

# **Integrating Induction and Deduction for Verification and Synthesis**

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# Bob's Vision: Exploit Synergies between Verification and Synthesis

- **VIS: Verification Interacting with Synthesis**
  - Continued in the ABC project
- **Techniques useful in formal verification (e.g., SAT) help in logic synthesis, and vice-versa (logic optimization helping verification engines)**
  - Our work: Beaver SMT solver uses ABC to optimize SAT encoding
- **The connection between verification and synthesis goes deeper...**

# Artifacts *synthesized* in Verification

**Hard part of verification is often a *synthesis* step**

- Specifications (e.g., inductive invariants, pre/post-conditions, function summaries)
- Environment assumptions / Env model / interface specifications
- Abstraction functions / abstract models
- Interpolants
- Ranking functions
- Intermediate lemmas for compositional reasoning
- ...

**Human Insight often needed to synthesize the “right” artifact**

# Strategy for Synthesis

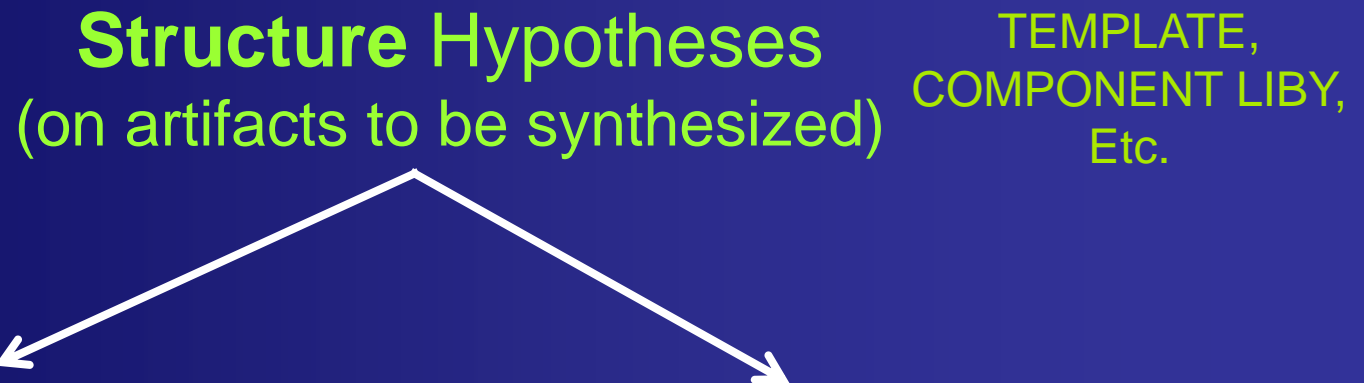
## ■ Some Human Input / Insight

- Hypothesis on the form of artifact to be synthesized
- “**Structure Hypothesis**”
- Example:
  - Template for invariant (e.g., “equivalences”)
  - Simulations may suggest likely equivalences [ABC strategy]

## ■ Induction + Deduction

- Induction: specific examples → general rules
  - Learning from examples
- Deduction: general rules → specific conclusions
  - Logical inference and constraint solving
- Purely deductive approach is inefficient / inapplicable
- Purely inductive approach gives no guarantees

# Structure-Constrained Induction and Deduction



**Deductive Procedure**  
“Lightweight”: solves lower complexity problem or special case of original decision problem

**Inductive Procedure**  
Active Learning: selects examples to learn from

S. A. Seshia, “Sciduction: Combining Induction, Deduction, and Structure for Verification and Synthesis”, DAC 2012

# Rest of the Talk

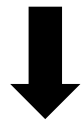
- **Synthesizing Environment Assumptions for Reactive Synthesis from Linear Temporal Logic (LTL)**
- **Conclusions and Future Directions**

# Reactive Synthesis from LTL

LTL Specification



Synthesis  
Tool



Finite-State Transducer  
(reactive program)

