CS 172: Computability and Complexity, Spring 2010 **S. A. Seshia & O. Etesami**

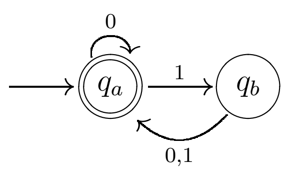
HW 3: Regexps, Non-Regular Languages and Minimization

Assigned: February 8, 2010 *Due in 283 Soda by 5 pm: February 16, 2010*

Note: Take time to write clear and concise solutions. Confused and long-winded answers may be penalized. Consult the course webpage for course policies on collaboration.

1. (4 points)

   Use the procedure discussed in class to convert the following DFA into a regular expression. First eliminate state q_b, then eliminate state q_a when simplifying the GNFA. Include all steps in your work.

2. (6 points)

   Prove that the language $L = \{wtw \mid w, t \in \{0,1\}^+\}$ is not regular.

   Note: The "+" operator means one or more repetitions of the pattern.

3. (8 points) Let L_n be the set of binary strings whose nth digit from the end is 1.

   (a) Show that L_n can be recognized by an NFA with $n + 1$ states.

   (b) Prove carefully that any DFA for L_n must have at least 2^n states. [Hint: use the concepts defined in the DFA minimization lecture]

4. (6 points)

   Let $\Sigma = \{\texttt{0}, \texttt{1}, \texttt{+}, \texttt{=}\}$ and

   $$\text{ADD} = \{x \texttt{=} y \texttt{+} z \mid x, y, z \text{ are binary integers, and } x \text{ is the sum of } y \text{ and } z\}.$$

   Show that ADD is not regular.

5. (6 points) In this homework problem, you will use a method other than the pumping lemma to prove that a language is non-regular.

   Recall the definition of $\sim_L$ from the lecture on the Myhill-Nerode theorem and DFA Minimization:

   $$\text{For } x, y \in \Sigma^*, x \sim_L y \quad \text{iff} \quad \forall z \in \Sigma^*, xz \in L \Leftrightarrow yz \in L.$$

   Recall also that if L is regular, $\sim_L$ has only finitely many equivalence classes.

   Prove that the language $L = \{0^n 1^n \mid n \geq 0\}$ is non-regular by showing that $\sim_L$ has infinitely many equivalence classes. (Precisely describe what these equivalence classes are.)