An Introduction to Satisfiability Modulo Theories

Clark Barrett and Sanjit Seshia

Example of Lazy SMT

$$\underbrace{g(a) = c}_{1} \quad \wedge \quad \underbrace{f(g(a)) \neq f(c)}_{\overline{2}} \vee \underbrace{g(a) = d}_{3} \quad \wedge \quad \underbrace{c \neq d}_{\overline{4}} \vee \underbrace{g(a) \neq d}_{\overline{3}}$$

$$\emptyset \parallel \quad 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3}$$

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$$\emptyset \parallel \quad 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} \qquad \Longrightarrow \quad (\text{UnitProp})$$

$$1 \parallel \quad 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3}$$

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$$\begin{array}{rcll} \emptyset & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(UnitProp)} \\ 1 & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(Decide)} \\ 1\ \overline{2}^{\mathsf{d}} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \end{array}$$

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1\ \overline{2}^{\mathsf{d}}\ \overline{4}^{\mathsf{d}} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad (\text{Theory Learn}) \\
1\ \overline{2}^{\mathsf{d}}\ \overline{4}^{\mathsf{d}} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2 \vee 4 &
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1\ \overline{2}^{\mathsf{d}}\ 4 & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2 \vee 4 &
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$\emptyset$	$\parallel$	$1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3}$	$\Longrightarrow$ (UnitProp)
1	$\parallel$	$1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3}$	$\Longrightarrow$ (Decide)
$1\ \overline{2}^{\mathsf{d}}$	$\parallel$	$1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3}$	$\Longrightarrow$ (Decide)
$1\ \overline{2}^{\mathsf{d}}\ \overline{4}^{\mathsf{d}}$	$\parallel$	$1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3}$	$\Longrightarrow$ (Theory Learn)
$1\ \overline{2}^{\mathsf{d}}\ \overline{4}^{\mathsf{d}}$	$\parallel$	$1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2 \vee 4$	$\Longrightarrow$ (Backjump)
$1\ \overline{2}^{\mathsf{d}}\ 4$	$\parallel$	$1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2 \vee 4$	$\Longrightarrow$ (UnitProp)
$1\ \overline{2}^{\mathsf{d}}\ 4\ \overline{3}$	$\parallel$	$1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2 \vee 4$	

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$1\ \overline{2}^{d}$	$\parallel$	$1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3}$	$\Longrightarrow$ (Decide)
$1\ \overline{2}^{d}\ \overline{4}^{d}$	$\parallel$	$1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3}$	$\Longrightarrow$ (Theory Learn)
$1\ \overline{2}^{d}\ \overline{4}^{d}$	$\parallel$	$1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2 \vee 4$	$\Longrightarrow$ (Backjump)
$1\ \overline{2}^{d}\ 4$	$\parallel$	$1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2 \vee 4$	$\Longrightarrow$ (UnitProp)
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$1\ \overline{2}^{d}\ 4\ \overline{3}$	$\parallel$	$1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2 \vee 4,\ \overline{1} \vee 2 \vee \overline{4} \vee 3$	$\Longrightarrow$ (Backjump)
$1\ 2$	$\parallel$	$1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2 \vee 4,\ \overline{1} \vee 2 \vee \overline{4} \vee 3$	$\Longrightarrow$ (UnitProp)
$1\ 2\ 3\ \overline{4}$	$\parallel$	$1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2 \vee 4,\ \overline{1} \vee 2 \vee \overline{4} \vee 3$	$\Longrightarrow$ (Theory Learn)
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\mathit{fail} & & &
\end{array}
$$

Example with Minimized Learned Clauses

$$\underbrace{g(a) = c}_{1} \quad \wedge \quad \underbrace{f(g(a)) \neq f(c)}_{\overline{2}} \vee \underbrace{g(a) = d}_{3} \quad \wedge \quad \underbrace{c \neq d}_{\overline{4}} \vee \underbrace{g(a) \neq d}_{\overline{3}}$$