Church 1957



Rabin 1972



Pnueli-Rosner 1989



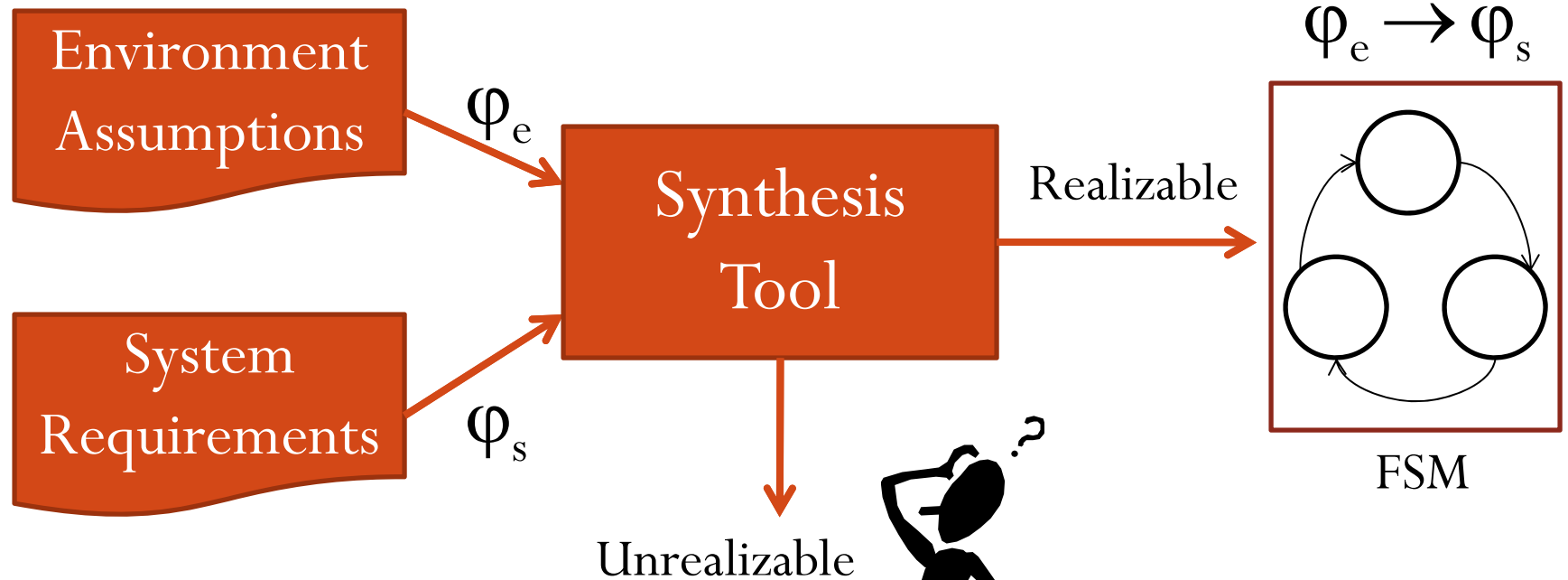
2013: many potential applications  
in Robotics & Embedded Systems

# A Personal Story of Synthesis from LTL

- Verified Electronic Voting Machine
  - Verilog design, list of LTL properties [Sturton et al., CCS 2009]
  - <http://uclid.eecs.berkeley.edu/vvm/>
- Attempt 1: Synthesizer *ran for more than a week*, no output
- Idea: Compositional Synthesis! Synthesize individual modules (selections within contests, navigation between contests, etc.)
- Attempt 2,3,...: Unrealizable! Too many *environment assumptions* needed (at interfaces between modules)



# Problem



**Often due to incomplete environment assumption!**

# Satisfiability and Realizability

- A LTL formula  $\varphi$  is **satisfiable** if there exists an infinite word (i.e. sequence of inputs and outputs) that satisfies  $\varphi$ .
- A LTL specification  $\varphi$  is **realizable** if for all inputs, there exists a finite-state transducer  $M$  (e.g. a Moore machine) which generates computations that satisfies  $\varphi$ .

