
2. Set of grammars $G_1, G_2, \ldots, G_k$ – each f_i is to be synthesized from $L(G_i)$
3. Specification ϕ – SMT formula in $T \cup (EUF)$ — $\phi(\vec{f}, \vec{X})$

Find: Expressions $e_i \in L(G_i)$ s.t. $\phi[f_i \leftarrow e_i,\ i=1,\ldots,k]$ is valid in T

If these exist, realizable.

if not, unrealizable.

$$\exists \vec{f} \in L(\vec{G})\ \forall \vec{X}\ .\ \phi(\vec{f}, \vec{X})$$

Example

$\phi(f, x, y)$ f : integer x integer → integer

Specification: (x ≤ f(x,y)) & (y ≤ f(x,y)) & (f(x,y) = x | f(x,y) = y) — QF_LIA

Set E: All expressions built from x,y,0,1, Comparison, +, If-Then-Else

L(G)

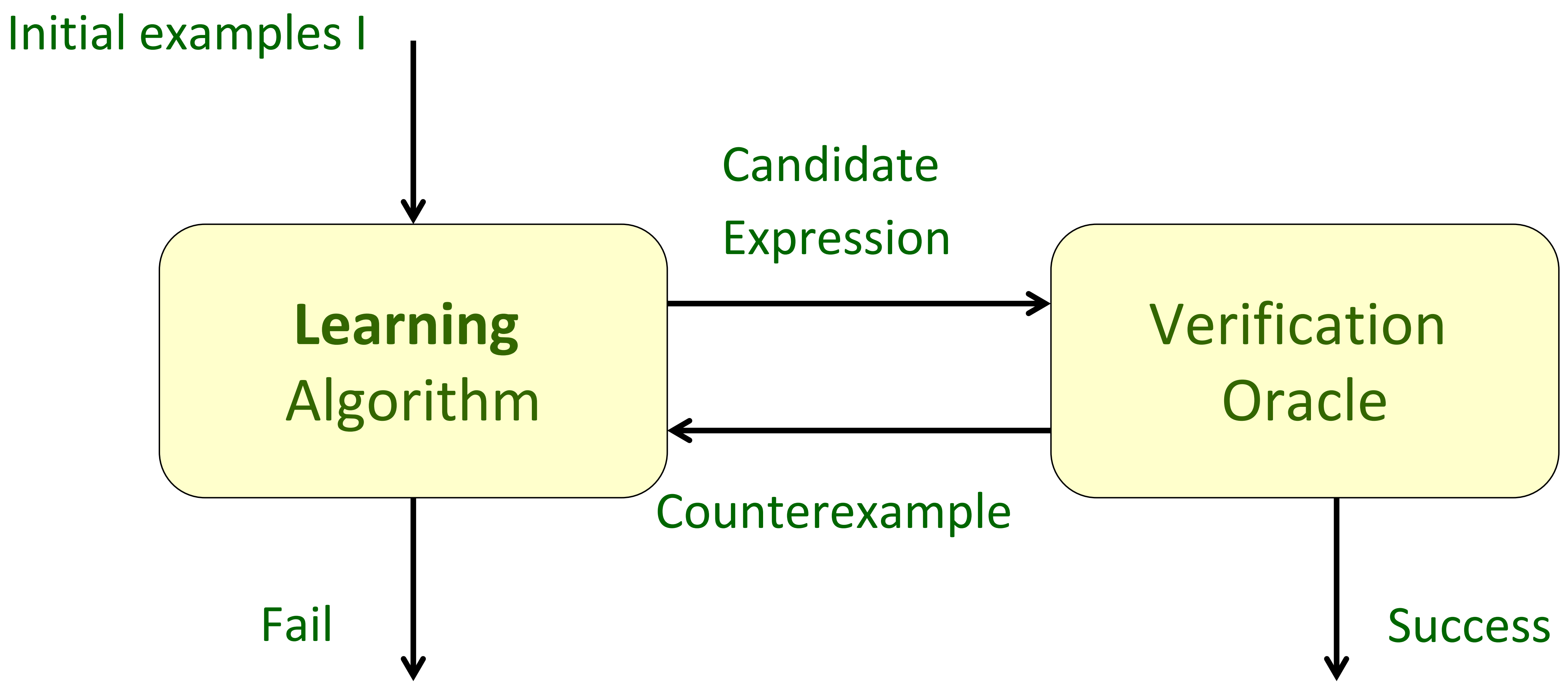
$f(x,y) = \text{ITE}(x \geq y, x, y)$

$x > y$

max

$(x \leq 0) \land (y \leq 0) \land (0 = x \mid 0 = y)$ X

SyGuS solved through Counterexample-Guided Inductive Synthesis (Counterexample-Guided Learning)