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\begin{array}{rcll}
\emptyset & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(UnitProp)} \\
1 & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(Decide)} \\
1\ \overline{2}^{\mathsf{d}} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(Decide)} \\
1\ \overline{2}^{\mathsf{d}}\ \overline{4}^{\mathsf{d}} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(Theory Learn)} \\
1\ \overline{2}^{\mathsf{d}}\ \overline{4}^{\mathsf{d}} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2 & \Longrightarrow \quad \text{(Backjump)} \\
1\ 2 & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2 & \Longrightarrow \quad \text{(UnitProp)} \\
1\ 2\ 3\ \overline{4} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2 & \Longrightarrow \quad \text{(Theory Learn)} \\
1\ 2\ 3\ \overline{4} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2,\ \overline{1} \vee \overline{3} \vee 4 &
\end{array}
$$

Example with Minimized Learned Clauses

$$\underbrace{g(a) = c}_{1} \;\wedge\; \underbrace{f(g(a)) \neq f(c)}_{\overline{2}} \;\vee\; \underbrace{g(a) = d}_{3} \;\wedge\; \underbrace{c \neq d}_{\overline{4}} \;\vee\; \underbrace{g(a) \neq d}_{\overline{3}}$$

$$\begin{array}{rcll}
\emptyset & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(UnitProp)} \\
1 & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(Decide)} \\
1\ \overline{2}^{\mathsf{d}} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(Decide)} \\
1\ \overline{2}^{\mathsf{d}}\ \overline{4}^{\mathsf{d}} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(Theory Learn)} \\
1\ \overline{2}^{\mathsf{d}}\ \overline{4}^{\mathsf{d}} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2 & \Longrightarrow \quad \text{(Backjump)} \\
1\ 2 & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2 & \Longrightarrow \quad \text{(UnitProp)} \\
1\ 2\ 3\ \overline{4} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2 & \Longrightarrow \quad \text{(Theory Learn)} \\
1\ 2\ 3\ \overline{4} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2,\ \overline{1} \vee \overline{3} \vee 4 & \Longrightarrow \quad \text{(Fail)} \\
\mathit{fail} & & &
\end{array}$$

Example with Early Conflict Detection

$$\underbrace{g(a) = c}_{1} \quad \wedge \quad \underbrace{f(g(a)) \neq f(c)}_{\overline{2}} \vee \underbrace{g(a) = d}_{3} \quad \wedge \quad \underbrace{c \neq d}_{\overline{4}} \vee \underbrace{g(a) \neq d}_{\overline{3}}$$

$$\emptyset \parallel \quad 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3}$$

Example with Early Conflict Detection

$$\underbrace{g(a) = c}_{1} \quad \wedge \quad \underbrace{f(g(a)) \neq f(c)}_{\overline{2}} \vee \underbrace{g(a) = d}_{3} \quad \wedge \quad \underbrace{c \neq d}_{\overline{4}} \vee \underbrace{g(a) \neq d}_{\overline{3}}$$

$$\emptyset \parallel \quad 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} \qquad \Longrightarrow \quad \text{(UnitProp)}$$

$$1 \parallel \quad 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3}$$

Example with Early Conflict Detection

$$\underbrace{g(a) = c}_{1} \quad \wedge \quad \underbrace{f(g(a)) \neq f(c)}_{\overline{2}} \vee \underbrace{g(a) = d}_{3} \quad \wedge \quad \underbrace{c \neq d}_{\overline{4}} \vee \underbrace{g(a) \neq d}_{\overline{3}}$$

$$\begin{array}{rcll}
\emptyset & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(UnitProp)} \\
1 & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(Decide)} \\
1\ \overline{2}^{\mathsf{d}} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} &
\end{array}$$

Example with Early Conflict Detection

$$\underbrace{g(a) = c}_{1} \quad \wedge \quad \underbrace{f(g(a)) \neq f(c)}_{\overline{2}} \vee \underbrace{g(a) = d}_{3} \quad \wedge \quad \underbrace{c \neq d}_{\overline{4}} \vee \underbrace{g(a) \neq d}_{\overline{3}}$$

$$\begin{array}{rcll}
\emptyset & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(UnitProp)} \\
1 & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(Decide)} \\
1\ \overline{2}^{\mathsf{d}} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(Theory Learn)} \\
1\ \overline{2}^{\mathsf{d}} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2 &
\end{array}$$

