CS 172: Computability and Complexity

DFAs and NFAs

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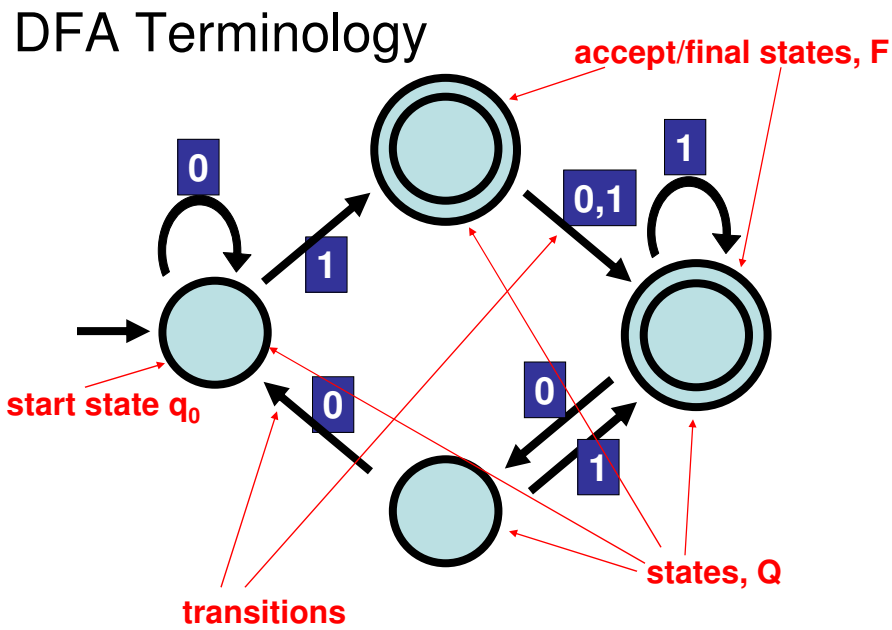
What we'll do today

- Introduction to
  - DFA : Deterministic Finite Automata
  - NFA : Non-deterministic Finite Automata
- DFAs to/from regular languages
- Operations on regular languages
- Non-determinism and NFAs
- HW1 is out today

DFA Terminology



A finite automaton is a 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$

- **Q is the set of states**
- **Σ is the alphabet**
- **$\delta : Q \times \Sigma \rightarrow Q$ is the transition function**
- **$q_0 \in Q$ is the start state**
- **$F \subseteq Q$ is the set of accept states**

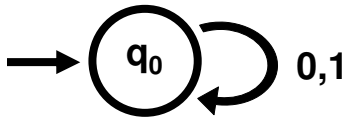
$L(M)$ = the language of machine M
= set of all strings machine M accepts

Acceptance (Formal Def.)

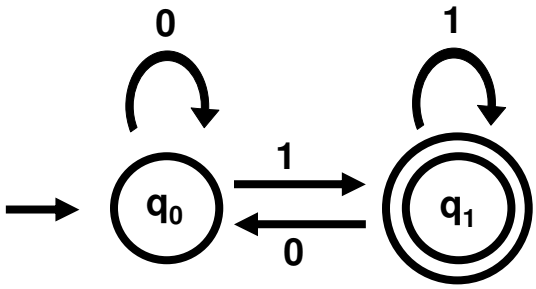
- M is a DFA with alphabet Σ
- An input string (word) w is a sequence $w_1\ w_2\ w_3 \ldots w_n$ where each $w_i \in \Sigma$
  - ε denotes the empty string
- M accepts w if "after fully reading w it ends in an accept state"
  - If there is a sequence of states $s_0, s_1, \ldots s_n$ such that
    - $s_0 = q_0$
    - $s_i = \delta(s_{i-1}, w_i)$
    - $s_n \in F$

Regular Language

- M is a DFA with alphabet Σ
- L(M) is the set of strings accepted by M
  - L(M) is the language of M
  - M recognizes L(M)
- Regular language: set of strings that is the language of *some* DFA
- The set of all possible strings is denoted Σ^*
- Is Σ^* regular?



L(M) = ?



L(M) = ?

Design a DFA

L(M) = { w | w has an even number of 1s}

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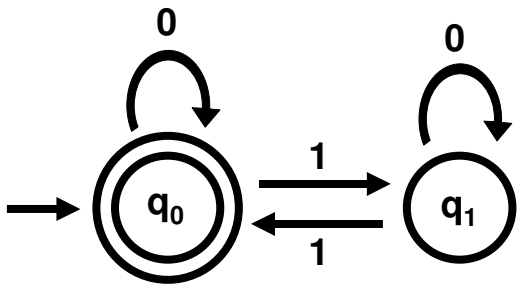


Sample Regular Languages

- Σ^*
- $\emptyset$
- **{ w | w ends in a 1}**
- **{ w | w has an even number of 1s}**