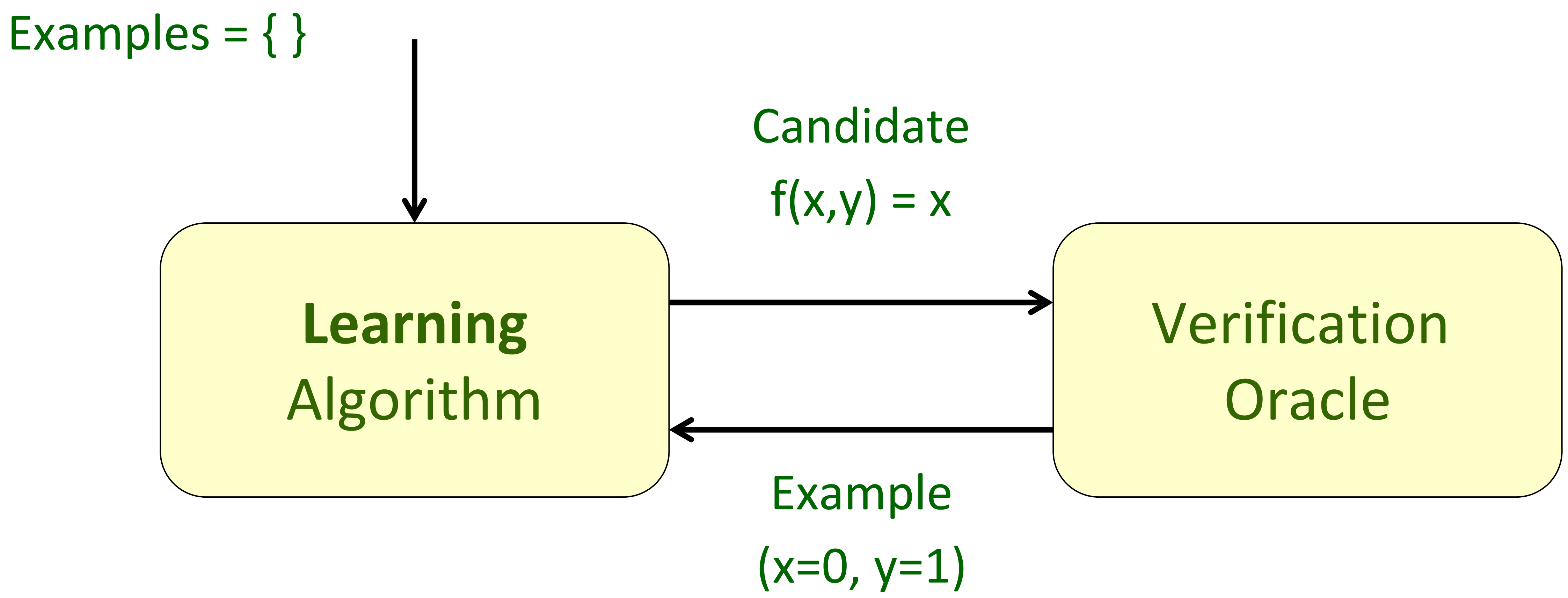
Concept class: Set E of expressions

Examples: Concrete input values

(slide adapted from one by R. Alur)

CEGIS Example

- Specification: $(x \le f(x,y))$ & $(y \le f(x,y))$ & $(f(x,y) = x \mid f(x,y) = y)$
- Set E: All expressions built from x,y,0,1, Comparison, +, If-Then-Else



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CEGIS Example

- Specification: $(x \le f(x,y))$ & $(y \le f(x,y))$ & $(f(x,y) = x \mid f(x,y) = y)$

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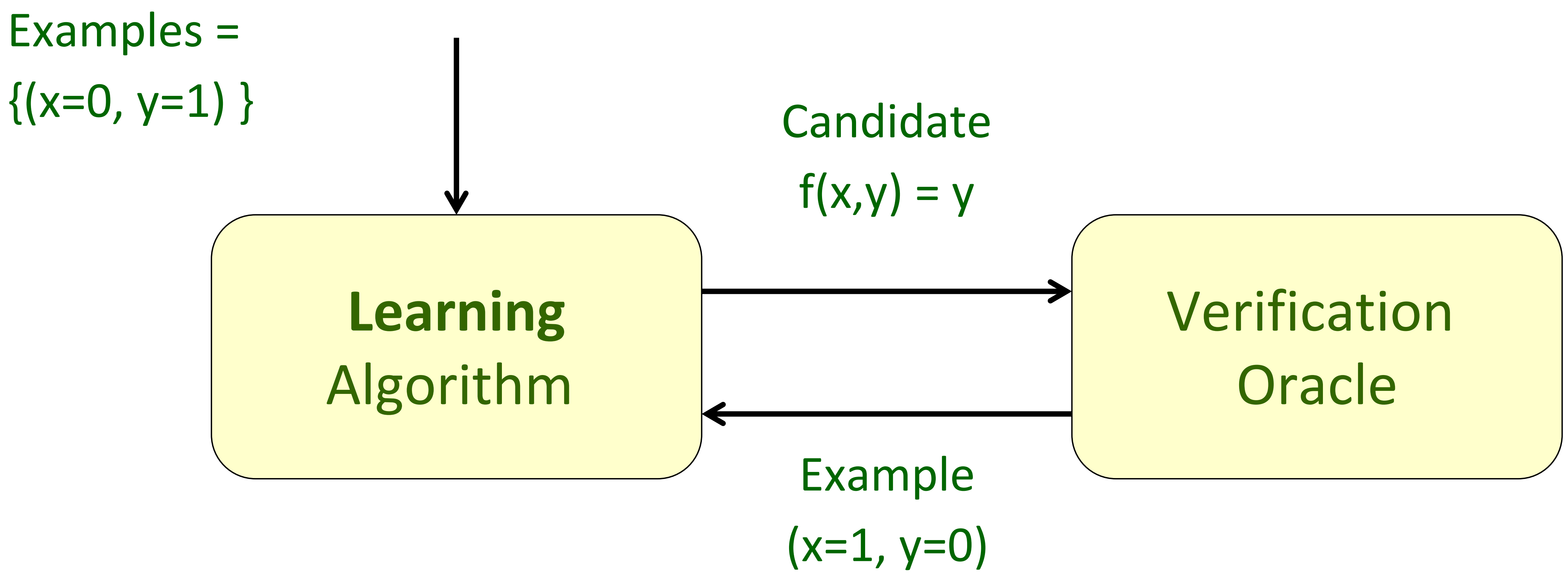