# Problem Description

- Goal:  
Generate additional assumptions to enable synthesis
- Context:  
Original specification is *satisfiable* but *unrealizable*
- Assume:
  - Given only a few interesting user scenarios (satisfying traces)
  - Specifications are in the GR(1) class
- Challenge:
  - Space of possible additional assumptions is huge
  - Want assumptions that can be understood and analyzed by a human user

# Example

- Inputs: request  $r$  and cancel  $c$
- Outputs: grant  $g$
- System specification  $\varphi_s$ :
  - $\mathbf{G}(r \rightarrow \mathbf{X F} g)$
  - $\mathbf{G}(c \vee g \rightarrow \mathbf{X} \neg g)$
- Environment assumption  $\varphi_e$ :
  - True
- No user scenarios.
- **Not realizable** because the environment can force  $c$  to be high all the time

# Our Contribution

[Wenchao Li et al., MEMOCODE 2011]

## *Counterstrategy-guided synthesis of environment assumptions*

- Demonstrated to generate useful/intuitive environment assumptions for digital circuits and robotic controllers

# Approach for Synthesizing Environment Assumptions

**Structure Hypothesis:**  
Environment Assumptions are  
Restricted GR(1) properties

+

**Inductive Inference:**  
Version Space Learning

+

**Deductive Engine:**  
(Finite-state) Model Checking

# GR(1) Synthesis [Piterman, Pnueli, Saar]

- Formulas in the form:  $\varphi_e \rightarrow \varphi_s$ 
  - Input and output partitions  $I$  and  $O$ .
  - $\varphi_\alpha^i$ : initial state formulas.  $\alpha \in \{e, s\}$
  - $\varphi_\alpha^t$ : transition formulas, in the form of  $\mathbf{G} B$ , where  $B$  is a Boolean combination of variables in  $I \cup O$  and expressions  $\mathbf{X} u$ ,  $u \in I$  if  $\alpha = e$  and  $u \in I \cup O$  if  $\alpha = s$ .
  - $\varphi_\alpha^f$ : fairness formulas, in the form of  $\mathbf{G} \mathbf{F} B$ , where  $B$  is a Boolean formula over  $I \cup O$ .
- Synthesis as a turn-based two-player game between the system and the environment
  - Realizable if the system has a **winning strategy, otherwise env wins**; Strategy representable as finite-state transducer

# Counter-strategy and counter-trace

- Counter-strategy is a strategy for the environment to force violation of the specification.
- Counter-trace is a fixed input sequence such that the specification is violated regardless of the outputs generated by the system.



