

# Plug-and-Play Model Predictive Control for Electric Vehicle Charging and Voltage Control in Smart Grids

Somil Bansal, Melanie N. Zeilinger, and Claire J. Tomlin

**Abstract**—This paper presents a predictive controller for handling plug-and-play (P&P) charging requests of electric vehicles (EVs) in a distribution system. The proposed method uses a two-stage hierarchical control scheme based on a model predictive control (MPC) formulation for tracking periodic references. The first stage computes a reachable periodic reference that trades off deviation from the nominal voltage with the required generation control. The second stage computes a controller that tracks this reference and charges the EVs, while satisfying system constraints at all times. Under the assumption of a time-periodic load, it is shown that the proposed controller is recursively feasible and exponentially stable i.e. the EVs' state of charge (SOC) and bus voltages converge to the desired SOC and to the optimal periodic reference respectively. Finally, the proposed scheme is illustrated in a set of examples.

## I. INTRODUCTION

In a typical distribution system, changes in demand result in fairly mild and predictable voltage fluctuations, where capacitor banks are generally used to regulate voltages [1], [2], [3]. The increasing penetration of EVs in the distribution system, however, will introduce rapid and random fluctuations in the voltages. This, coupled with proliferation of renewable energy sources such as photovoltaic devices, poses some key challenges in terms of grid management and network operation. Capacitor banks alone are not sufficient to regulate voltages in such a scenario (see [4], [5] and references therein). Control schemes such as *inverter volt/var control* have been proposed to stabilize the voltages, in which reactive power is pushed into or pulled from the distribution system at a much faster rate compared to capacitor banks [6], [7], [8]. Since these control schemes are based on local generation at a bus, we will refer to them as *generation-based control* in this paper.

Although EVs represent an additional load on the distribution systems, this load is controllable and thereby offers an important opportunity. As per the smart grid initiatives in most countries that envision their integration with renewable energy sources as a dispatchable load [9], EVs not only represent a sustainable alternative to fuel-based automobiles but can also provide increased reliability of the distribution system [10], [5], [11]. Motivated by these ideas, our goal in this work is to develop a control scheme that integrates

EVs in the distribution network while taking into account additional loads and network requirements. There are three key challenges in designing a control scheme to achieve this integration. First, the proposed scheme should be able to charge all connected EV batteries to their desired SOC while minimizing the voltage fluctuations in the system. Second, it should be able to handle variations in the number of connected EVs, i.e. plugging in and out operations. In a real scenario, a user can request to connect or disconnect an EV at any desired time and bus. This will modify the overall distribution system, and the current controller may be infeasible and/or unstable for the modified system, requiring a controller redesign. Third, network constraints, or more specifically voltage constraints, should be satisfied at all times. Assuming that the remaining load profiles cannot be affected, this has to be achieved by scheduling EV charging accordingly. The smart grid literature (e.g. [9]) suggests that this functionality will be enabled by a communication infrastructure, allowing to control the charging power of individual EVs. In this work, we will address these three challenges and provide a constrained optimal control scheme that integrates charging and generation-based control. Under the proposed scheme, constraints on EVs as well as on the network are satisfied at all times. Moreover, a plug and play MPC (P&P MPC) approach is proposed to handle plugging in and out of EVs.

Although a generation-based control scheme is capable of stabilizing voltage fluctuations, this control input is limited by the capacity of the generator. Strain on the grid can increase significantly if several EVs are connected to the distribution system during peak hours and charged according to uncoordinated charging schedules (i.e. charging starts immediately after connection at a fixed charging power). Generation-based control alone may not be able to stabilize the voltages in such an environment. Integrating both control inputs, i.e. charging and generation-based control, in one control scheme allows to control the charging such that voltage constraints are satisfied with a given capacity of generators and hence is more cost efficient.