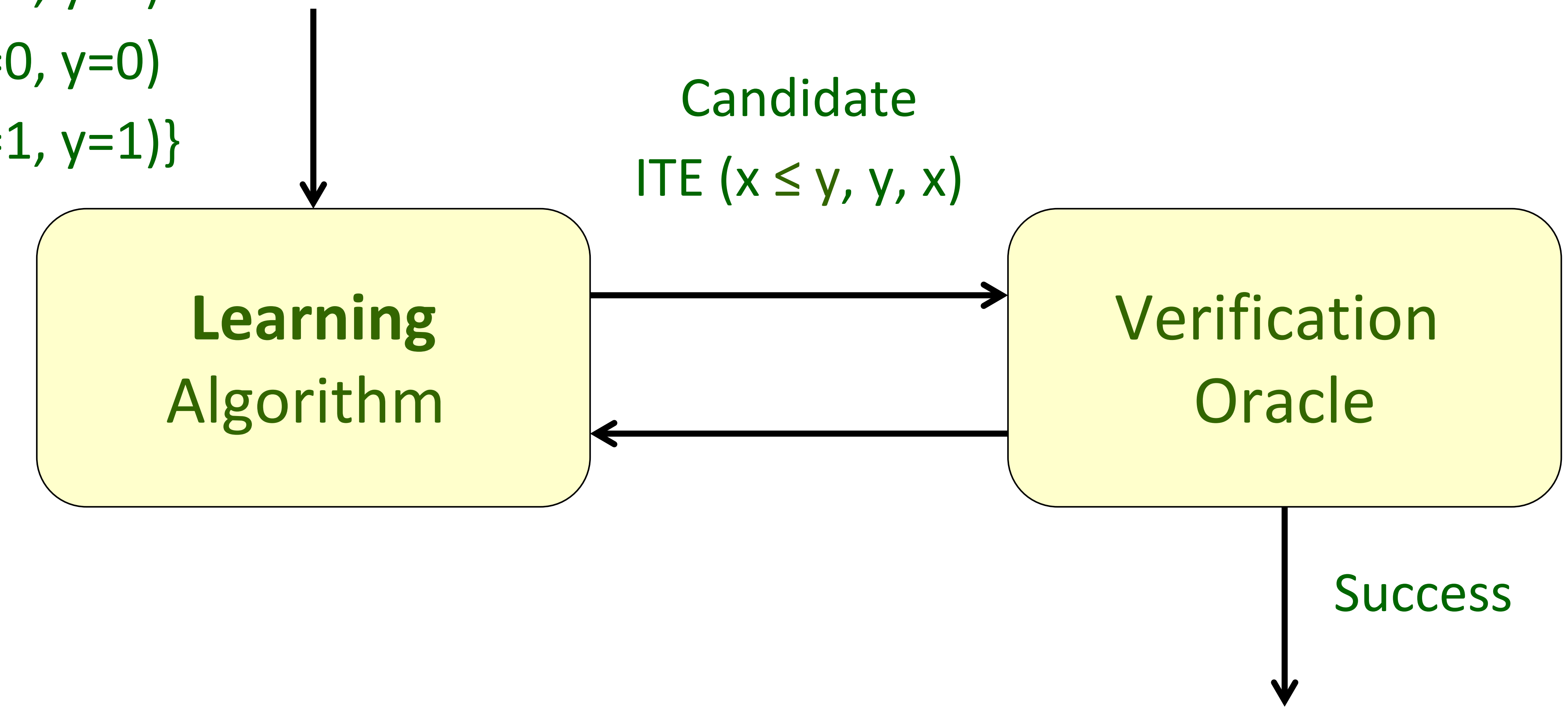
(slide adapted from one by R. Alur)

CEGIS Example

- Specification: (x ≤ f(x,y)) & (y ≤ f(x,y)) & (f(x,y) = x | f(x,y) = y)

- Set E: All expressions built from x,y,0,1, Comparison, +, If-Then-Else

Examples =
{(x=0, y=1)
(x=1, y=0)
(x=0, y=0)
(x=1, y=1)}



(slide adapted from one by R. Alur)