Points to Ponder

For a DFA $M = (Q, \Sigma, \delta, q_0, F)$

- Number of states, $|Q|$, is finite
- Is Σ a finite set?
- Is L(M) a finite set?
  - Does it have a finite representation?
- Suppose you are given the 5-tuple in advance, so you know $|Q|$, etc. Do you know how long you will have to run M on any input? Input length is finite, but unbounded by parameters of M

Operations on Regular Languages

Given: Two regular languages A and B

- Union: $A \cup B = \{w \mid w \in A \text{ or } w \in B\}$
- Intersection: $A \cap B = ?$
- Complementation: $\overline{A} = \{w \mid w \notin A\}$
- Reverse: $A^R = \{w_1 w_2 \ldots w_k \mid w_k w_{k-1} \ldots w_1 \in A\}$
- Concatenation:

  $A \cdot B = \{vw \mid v \in A \text{ and } w \in B\}$
- Star:

  $A^* = \{w_1 w_2 \ldots w_k \mid k \geq 0 \text{ and each } w_i \in A\}$

Closure

- If you perform an operation on one/more regular languages, is the result also a regular language?
- Example:

  Is $A \cup B$ regular if A and B are regular?

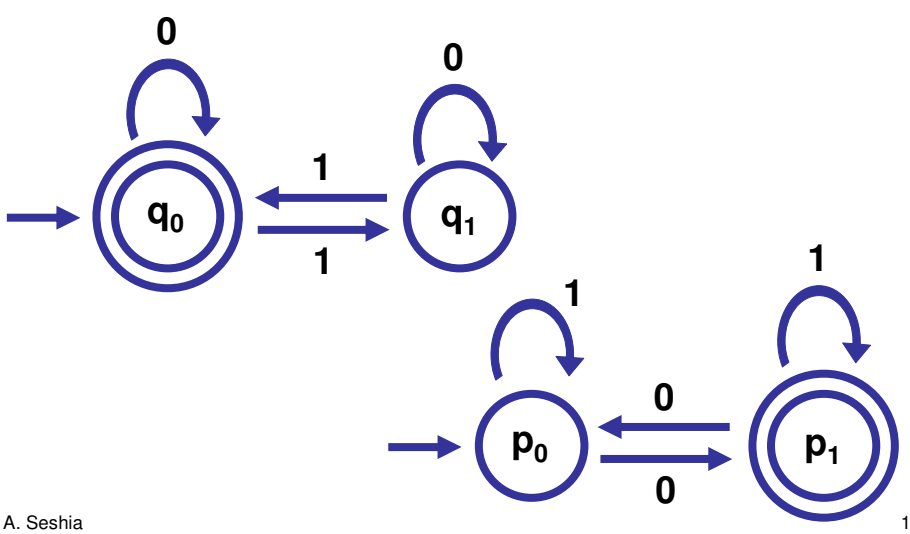
  YES

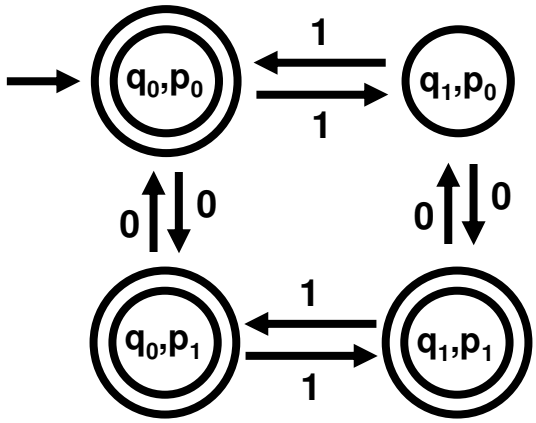
Union Theorem

The union of two regular languages is also a regular language

Proof (Hints)?