In this study, we will use a hierarchical Model Predictive Control (MPC) approach to design our controller. MPC is an attractive tool, being capable of minimizing a control objective while ensuring constraint satisfaction. Moreover the 'look ahead' characteristic of MPC provides improved performance and exploits load forecasts. Assuming that the load is (approximately) periodic, an MPC approach for tracking periodic references [12] is employed, which ensures that EVs are charged, while bus voltages track an optimal periodic

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trajectory minimizing both the deviation to the nominal voltage and the required generator control. Ideally we want bus voltages to track the nominal voltage exactly, but in general this may be undesirable due to expensive generation, or infeasible due to limits imposed by network constraints or model dynamics. A periodic reference is therefore calculated and tracked, which is the optimal tradeoff between deviation from the actual reference and required control effort, and consistent with system dynamics and constraints (from hereon called *optimal periodic reference*). We also employ the ideas of the P&P MPC concept introduced in [13] to deal with the connection and disconnection of EVs from the grid, which provides an online feasibility handling compared to other PP approaches (see, e.g., [14], [15] and references therein). A procedure for updating the controller together with a transition scheme is proposed, which prepares the system for the requested modifications. To summarize, the main contributions of our work are:

- system voltages are regulated to the optimal periodic reference, trading off deviation from nominal voltage and generation control effort;
- the load and network constraints are explicitly taken into account, and constraint satisfaction is guaranteed at all times;
- the control scheme ensures exponential convergence of voltages to the optimal periodic reference, and of EV batteries to their desired charge level;
- the proposed control scheme can handle the online plugging and unplugging of EVs;
- all results are established under a time-periodic load as opposed to a constant load considered in many studies.

The paper is organized as follows: Section II introduces the system model. In Section III, control objectives are defined and the hierarchical MPC controller is proposed together with an extension for handling plug and play requests. Section IV presents numerical simulations demonstrating the advantages of the proposed control scheme and Section V provides concluding remarks.

## II. PRELIMINARIES

### A. Electric Vehicle Charging Model

We adopt a linear state space model governing the battery SOC of an EV:

$$e(k+1) = e(k) + Tc(k) \quad (2)$$

where  $e \in \mathbb{R}$  is the battery SOC (kWh),  $c \in \mathbb{R}$  is the charging power supplied for charging the device (kW), and  $T$  is the sampling time, which is assumed to be fixed. For EVs, the battery storage capacity and the maximum charging power are limited by the following constraints:

$$e_{min} \leq e(k) \leq e_{max} \quad (3)$$

$$0 \leq c(k) \leq c_{max} \quad (4)$$

*Remark 1:* The EV charging model introduced in this section has been chosen for the sake of simplicity. However, more advanced linear models can be similarly considered (e.g. including some efficiency of charging as in [16]).

### B. Network Model

We consider a radial distribution network, which is a structure commonly considered in the power systems literature. To characterize the power flow in this network we adopt the DistFlow equations first introduced in [1]. We adopt the notation introduced in [7], restated here for completeness.

TABLE I  
VARIABLES FOR A RADIAL DISTRIBUTION NETWORK

$\mathcal{N}$	Set of buses, $\mathcal{N} := \{1, \dots, n\}$
$\mathcal{L}$	Set of lines between the buses in $\mathcal{N}$
$\mathcal{L}_i$	Set of lines connecting bus 0 to bus $i$
$p_i^l, p_i^v$	Real power consumption by load and EVs at bus $i$
$q_i^l, q_i^g$	Reactive power consumption and generation at bus $i$
$r_{ij}, x_{ij}$	Resistance and reactance of line $(i, j) \in \mathcal{L}$
$P_{ij}, Q_{ij}$	Real and reactive power flows from bus $i$ to $j$
$v_i$	Voltage magnitude at bus $i$
$l_{ij}$	Squared magnitude of complex current from bus $i$ to $j$
$M_i$	Number of EVs connected at bus $i$