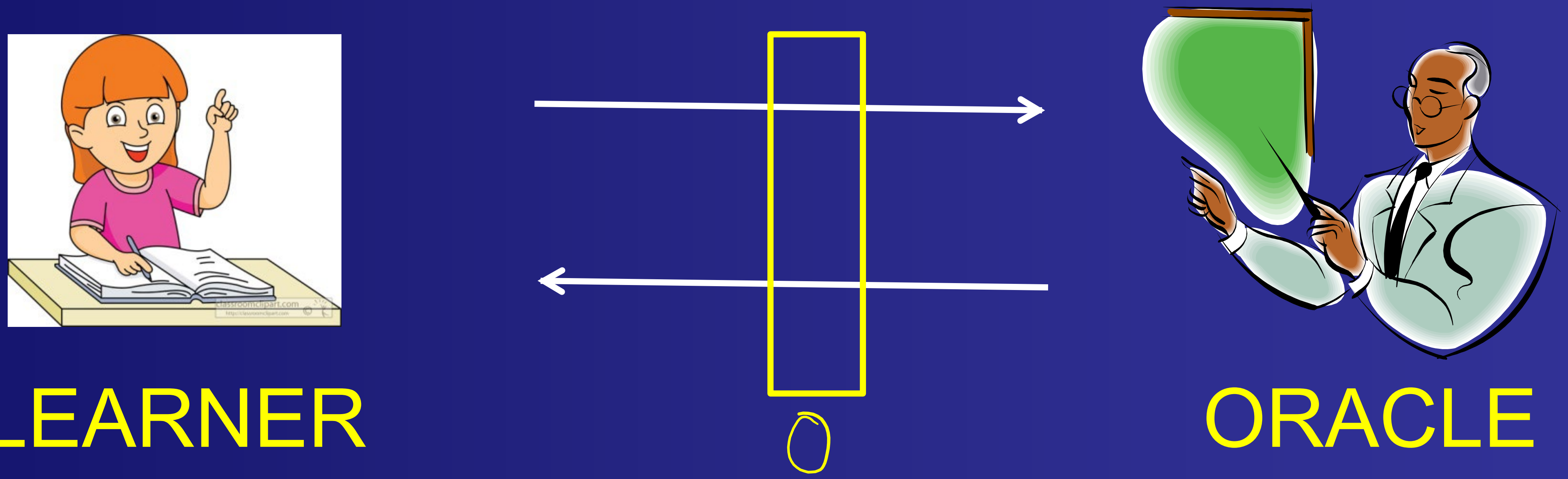
Formal Inductive Synthesis & Oracle-Guided Inductive Synthesis

Formal Inductive Synthesis

- Given:
  - Class of Artifacts C -- Formal specification ϕ
  - Domain of examples D
  - Oracle Interface O
    - Set of (query, response) types
- Find using only O an $f \in C$ that satisfies ϕ
  - i.e. no direct access to D or ϕ

