1. (4 points)

Use the procedure discussed in class to convert the following DFA into a regular expression. First eliminate state $q_b$, then eliminate state $q_a$ when simplifying the GNFA. Include all steps in your work.

![DFA Diagram]

2. (6 points)

Prove that the language $L = \{wtw \mid w, t \in \{0, 1\}^+\}$ is not regular.

Note: The “$+$” operator means one or more repetitions of the pattern.

3. (8 points) Let $L_n$ be the set of binary strings whose $n$th digit from the end is 1.

   (a) Show that $L_n$ can be recognized by an NFA with $n + 1$ states.

   (b) Prove carefully that any DFA for $L_n$ must have at least $2^n$ states. [Hint: use the concepts defined in the DFA minimization lecture]

4. (6 points)

Let $\Sigma = \{0, 1, +, =\}$ and

$ADD = \{x = y + z \mid x, y, z$ are binary integers, and $x$ is the sum of $y$ and $z\}$.

Show that $ADD$ is not regular.
5. (6 points) In this homework problem, you will use a method other than the pumping lemma to prove that a language is non-regular.

Recall the definition of $\sim_L$ from the lecture on the Myhill-Nerode theorem and DFA Minimization:

For $x, y \in \Sigma^*$, $x \sim_L y$ iff $\forall z \in \Sigma^*$, $xz \in L \iff yz \in L$.

Recall also that if $L$ is regular, $\sim_L$ has only finitely many equivalence classes.

Prove that the language $L = \{0^n1^n \mid n \geq 0\}$ is non-regular by showing that $\sim_L$ has infinitely many equivalence classes. (Precisely describe what these equivalence classes are.)