The power flow equations for a radial distribution network can be written as the following DistFlow equations [2]:

$$P_{ij} = p_j^l + p_j^v + r_{ij}l_{ij} + \sum_{k:(j,k) \in \mathcal{L}} P_{jk} \quad (5a)$$

$$Q_{ij} = q_j^l - q_j^g + x_{ij}l_{ij} + \sum_{k:(j,k) \in \mathcal{L}} Q_{jk} \quad (5b)$$

$$v_j^2 = v_i^2 + (r_{ij}^2 + x_{ij}^2)l_{ij} - 2(r_{ij}P_{ij} + x_{ij}Q_{ij}) \quad (5c)$$

$$l_{ij}v_i^2 = P_{ij}^2 + Q_{ij}^2 \quad (5d)$$

where  $P_{ij}$ ,  $Q_{ij}$ ,  $v_j$  and  $l_{ij}$  are defined in Table I. Following [3], [7] we neglect the higher order real and reactive power loss terms with respect to power flows  $P_{ij}$  and  $Q_{ij}$ . This approximation introduces a small relative error, typically on the order of 1% [7]. With this approximation, (5) can be reduced to the following linear equation (see [7] for more details on this derivation):

$$v = \bar{v}_0 - R(p^l + p^v) - X(q^l - q^g), \quad (6)$$

where

$$R_{ij} = \sum_{(h,k) \in \mathcal{L}_i \cap \mathcal{L}_j} r_{hk} \quad (7a)$$

$$X_{ij} = \sum_{(h,k) \in \mathcal{L}_i \cap \mathcal{L}_j} x_{hk} \quad (7b)$$

and  $\bar{v}_0 = (v_0, \dots, v_0) \in \mathbb{R}^n$ . The other variables in equation (6) are the generation input (column) vector  $q^g := (q_1^g, \dots, q_n^g) \in \mathbb{R}^n$  and the EV load vector  $p^v := (p_1^v, \dots, p_n^v) \in \mathbb{R}^n$ , where  $p_i^v \in \mathbb{R}$ , denotes the net load of all vehicles charging at bus  $i$ , i.e.  $p_i^v = \sum_{j=1}^{M_i} c_j$ . Note that  $M_i$  can vary over time due to plugging and unplugging operations.

We assume that the substation voltage  $v_0$  is given and is constant. Furthermore, load profiles  $p^l$  and  $q^l$  are time-varying but their 24 hour forecast is assumed to be given. To establish the desired results we will make the following assumption on the load profile:

*Assumption 1:* The load profiles  $p^l$  and  $q^l$  are time periodic with period length 24h.

This assumption is a tradeoff between a freely varying load (real scenario) and a constant load (a stringent assumption). However, it is reasonable to expect the load profile to be approximately periodic with a period of 1 day, because the load on the distribution system is likely to be similar at a certain time of the day on two consecutive days.

Let  $v_{nom}$  denote the nominal value of bus voltage. Also, define  $\tilde{v} := \bar{v}_0 - Rp^l - Xq^l$ , which is periodic due to Assumption 1. Then the model (6) reduces to:

$$v = Xq^g - Rp^v + \tilde{v}. \quad (8)$$

### C. Network Constraints

Depending on the load, bus voltages can fluctuate significantly. For reliable operation of the distribution network it is required to maintain the bus voltages  $v$  within a tight range around the nominal value at all times:

$$v_{min} \leq v - v_{nom} \leq v_{max}. \quad (9)$$

In addition, there are inherent physical limitations on the generator control input, which is limited to:

$$q_{min} \leq q^g \leq q_{max}. \quad (10)$$

*Remark 2:* The generation power output has both active and reactive power components. However, due to inverters connected at the generator output, one can always control it to output a constant active power and varying reactive power [7]. In our work this constant active power is merged into the total active load  $p^l$ .