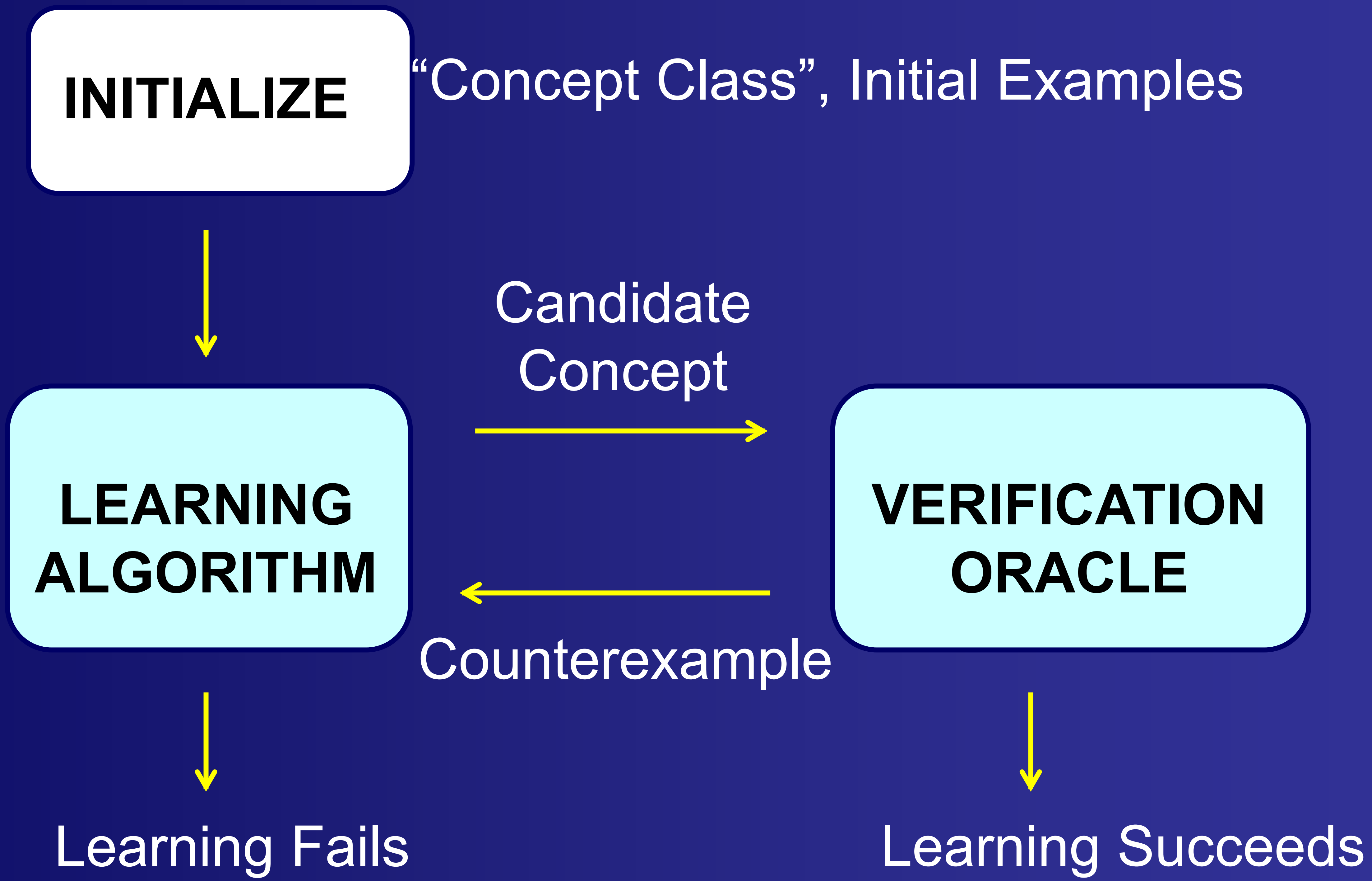
Oracle Interface

- Generalizes the simple model of sampling positive/negative examples from a corpus of data

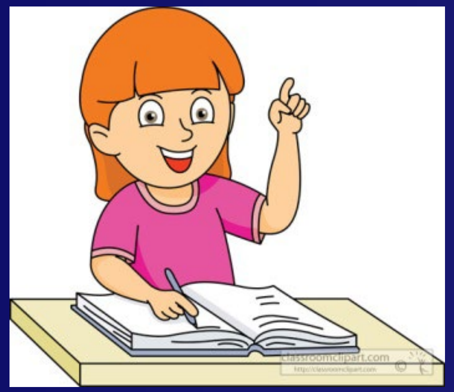


- Specifies WHAT the learner and oracle do
- Does *not* specify HOW the oracle/learner is implemented

CEGIS = Learning from Examples & Counterexamples



Common Oracle Query Types (for trace property ϕ)



Positive Witness

$x \in \phi$, if one exists, else $\perp$

Negative Witness

$x \notin \phi$, if one exists, else $\perp$

Membership: Is $x \in \phi$?

Yes / No

Equivalence: Is $f = \phi$?

Yes / No + $x \in \phi \oplus f$

Subsumption/Subset: Is $f \subseteq \phi$?

Yes / No + $x \in f \setminus \phi$

Distinguishing Input: f, $X \subseteq f$

f' s.t. $f' \neq f \wedge X \subseteq f'$, if it exists; o.w. $\perp$

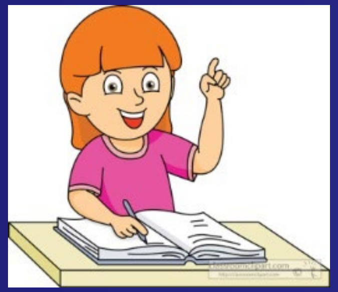
LEARNER

ORACLE

Formal Inductive Synthesis

- Given:
  - Class of Artifacts C -- Formal specification ϕ
  - Domain of examples D
  - Oracle Interface O
    - Set of (query, response) types
- Find using only O an $f \in C$ that satisfies ϕ
  - i.e. no direct access to D or ϕ
- How do we solve this?

Design/Select:



Oracle-Guided Inductive Synthesis (OGIS)

- A **dialogue** is a sequence of (query, response) conforming to an oracle interface O
- An **OGIS engine** is a pair **<L, T>** where
  - L is a learner, a non-deterministic algorithm mapping a dialogue to a concept c and query q
  - T is an oracle/teacher, a non-deterministic algorithm mapping a dialogue and query to a response r
- An OGIS engine <L,T> solves an FIS problem if **there exists a dialogue between L and T that converges** in a concept $f \in C$ that satisfies ϕ

[See Jha & Seshia, Acta Informatica 2017 for details]

Examples of OGIS

- **L* algorithm to learn DFAs: counterexample-guided**
  - **Membership + Equivalence queries**
- **CEGIS used in SyGuS solvers**
  - **(positive) Witness + Counterexample/Verification queries**
- **CEGIS for Hybrid Systems**
  - **Requirement Mining** [Jin et al., HSCC 2013]
  - **Reactive Model Predictive Control** [Raman et al., HSCC 2015]
- **Two different examples:**
  - **Learning Programs from Distinguishing Inputs** [Jha et al., ICSE 2010]
  - **Learning LTL Properties for Synthesis from Counterstrategies** [Li et al., MEMOCODE 2011]

More Examples

- CEGIS(T) [3]

Is my program “x + 5” correct?