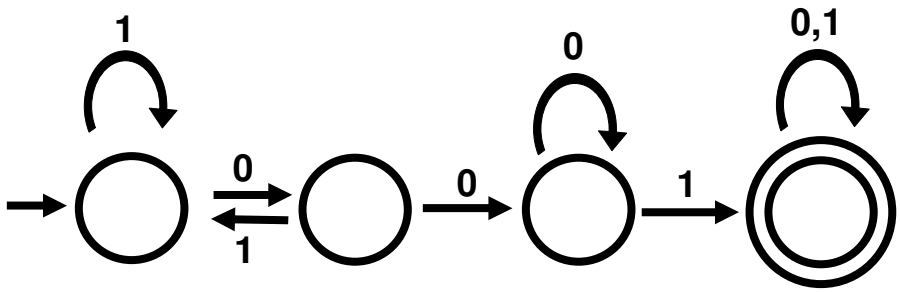
An Example



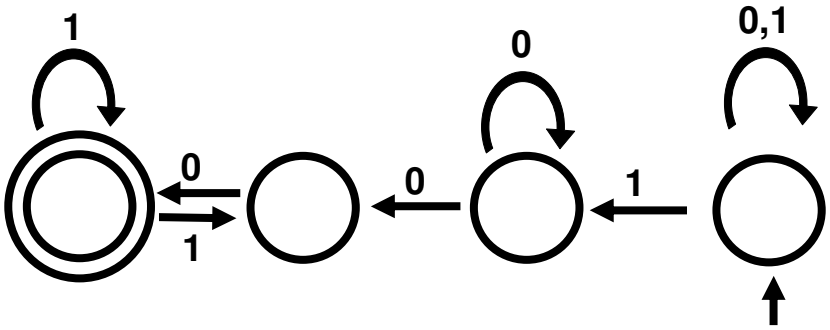


Closure under Reverse

- Reverse: $A^R = \{w_1 w_2 \ldots w_k \mid w_k w_{k-1} \ldots w_1 \in A\}$
- Regular languages are closed under reverse. Here's an attempt to prove it:
  - Given M that recognizes A
  - What if you could "run it backwards"?
  - Construct M^R as M with all arrows reversed & accept state becomes start state
  - Is M^R guaranteed to be a DFA?



Non-determinism



What happens with 100?

We will say that the NFA accepts if
there is a way to make it reach an accept state

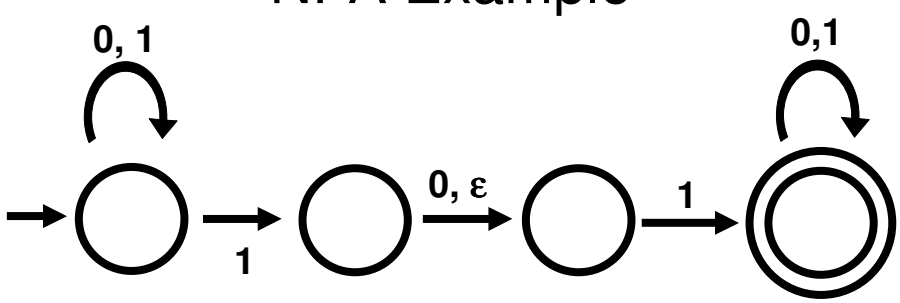
A non-deterministic finite automaton (NFA) is also a 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$

- **Q is the set of states**
- **Σ is the alphabet**
- **$\delta : Q \times \Sigma_\varepsilon \rightarrow 2^Q$ is the transition function**
- **$q_0 \in Q$ is the start state**
- **$F \subseteq Q$ is the set of accept states**

2^Q is the set of subsets of Q and $\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$

L(M) defined in the same way as for DFAs

NFA Example



How does this differ from the DFAs we've seen?

- Epsilon transitions
- Missing transitions from some states
- More than 1 transition from a state on same symbol

What's its language?

NFAs and DFAs

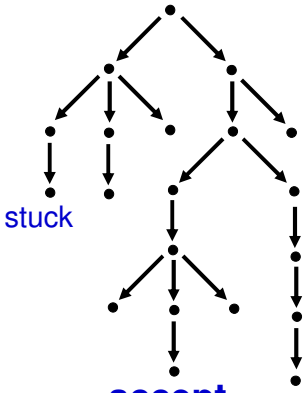
- Is every DFA also an NFA?
- Is every NFA a DFA?

Tree of Computations

Deterministic Computation

accept or reject

Non-Deterministic Computation



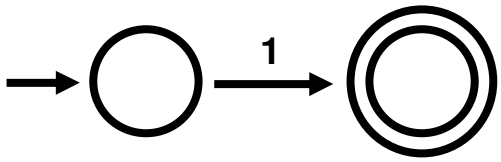
accept **reject**

Where does non-determinism arise?

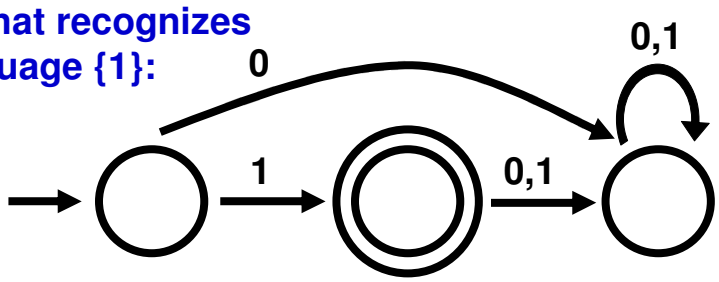
- In practice: From lack of information
  - A precise model might use randomness (probabilistic transitions between states), but we might not know the probabilities
  - Important! Non-determinism is *not* the same as probabilistic computation.
- NFAs and DFAs are "equivalent"!
  - Their languages are the same
    - More on this next time
  - But NFAs are simpler

NFAs are simpler than DFAs

An NFA that recognizes the language {1}:



A DFA that recognizes the language {1}:



Union Theorem for NFAs

- If you have two NFAs N_1 and N_2, with languages L_1 and L_2, how can we get the NFA for $L_1 \cup L_2$?

Next Steps

- Read Sipser 1.2 in preparation for next lecture