### D. System as an input-output model

In this section we will represent the overall system in the standard linear input-output system form with bus voltages as outputs and SOC<sub>s</sub> as states. Recalling that  $p_i^v = \sum_{j=1}^{M_i} c_j$ , (8) can be rewritten as:

$$v = Xq^g - RKu_2 + \tilde{v} \quad (11)$$

where  $u_2 := (c_1, \dots, c_{M_1}, \dots, c_M)^T \in \mathbb{R}^M$ ,  $K \in \mathbb{R}^{n \times M}$  and  $M$  is the total number of EVs connected to the grid, i.e.  $M = \sum_{j=1}^n M_j$ .  $K_{ij} = 1$  if and only if EV  $j$  is connected to bus  $i$  and 0 otherwise. The control objective of  $v$  tracking  $v_{nom}$  is therefore equivalent to  $(v - \tilde{v})$  tracking the periodic reference  $r := v_{nom} - \tilde{v}$ . Note that  $r$  is the actual reference that we would ideally like to track and should not be confused with the optimal periodic reference.

EVs are connected to the grid to charge them to a desired SOC,  $e_{des}$  (specified by the user at the time of connection). Rewriting the SOC dynamic (2) in terms of  $\bar{e} := e_{des} - e$ :

$$\bar{e}(k+1) = \bar{e}(k) - Tc(k) \quad (12)$$

Combining equation (12) for all EVs leads to the overall system model.

$$x(k+1) = Ax(k) + Bu(k) \quad (13a)$$

$$y(k) = Cx(k) + Du(k) \quad (13b)$$

$$(x(k), u(k), y(k)) \in \mathcal{Z}_k \quad (14)$$

where

$$x = (\bar{e}_1, \dots, \bar{e}_{M_1}, \dots, \bar{e}_M)^T, \quad u = [q^g \ u_2]^T, \quad y = (v - \tilde{v})$$

$$A = I, \quad B = [0 \quad -T], \quad C = 0, \quad D = [X \quad -RK]$$

$$\mathcal{Z}_k = \left\{ \begin{array}{l} (x(k), u(k), y(k)) : e_{min} \leq e_{des} - x(k) \leq e_{max} \\ q_{min} \leq q^g(k) \leq q_{max}, \quad 0 \leq u_2(k) \leq c_{max} \\ v_{min} - \tilde{v}(k) \leq y(k) - v_{nom} \leq v_{max} - \tilde{v}(k) \end{array} \right\}$$

Note that due to the fact that  $\tilde{v}(k)$  is time-periodic, constraint set  $\mathcal{Z}_k$  is also time-varying and periodic.

## III. CONTROLLER DESIGN

The goal is to design a controller that captures three control objectives: bus voltages to track the optimal periodic reference, to charge the EVs to their desired SOC, and to enable P&P operations. The designed controller should achieve these goals subject to the system constraints. The proposed control scheme addresses these three objectives by solving the following three subproblems subject to (13) and (14):

- computation of a periodic reference, which is an optimal tradeoff between deviation from nominal voltage and the required generation control input;
- computation of a controller, which ensures that bus voltages track the optimal periodic reference and EVs are charged to their desired SOC<sub>s</sub>;
- if a modification is requested, determination of a feasible P&P time, preparation of the system for the modification and redesign/update of controller such that it is feasible for the modified system.

In Subsection III-A, we propose a method for subproblems 1 and 2 using a two-stage hierarchical control scheme based on an MPC approach for tracking periodic references [12], [17]. Subproblem 3 is considered in Subsection III-B using the P&P MPC concept introduced in [13].

### A. Periodic Reference Tracking MPC Integrating EV Charging and Generation-based Control

For the purpose of this section we assume that the number of EVs connected to the grid is constant, i.e. no new EVs are connected to or disconnected from the system. We will relax this assumption in the next section. In general, in a reference tracking problem, it may be impossible to track the given reference signal due to limits imposed by the constraints (see [12], [17] and references therein). It is therefore common practice to solve the tracking problem in a hierarchical way, where first a reachable trajectory is calculated, in our case periodic, which is the closest trajectory to the given reference signal (with respect to some objective function) satisfying system dynamics and constraints, and then, the optimal reachable trajectory is tracked instead [12]. The optimization problem for computing the optimal reachable reference  $(x^s(k), u^s(k), y^s(k))$  with period  $N_r$ , given the

reference  $r_k$ , (referred to as *stage-1* in this paper) is given by:

$$(x^s(k), u^s(k), y^s(k)) := \underset{\tilde{x}, \tilde{u}, \tilde{y}}{\operatorname{argmin}} V_1(r_k; \tilde{x}, \tilde{u}, \tilde{y}) \quad (15a)$$

$$\text{s.t.} \quad \tilde{x}(i+1) = A\tilde{x}(i) + B\tilde{u}(i) \quad (15b)$$

$$\tilde{y}(i) = C\tilde{x}(i) + D\tilde{u}(i) \quad (15c)$$

$$\tilde{x}(0) = A\tilde{x}(N_r - 1) + B\tilde{u}(N_r - 1) \quad (15d)$$

$$(\tilde{x}(i), \tilde{u}(i), \tilde{y}(i)) \in \mathcal{Z}_i; \quad i = 0, \dots, N_r - 1 \quad (15e)$$

where,

$$V_1(r_k; \tilde{x}, \tilde{u}, \tilde{y}) = \sum_{i=0}^{N_r-1} \|\tilde{y}(i) - r(k+i)\|_{T_1}^2 + \|\tilde{u}(i)\|_{T_2}^2 + \|\tilde{x}(i)\|_{T_3}^2.$$

$T_1$  is positive definite and  $T_2, T_3$  are positive semi-definite weight matrices of appropriate dimensions. Throughout this paper we use the term optimal periodic reference to denote both  $y^s$  and  $v^s := y^s + \tilde{v}$  interchangeably.  $v^s$  represents an optimal voltage profile, trading off deviation from  $v_{nom}$  and required generation control.

*Remark 3:* (1) If the given reference  $r_k$  is reachable and has period  $N_r$ , then it can be obtained as the optimal reachable reference by setting  $T_2$  in (15) to zero. (2) From (13) it is clear that  $x$  is monotonically decreasing in  $u_2$ . Due to the periodicity constraint on  $x$  in the first stage, the optimal solution is therefore  $x^s, u_2^s \equiv 0$ . (3) If the reference trajectory  $r$  does not vary and is periodic with period  $N_r$ , then stage-1 only has to be solved once.

At the second level, a predictive controller is designed to track the calculated reachable trajectory  $(x^s(k), u^s(k), y^s(k))$ . We propose the following MPC problem (referred to as *stage-2* in this paper):

$$\underset{\bar{x}, \bar{u}, \bar{y}}{\min} V_2(x(k), x^s(k), u^s(k), y^s(k); \bar{x}, \bar{u}, \bar{y}) \quad (16a)$$

$$\text{s.t.} \quad \bar{x}(i+1) = A\bar{x}(i) + B\bar{u}(i) \quad (16b)$$

$$\bar{y}(i) = C\bar{x}(i) + D\bar{u}(i) \quad (16c)$$

$$\bar{x}(0) = x(k) \quad (16d)$$

$$\bar{x}(N) = x^s(N|k) \quad (16e)$$

$$(\bar{x}(i), \bar{u}(i), \bar{y}(i)) \in \mathcal{Z}_i; \quad i = 0, \dots, N - 1 \quad (16f)$$

where,

$$V_2(x, x^s, u^s, y^s; \bar{x}, \bar{u}, \bar{y}) = \sum_{i=0}^{N-1} \|\bar{y}(i) - y^s(i)\|_{R_1}^2 + \|\bar{u}(i) - u^s(i)\|_{R_2}^2 + \|\bar{x}(i)\|_{R_3}^2$$

$R_1, R_2, R_3$  are positive definite weight matrices of appropriate dimensions and  $N$  is the prediction horizon of the MPC problem. MPC problem (16) is solved at every sampling time, returning the optimal control sequence  $\mathbf{u}^*(x)$  (we omit the dependence of  $\mathbf{u}^*$  on the optimal reference for ease of notation). The optimal control law is defined in a receding

horizon fashion by  $\kappa(x) = u_0^*(x)$ , where  $u_0^*(x)$  denotes the first control input of the sequence.