No, but if you replace 5 with 1, it would be

- ICE learning [4]

Is my invariant “x > 5 ” correct?

No, and $inv(6) \implies inv(5)$

No, and $inv(100) = false$

No, and $inv(0) = true$

[3] Counterexample Guided Inductive Synthesis modulo Theories - Abate et al
[4] ICE: A robust framework for learning invariants - Garg et al

(slide due to E. Polgreen)

Satisfiability and Synthesis Modulo Oracles

(some slide material due to E. Polgreen)

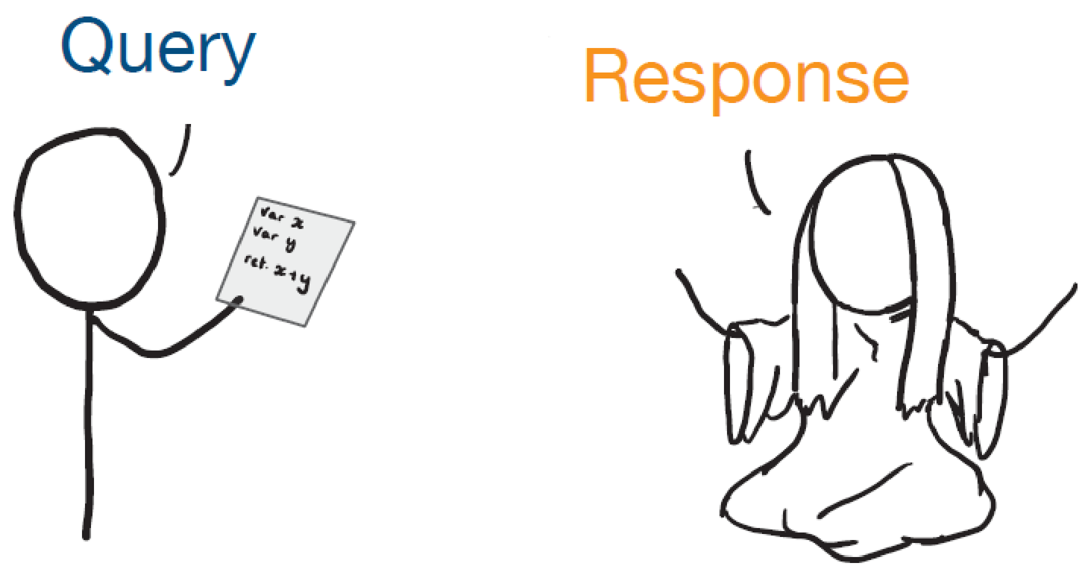
Logic Constraint Solvers → Oracle-Based Solvers

- Current SMT solvers require *all constraints to be encoded as logical formulas*
- Limiting for *complex* components, or those that may only be available as *executables* or via interaction with *humans*
- Our Contribution: [Polgreen et al., VMCAI'22]
  - Satisfiability Modulo Theories *and Oracles* (SMTO)
  - Synthesis Modulo *Oracles* (SMO)
  - Key idea: Oracle Interface expanded by oracle using "assumption generator" and "constraint generators"

$$I = \begin{cases} Q & : (y_1\, \sigma_1), \ldots, (y_j\, \sigma_j) \\ R & : (z_1\, \sigma'_1), \ldots, (z_k\, \sigma'_k) \\ \alpha_{gen} & : \textit{assumption generator} \\ \beta_{gen} & : \textit{constraint generator} \end{cases}$$

Formalized Oracle Interface

- We define how the oracle is queried by defining an interface



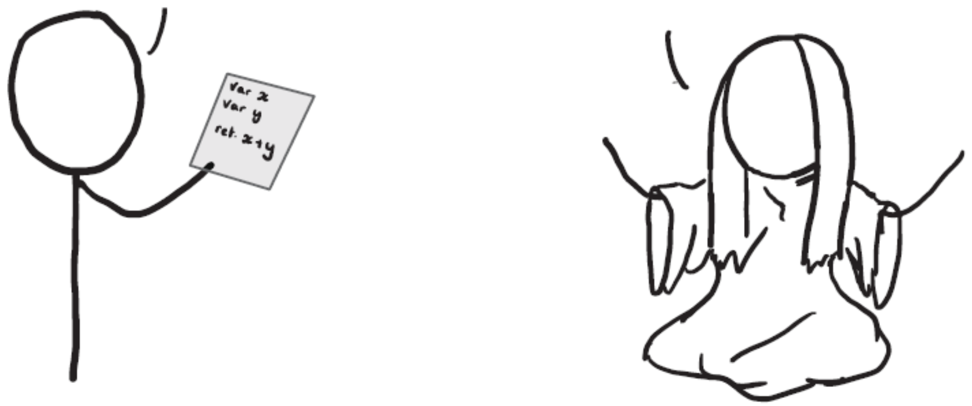
$\vec{y}$: query domain
$\vec{z}$: response co-domain
α_{gen} : assumption generator
β_{gen} : constraint generator

- and assumption and constraint generators, which generate:
  - assumptions the solver is allowed to make
  - and constraints the solver must abide by

Oracle Function Symbols

Is this number y prime?