# Example

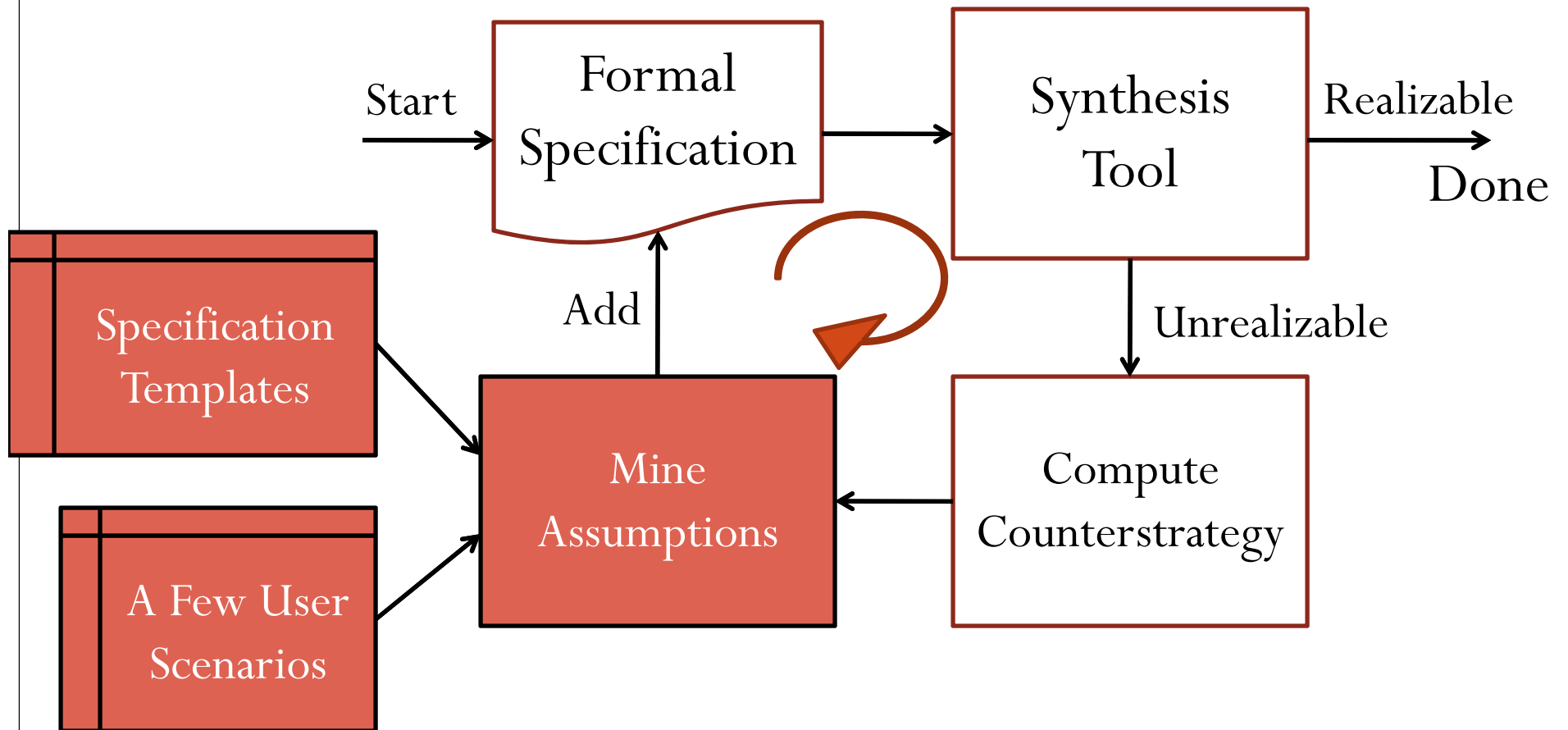
- System  $\varphi_s$ :
  - $\mathbf{G} (r \rightarrow \mathbf{X F} g)$
  - $\mathbf{G}(c \vee g \rightarrow \mathbf{X} \neg g)$

- A counter-trace:

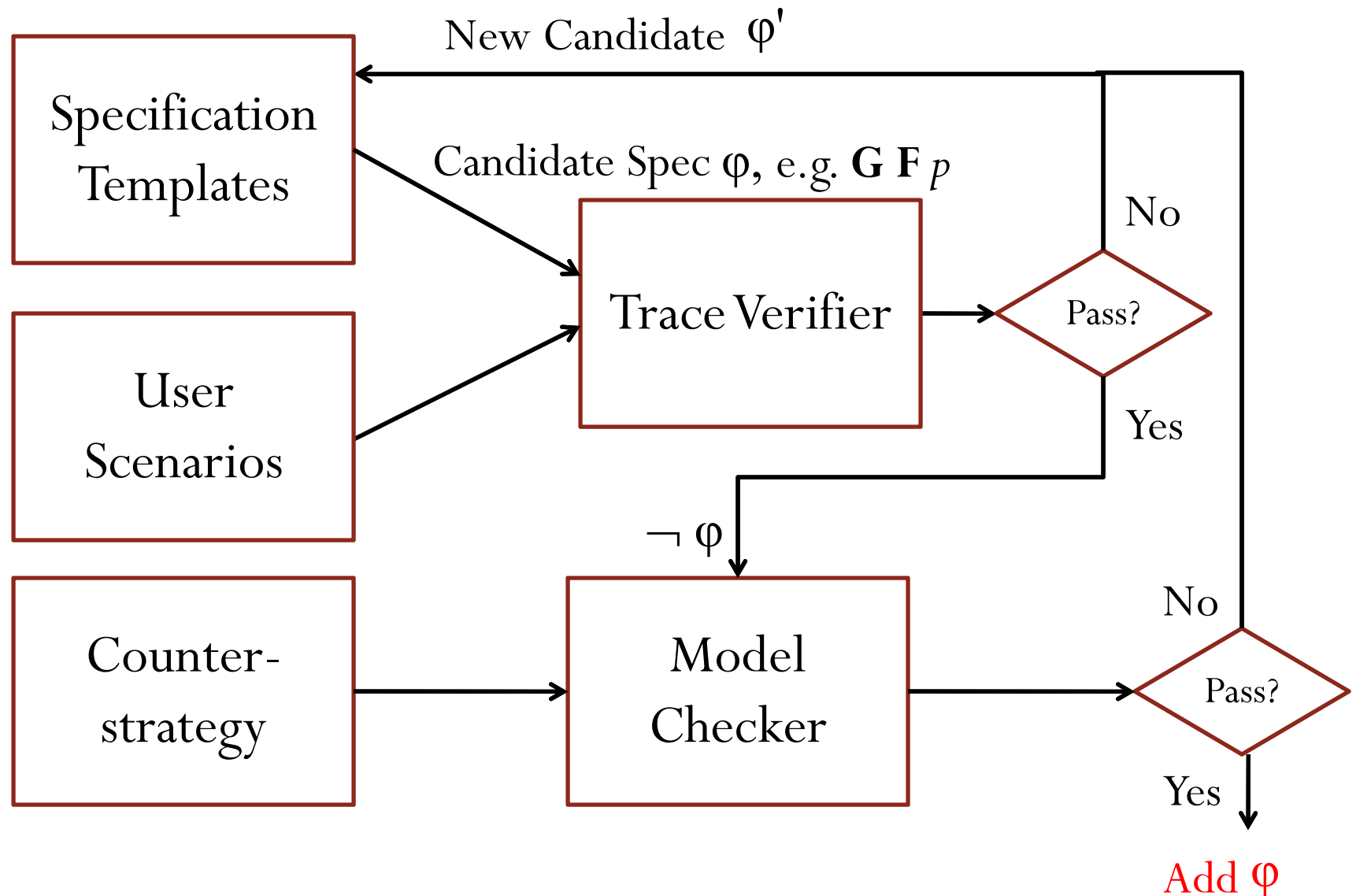
$r$ : 1 1 (1)

$c$ : 1 1 (1)