*Remark 4:* (1) In stage-2, a cost function is chosen that penalizes the deviation of voltage from the nominal voltage, the difference between the current SOC and the desired SOC and the generation control input. The weight on the EV input should be chosen very small, as the total amount of energy required to charge the EV is fixed. (2) If there is a penalty on the input in stage-1 (i.e.,  $T_2$  is positive definite), the above hierarchical control structure ensures that the bus voltages track nominal voltage as close as possible with minimum control input while charging EVs.

We will conclude this section with formally establishing the exponential convergence of bus voltages to the optimal reference trajectory (i.e.  $v \rightarrow v^s$ ), and of the SOCs to their desired SOCs (i.e.  $x \rightarrow 0$ ).

*Theorem 1:* Assume that the reference trajectory  $r$  is periodic with period  $N_r$  and let  $(x^s, u^s, y^s)$  be the corresponding optimal reachable reference. Let  $\mathcal{X}_N$  denote the set of initial states for which (16) is feasible. For any  $x \in \mathcal{X}_N$ , the proposed control law  $\kappa(x)$  ensures that the system constraints are satisfied at all times and bus voltages and SOC converge exponentially to  $v^s$  and desired SOC, respectively.

**Proof.** We first prove that the stage-2 problem is feasible at all times if the initial state is feasible. This will ensure that system constraints are satisfied at all times.

Noting that the constraint set  $\mathcal{Z}_i$  is periodic, recursive feasibility is ensured by (16e), and can be established by using the shifted control sequence proposed in Theorem 2 in [17]. With this control sequence, we obtain

$$V_2(k+1) - V_2^*(k) = -\|\bar{y}(k) - y^s(0|k)\|_{R_1}^2 - \|\bar{x}(k)\|_{R_3}^2 - \|\bar{u}(k) - u^s(0|k)\|_{R_2}^2 \quad (17a)$$

Exponential convergence of  $y \rightarrow y^s$  (equivalent to  $v \rightarrow v^s$ ) and  $x \rightarrow 0$  follows standard arguments using the Lyapunov exponential stability theorem, proving the result. ■

*Remark 5:* We have assumed periodic references in Theorem 1 for simplicity of presentation. However, using the formulation in [17], these results can be extended to varying periodic references.

## B. Plug-And-Play EV Charging

In real distribution systems users can connect or disconnect their EVs randomly. This changes the overall load on the system and can affect bus voltages significantly. This section extends the MPC scheme to the case where the system dynamics in (13) change due to EVs joining or leaving the network by employing the concept of P&P MPC in [13]. The introduction of P&P capabilities poses two key challenges [13], [18]: 1. Feasibility of the network change has to be assessed and the system must be prepared for this change; 2. The control law has to be redesigned for the modified dynamics. In the considered case, the problem is reduced to the first issue since the controller is computed centrally and the new controller is directly given by solving the MPC problem in Section III-A with the dynamics replaced by the modified dynamics. In this section, we address the first

challenge by means of a preparation phase ensuring recursive feasibility and stability during P&P operation.

As discussed earlier, sudden changes in the system may lead to violation of constraints (14). Consider for example the scenario, where a large number of EVs are connected to an already heavily loaded bus. In this case, the bus voltage may fluctuate significantly and it may not be possible for the system to satisfy voltage constraint (9) given the current system state and load. This problem is addressed by using the concept of a transition phase (first introduced in [13]), where first a steady-state is computed that allows P&P operation and then the system is controlled to this steady-state. After reaching this steady-state the P&P operation is performed and the new controller is applied to the modified system.