No, z=false, it is not prime.



An oracle function symbol is a symbol whose behaviour is defined to be the same as an external oracle.

Note: oracle must be functional

$$\vec{y} : (y : integer)$$
$$\vec{z} : (z : bool)$$
$$\alpha_{gen} : prime(y) = z$$
$$\beta_{gen} : \emptyset$$

prime is an oracle function symbol

Satisfiability Modulo Theories and Oracles (SMTO)

An SMTO problem is a tuple:

- $\vec{f}$: a set of ordinary function symbols
- $\vec{\theta}$: a set of oracle function symbols
- ρ : a formula in a background theory
- $\vec{\mathcal{O}}$: a set of oracle interfaces

$\vec{f} : \{f_1, f_2\}$

$\vec{\theta} : \{prime\}$

$\rho : prime(f_1) \wedge prime(f_2) \wedge (f_1 * f_2 = 24)$

$\vec{\mathcal{O}} : \{\mathcal{O}_{prime}\}$

$\mathcal{O}_{prime}$

$\vec{y}$: $(y : integer)$

$\vec{z}$: $(z : bool)$

α_{gen} : $prime(y) = z$

β_{gen} : $\emptyset$

Is this satisfiable? What is a valid assignment to f_1 and f_2?

Satisfiability Modulo Theories and Oracles (SMTO)

SAT?

$$prime(f_1) \wedge prime(f_2) \wedge (f_1 * f_2 = 24)$$

$\mathcal{O}_{prime}$

$\vec{y}$: $(y : integer)$
$\vec{z}$: $(z : bool)$
α_{gen} : $prime(y) = z$
β_{gen} : $\varnothing$

Conjunction of assumptions. True if no assumptions

Satisfiable iff $\exists f_1, f_2 \,.\, \forall prime \,.\, A \implies \rho$ is satisfiable

Unsatisfiable iff $\exists f_1, f_2 \,.\, \exists prime \,.\, A \wedge \rho$ is unsatisfiable

Unknown otherwise

Restrict to *Definitional SMTO*

Satisfiability Modulo Theories and Oracles (SMTO)

SAT?

$$prime(f_1) \wedge prime(f_2) \wedge (f_1 * f_2 = 24)$$

$\mathcal{O}_{prime}$

$\overrightarrow{y}$: $(y : integer)$
$\vec{z}$: $(z_1 : bool, z_2 : integer)$
α_{gen} : $prime(y) = z$
β_{gen} : $f_1 < z_2$

Constraints must be obeyed by the solver:

Conjunction of constraints. True if no constraints.

Satisfiable iff $\exists f_1, f_2 . \forall prime . A \implies (\rho \wedge B)$ is satisfiable

Unsatisfiable iff $\exists f_1, f_2 . \exists prime . A \wedge \rho \wedge B$ is unsatisfiable

Unknown otherwise

Synthesis Modulo Oracles (SyMO)

A SyMO problem is a tuple:

$\vec{f}$: a tuple of functions to synthesise

$\vec{\theta}$: a set of oracle function symbols

$\forall \vec{x} \,.\, \phi$: a formula in a background theory, where ϕ is quantifier-free