# Assumption Mining to Assist Synthesis



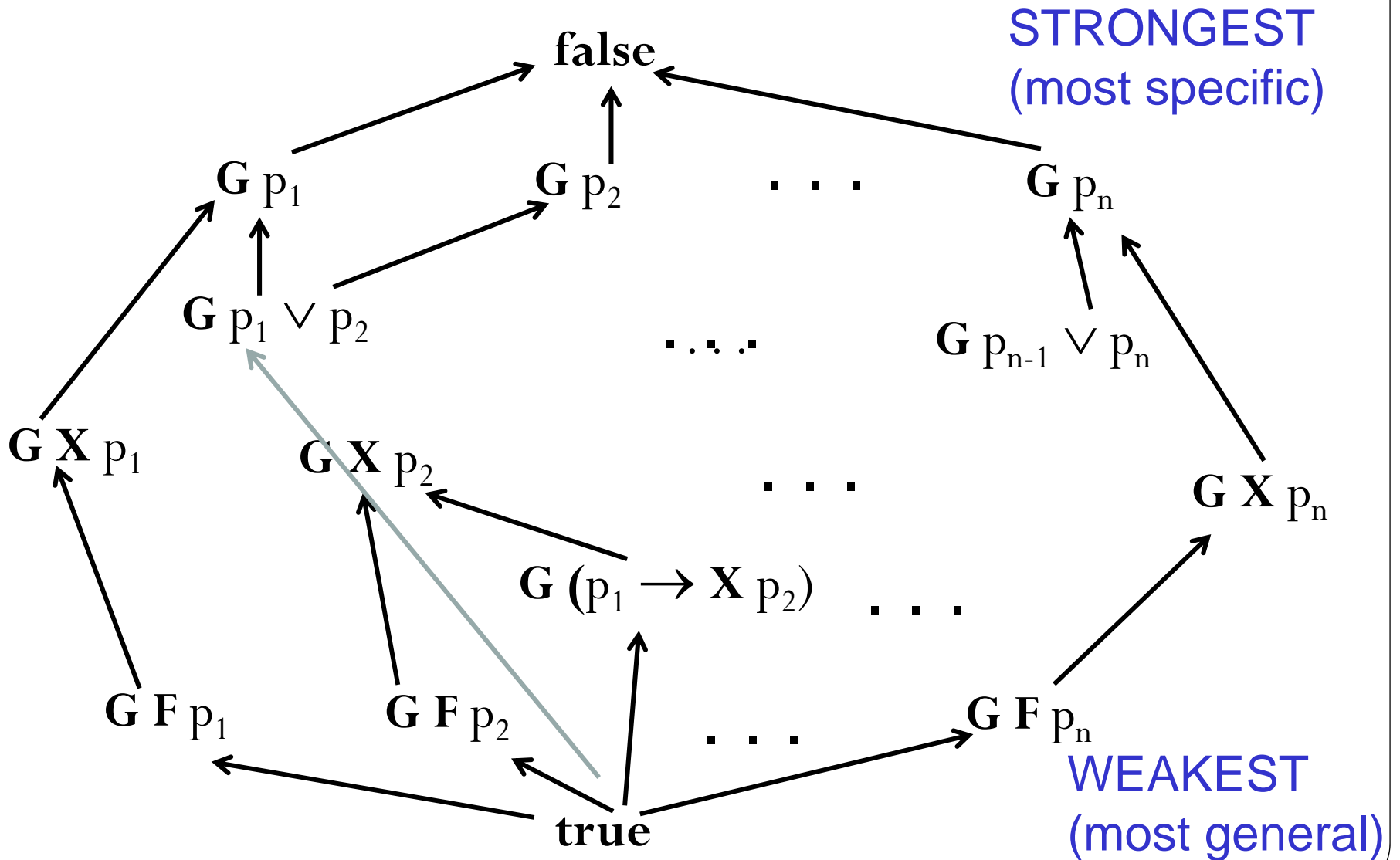
# Mining Algorithm



# Assumption Templates follow GR(1)

- $\gamma^1: \mathbf{G F } ?b$ , where  $b \in I$
- $\gamma^2: \mathbf{G } (?b_1 \rightarrow \mathbf{X } ?b_2)$ , where  $b_1 \in I \cup O$  and  $b_2 \in I$
- $\gamma^3: \mathbf{G } (?b_1 \vee ?b_2)$ , where  $b_1, b_2 \in I$
  
- The specification remains in GR(1) after the addition of new assumptions.
- How to pick candidate assumptions: (next slide)
  - Weakest to Strongest, with heuristics
  - $\mathbf{G F } b$  weaker than  $\mathbf{G X } b$  weaker than  $\mathbf{G } b$
  - IMPORTANT: Check each assumption for consistency with existing set

