HW 2: Equivalence of DFAs & NFAs; Regular Expressions

Assigned: February 1, 2010
Due: February 8, 2010

Note: Take time to write clear and concise solutions. Confused and long-winded answers may be penalized. Consult the course webpage for course policies on collaboration.

1. (8 points)
   (a) Give an NFA recognizing all binary strings which both do not contain 11 as a substring and do not end with 1.
   (b) Following the equivalence proof, convert this to a DFA.
   (c) Simplify the DFA given in part (b) so that every state is reachable by some possible computation of the machine.
   (d) Give a regular expression for this language. You are not required to use the equivalence proof.

2. (6 points) If \( w = w_1w_2 \ldots w_n \) is a string, define \( w^{1/2} \) to be the first half of \( w \). I.e. \( w^{1/2} = w_1w_2w_3 \ldots w_{\lfloor \frac{n}{2} \rfloor} \). For any language \( L \), define \( L^{1/2} = \{ w^{1/2} : w \in L \} \).
   Prove that if \( L \) is regular, then \( L^{1/2} \) is regular.

   Hint: If a DFA recognizing \( L \) has state space \( Q \), construct an NFA with state space \( Q \times Q \) where the first coordinate goes forward and the second coordinate goes backward.

3. (4 points)
   (a) Assume \( A \) is a language recognized by a DFA \( M \), and let \( L \) be an arbitrary language. Define \( B \) to be the set of all strings that can be padded by a string in \( L \) and end up in \( A \). Formally \( B = \{ x : \exists y \in L, xy \in A \} \). Show that \( B \) is also recognizable by a DFA \( M_L \) that has the same states and transition function as \( M \).
   (b) Is the same true for NFAs?
4. (4 points) Explain in words what languages the following regular expressions represent.

   Let $\Sigma = \{0, 1\}$.

   (a) $0\Sigma\Sigma^*1\Sigma \cup 1\Sigma\Sigma^*0\Sigma$

   (b) $(0^*10^*10^*)^*$

5. (8 points)

   (a) Two regular expressions are equivalent if their corresponding languages are the same.

       Let $A$ be a regular expression. Prove that $A \circ \varepsilon = A$.

   (b) Let $L$ be a language comprising all strings $w$ such that $w$ contains an even number of $1$s, an odd number of $0$s, and no occurrences of the sub-string $10$. Write down a regular expression that generates $L$. Justify your answer.

   (c) In the C programming language, comments appear between delimiters such as /* and */. Let $L$ denote the language of all valid delimited comment strings. A member of $L$ must begin with /* and end with */ but have no intervening */. For simplicity, assume that the characters within the comments that are not / or * are only the symbols 0 and 1, so the alphabet $\Sigma$ is $\{0, 1, *, /\}$.

       Give a regular expression that generates $L$. 