$\vec{\mathcal{O}}$: a set of oracle interfaces

Generalizes SyGuS with richer oracle interfaces

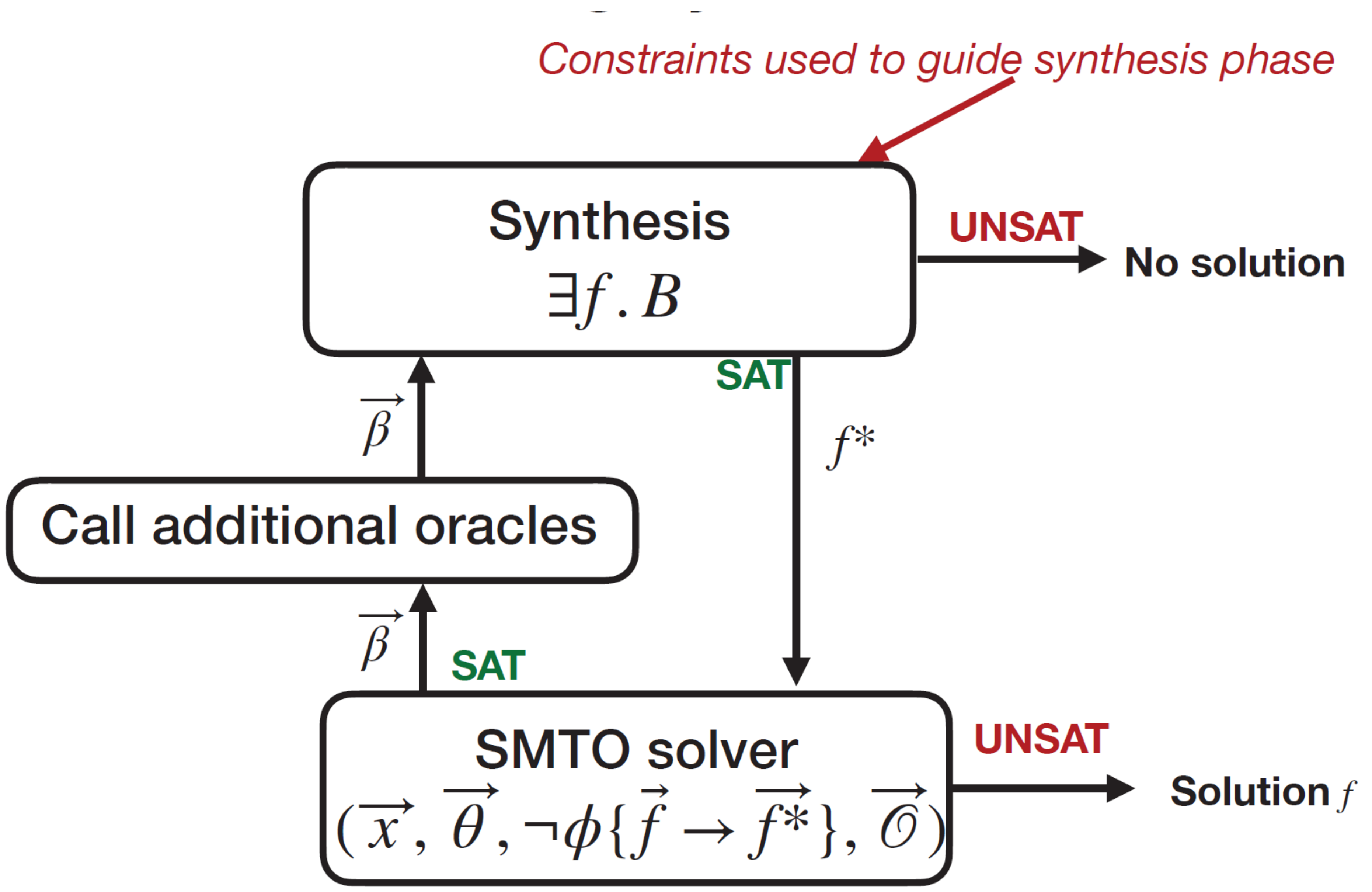
And:

- All **assumption** generators define **oracle function symbols**
- All oracles are **functional**

$\Longrightarrow$ checking $\vec{f}$ is valid is now **definitional SMTO**

Synthesis Modulo Oracles (SyMO)

- Synthesis solver calls SMTO solver to check correctness of the synthesized functions
- It can additionally invoke other oracles to guide the search
  - E.g. answering membership queries, provide labeled examples, demonstrations, preferences, etc.



Some Experimental Results with SyMO/SMTO

Approximate model

	Problem		Delphi (oracles)		CVC5 (no oracles)	
		#	#	s	#	s
SyMO	Images	10	9	21.6s	0	
	Control stability	112	104	29.3s	16	19.4s
	Control safety	112	31	59.9s	0	
	PBE	150	148	0.5s	150	<0.5s
SMTO	Math	12	9	<0.5s	5	2.2s

Oracle-Guided Reasoning with UCLID5

Latest version of UCLID5 has support for Satisfiability and Synthesis Modulo Oracles

Used it for several tasks including algorithmically synthesizing a stabilizing controller

```
module main {
  var x0, x1: float;
  group states : float = {x0, x1};
  <..LTI system spec vars decls..>
  oracle function [isstable] isStable
    (s00:float, s01:float, s10:float, s11:float) : boolean;
  synthesis function k0 (): float;
  synthesis function k1 (): float;

  // LTI system spec values
  axiom A:(a00==0.901224922471 && a01==0.000000013429 && a10==0.000000007451 &&
     a11==0.0);
  axiom B: (b0==128.0 && b1==0.0);
  axiom ax1: ABK00 == a00 - b0*k0());
  <...>
  axiom ax4: ABK11 == a11 - b1*k1());

  init { // bound initial states
    assume (finite_forall (s: float) in states :: s<0.1 && s>-0.1);
  }
  next { // step the system
    x0' = ABK00*x0 + ABK01*x1;
    x1' = ABK10*x0 + ABK11*x1;
  }
  // the safety condition
  invariant stability: isStable(ABK00, ABK01, ABK10, ABK11);
  invariant safety: finite_forall (s: float) in states :: s < 1.0&&s > -1.0;

  control {
    unroll(10); // fix safety bound
    check;
  }
}
```

Summary

- Formal Synthesis
- Verification by Reduction to Synthesis
- Syntax-Guided Synthesis
- Formal Inductive Synthesis
  - Counterexample-guided inductive synthesis (CEGIS)
  - General framework for solution methods: Oracle-Guided Inductive Synthesis (OGIS)
  - Theoretical analysis (see Jha & Seshia, 2017)
- Satisfiability and Synthesis Modulo Oracles
  - A generic approach to solve OGIS problems
- Lots of potential for future work!