Example with Early Conflict Detection

$$\underbrace{g(a) = c}_{1} \quad \wedge \quad \underbrace{f(g(a)) \neq f(c)}_{\overline{2}} \vee \underbrace{g(a) = d}_{3} \quad \wedge \quad \underbrace{c \neq d}_{\overline{4}} \vee \underbrace{g(a) \neq d}_{\overline{3}}$$

$$\begin{array}{rcll}
\emptyset & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(UnitProp)} \\
1 & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(Decide)} \\
1\ \overline{2}^{\mathsf{d}} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(Theory Learn)} \\
1\ \overline{2}^{\mathsf{d}} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2 & \Longrightarrow \quad \text{(Backjump)} \\
1\ 2 & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2 &
\end{array}$$

Example with Early Conflict Detection

$$\underbrace{g(a) = c}_{1} \quad \wedge \quad \underbrace{f(g(a)) \neq f(c)}_{\overline{2}} \vee \underbrace{g(a) = d}_{3} \quad \wedge \quad \underbrace{c \neq d}_{\overline{4}} \vee \underbrace{g(a) \neq d}_{\overline{3}}$$

$\emptyset$	$\parallel \quad 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3}$	$\Longrightarrow$	(UnitProp)
1	$\parallel \quad 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3}$	$\Longrightarrow$	(Decide)
$1\ \overline{2}^{\mathsf{d}}$	$\parallel \quad 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3}$	$\Longrightarrow$	(Theory Learn)
$1\ \overline{2}^{\mathsf{d}}$	$\parallel \quad 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2$	$\Longrightarrow$	(Backjump)
$1\ 2$	$\parallel \quad 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2$	$\Longrightarrow$	(UnitProp)
$1\ 2\ 3\ \overline{4}$	$\parallel \quad 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2$		

Example with Early Conflict Detection

$$\underbrace{g(a) = c}_{1} \quad \wedge \quad \underbrace{f(g(a)) \neq f(c)}_{\overline{2}} \quad \vee \quad \underbrace{g(a) = d}_{3} \quad \wedge \quad \underbrace{c \neq d}_{\overline{4}} \quad \vee \quad \underbrace{g(a) \neq d}_{\overline{3}}$$

$$\begin{array}{rcll}
\emptyset & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(UnitProp)} \\
1 & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(Decide)} \\
1\ \overline{2}^{\mathsf{d}} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(Theory Learn)} \\
1\ \overline{2}^{\mathsf{d}} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2 & \Longrightarrow \quad \text{(Backjump)} \\
1\ 2 & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2 & \Longrightarrow \quad \text{(UnitProp)} \\
1\ 2\ 3\ \overline{4} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2 & \Longrightarrow \quad \text{(Theory Learn)} \\
1\ 2\ 3\ \overline{4} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2,\ \overline{1} \vee \overline{3} \vee 4 &
\end{array}$$

Example with Early Conflict Detection

$$\underbrace{g(a) = c}_{1} \quad \wedge \quad \underbrace{f(g(a)) \neq f(c)}_{\overline{2}} \vee \underbrace{g(a) = d}_{3} \quad \wedge \quad \underbrace{c \neq d}_{\overline{4}} \vee \underbrace{g(a) \neq d}_{\overline{3}}$$

$$
\begin{array}{rcll}
\emptyset & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad (\text{UnitProp}) \\
1 & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad (\text{Decide}) \\
1\ \overline{2}^{\mathsf{d}} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad (\text{Theory Learn}) \\
1\ \overline{2}^{\mathsf{d}} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2 & \Longrightarrow \quad (\text{Backjump}) \\
1\ 2 & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2 & \Longrightarrow \quad (\text{UnitProp}) \\
1\ 2\ 3\ \overline{4} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2 & \Longrightarrow \quad (\text{Theory Learn}) \\
1\ 2\ 3\ \overline{4} & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3},\ \overline{1} \vee 2,\ \overline{1} \vee \overline{3} \vee 4 & \Longrightarrow \quad (\text{Fail}) \\
\mathit{fail} & & &
\end{array}
$$