# Version Space Learning



# Theoretical Results

- Theorem 1: A redundant environment assumption (implied by the existing assumptions) is never added.
  - Proof sketch: guaranteed by the counter-strategy guided approach.
- Corollary: The set of environment assumptions is minimal (but not minimum, in terms of number of properties).
  - Example: We may find  $G p$  and  $G q$  rather than  $G (p \wedge q)$
- Theorem 2: [Completeness] If there exist environment assumptions under our structure hypothesis that make the spec realizable, then the procedure finds them (terminates successfully).
  - “conditional completeness” guarantee
- Theorem 3: [Soundness] The procedure never adds inconsistent environment assumptions.

# Example

- System  $\varphi_s$ :
  - $\mathbf{G} (r \rightarrow \mathbf{X} \mathbf{F} g)$
  - $\mathbf{G}(c \vee g \rightarrow \mathbf{X} \neg g)$
- A counter-trace:  
 $r: 1 \ 1 \ (1)$   
 $c: 1 \ 1 \ (1)$
- Test assumption candidates by checking its negation:
  - $\mathbf{G} (\mathbf{F} c)$  ----- ×
  - $\mathbf{G} (\mathbf{F} \neg c)$  ----- ✓

- System  $\varphi_s$ :
  - $\mathbf{G} (r \rightarrow \mathbf{X} \mathbf{F} g)$
  - $\mathbf{G}(c \vee g \rightarrow \mathbf{X} \neg g)$
- Environment  $\varphi_e$ :
  - $\mathbf{G} (\mathbf{F} \neg c)$

Realizable!

# Experimental Results Summary

- Experiment Setup:
  - Remove assumptions from an originally realizable specification
  - Use a few (often a single) satisfying traces of the original specification as representative user scenarios
  - Mine additional assumptions until the specification is realizable
- Use Cadence SMV to generate the satisfying traces and model check the counter-strategies
- Use RATSY [Bloem et al. 2010] to check realizability of the specifications and compute the counter-strategies and counter-traces in case of unrealizability



# Experimental Results Summary

- Result Summary:
  - Case studies in existing literature: AMBA AHB, IBM Gen Buffer, robotic vehicle controller, etc.
  - Recovered the missing assumption in most cases
  - AMBA AHB Example:

Original assumption:

$G (HLOCK[0] = 1 \rightarrow HBUSREQ[0] = 1)$

Master 0 requests locked  
access to the bus



Mined assumption:

$G (F HLOCK[0] = 0)$

Master 0 requests  
access to the bus



# Related Work

- Constructing the “weakest” assumption needed for realizability from the game graph [Chatterjee et al. 2008]
  - Computes a safety assumption that removes a minimal set of environment edges from the game graph
  - Computes a liveness assumption that puts fairness on the remaining environment edges
  - The additional environment assumption is a single Büchi automaton which can be difficult for a normal human user to analyze or even understand.
- Computing counter-strategy for GR(1) synthesis [Konighofer et al. 2009]
  - Counter-strategies and counter-traces as explanations for unrealizability

# Discussion

- Our approach can generate useful environment assumptions to cope with unrealizability
  - More experimentation underway → compositional synthesis
- Limitation: choice of templates
  - Can extend same approach with broader set of templates
- Can the same idea of learning from counter-strategies and counter-traces be applied to other synthesis tasks?
  - i.e., not just synthesis from LTL

# Conclusion: Induction + Deduction + Structure

- **Verification “=” Synthesis**
- **Structure Hypothesis encodes human insight about form of artifact**
- **Synthesis procedure combines inductive inference with deductive reasoning**
- **Several demonstrations**
  - Abstraction-based verification of RTL designs [Brady et al, '11]
  - Synthesis of loop-free programs [Jha et al, '10]
  - Switching logic synthesis for safety and optimality [Jha et al '10,'11]
  - Environment assumptions for LTL synthesis [Li et al '11]
  - Fixed-point code from floating-point [Jha, Seshia, '11]
  - ...