Example with Theory Propagation

$$\underbrace{g(a) = c}_{1} \quad \wedge \quad \underbrace{f(g(a)) \neq f(c)}_{\overline{2}} \vee \underbrace{g(a) = d}_{3} \quad \wedge \quad \underbrace{c \neq d}_{\overline{4}} \vee \underbrace{g(a) \neq d}_{\overline{3}}$$

$$\emptyset \parallel \quad 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3}$$

Example with Theory Propagation

$$\underbrace{g(a) = c}_{1} \quad \wedge \quad \underbrace{f(g(a)) \neq f(c)}_{\overline{2}} \vee \underbrace{g(a) = d}_{3} \quad \wedge \quad \underbrace{c \neq d}_{\overline{4}} \vee \underbrace{g(a) \neq d}_{\overline{3}}$$

$$\begin{array}{lll} \emptyset \parallel & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(UnitProp)} \\ 1 \parallel & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \end{array}$$

Example with Theory Propagation

$$\underbrace{g(a) = c}_{1} \quad \wedge \quad \underbrace{f(g(a)) \neq f(c)}_{\overline{2}} \vee \underbrace{g(a) = d}_{3} \quad \wedge \quad \underbrace{c \neq d}_{\overline{4}} \vee \underbrace{g(a) \neq d}_{\overline{3}}$$

$$\begin{array}{rcll} \emptyset & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(UnitProp)} \\ 1 & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(Theory Propagate)} \\ 1\ 2 & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \end{array}$$

Example with Theory Propagation

$$\underbrace{g(a) = c}_{1} \quad \wedge \quad \underbrace{f(g(a)) \neq f(c)}_{\overline{2}} \vee \underbrace{g(a) = d}_{3} \quad \wedge \quad \underbrace{c \neq d}_{\overline{4}} \vee \underbrace{g(a) \neq d}_{\overline{3}}$$

$$\begin{array}{rcll}
\emptyset & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(UnitProp)} \\
1 & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(Theory Propagate)} \\
1\ 2 & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(UnitProp)} \\
1\ 2\ 3 & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} &
\end{array}$$

Example with Theory Propagation

$$\underbrace{g(a) = c}_{1} \quad \wedge \quad \underbrace{f(g(a)) \neq f(c)}_{\overline{2}} \vee \underbrace{g(a) = d}_{3} \quad \wedge \quad \underbrace{c \neq d}_{\overline{4}} \vee \underbrace{g(a) \neq d}_{\overline{3}}$$

$$\begin{array}{rcll}
\emptyset & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(UnitProp)} \\
1 & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(Theory Propagate)} \\
1\ 2 & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(UnitProp)} \\
1\ 2\ 3 & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad \text{(Theory Propagate)} \\
1\ 2\ 3\ 4 & \| & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} &
\end{array}$$

Example with Theory Propagation

$$\underbrace{g(a) = c}_{1} \quad \wedge \quad \underbrace{f(g(a)) \neq f(c)}_{\overline{2}} \vee \underbrace{g(a) = d}_{3} \quad \wedge \quad \underbrace{c \neq d}_{\overline{4}} \vee \underbrace{g(a) \neq d}_{\overline{3}}$$

$$\begin{array}{rll}
\emptyset \parallel & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad (\text{UnitProp}) \\
1 \parallel & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad (\text{Theory Propagate}) \\
1\ 2 \parallel & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad (\text{UnitProp}) \\
1\ 2\ 3 \parallel & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad (\text{Theory Propagate}) \\
1\ 2\ 3\ 4 \parallel & 1,\ \overline{2} \vee 3,\ \overline{4} \vee \overline{3} & \Longrightarrow \quad (\text{Fail}) \\
\mathit{fail} & &
\end{array}$$