The steady state  $(x^{ss}, u^{ss})$  is chosen such that it is reachable from the current system state under the previous dynamics and starting from the steady state, there exists a control sequence such that the optimization problem (16) is feasible for the modified system. In particular, let  $\mathcal{S}$  and  $\bar{\mathcal{S}}$  be the set of current EVs and the modified set of EVs (after the P&P operation) respectively. For any set  $\mathcal{D}$ , denote by  $x_{\mathcal{D}}$  the state of EVs in that set. System matrices are similarly denoted by  $A_{\mathcal{D}}, B_{\mathcal{D}}, C_{\mathcal{D}}$  and  $D_{\mathcal{D}}$ . Also let  $x(k)$  be the current state of the system. This results in the optimization problem:

$$\min_{\bar{x}^{ss}, \bar{u}^{ss}} \sum_{i=0}^{d-1} (\|x_{\mathcal{S}}(i)\|^2) \quad (18a)$$

$$\text{s.t.} \quad \bar{x}^{ss} = A_{\mathcal{S}}\bar{x}^{ss} + B_{\mathcal{S}}\bar{u}^{ss} \quad (18b)$$

$$x_{\mathcal{S}}(i+1) = A_{\mathcal{S}}x_{\mathcal{S}}(i) + B_{\mathcal{S}}u_{\mathcal{S}}(i) \quad (18c)$$

$$y(i) = C_{\mathcal{S}}x_{\mathcal{S}}(i) + D_{\mathcal{S}}u_{\mathcal{S}}(i) \quad (18d)$$

$$(x_{\mathcal{S}}(i), u_{\mathcal{S}}(i), y(i)) \in \mathcal{Z}_i \quad (18e)$$

$$x_{\mathcal{S}}(d+m+1) = A_{\bar{\mathcal{S}}}x_{\bar{\mathcal{S}}}(d+m) + B_{\bar{\mathcal{S}}}u_{\bar{\mathcal{S}}}(d+m) \quad (18f)$$

$$y(d+m) = C_{\bar{\mathcal{S}}}x_{\bar{\mathcal{S}}}(d+m) + D_{\bar{\mathcal{S}}}u_{\bar{\mathcal{S}}}(d+m) \quad (18g)$$

$$(x_{\bar{\mathcal{S}}}(d+m), u_{\bar{\mathcal{S}}}(d+m), y(d+m)) \in \mathcal{Z}_{d+m} \quad (18h)$$

$$x_{\mathcal{S}}(0) = x, \quad x_{\mathcal{S}}(d) = \bar{x}^{ss}, \quad x_{\bar{\mathcal{S}}}(d+N) = 0 \quad (18i)$$

$$i = 0, \dots, d; \quad m = 0, \dots, N-1$$

where  $d$  is determined to be as small as possible while providing feasibility of problem (18). If a P&P is ever possible for the distribution system, optimization problem (18) is feasible. Note that problem (18) not only prepares the system for the P&P request, but also minimizes the waiting time for the EVs before they can be charged by the system (i.e. minimizes the duration of the required transition phase). It is also important to note that due to the time varying loads, this plug and play problem really becomes a problem of when to plug-in/-out and it is not enough for safe P&P to be only at a given state, but at a given state at a given time. For example, the same steady-state may not allow a P&P operation when the system is heavily loaded.

Denote the optimal solution of (18) by  $(d^*, x^{ss}, u^{ss})$ , i.e. the P&P operation is performed after time  $d^*T$ . In order to

ensure that the system reaches the steady-state within the specified  $d^*$  time steps, the control sequence obtained in (18) can be applied open loop, or a shrinking horizon MPC scheme can be applied. Once the system reaches the steady-state, constraint satisfaction is guaranteed for the modified system by (18). Hence from this point on, we can use the method described in Section III-A to design a controller for the modified system that tracks  $y^s$  and exponential convergence is again ensured by the proposed technique.

*Remark 6:* In the P&P procedure, the optimal periodic reference does not have to be recomputed for the modified system as it does not depend on EVs (see Remark 3).

#### IV. NUMERICAL EXAMPLES

*Example 1:* We apply the method presented in Section III-A to a 9-branch linear feeder system, demonstrating the performance for controlling the bus voltages and charging EVs. We also show the effect of not using a penalty on the generation input in (15), i.e. the computed optimal periodic reference is that closest to the reference  $v_{nom}$  subject to system constraints.

The study is performed for the 9-bus network used in [1] with network data rescaled for the household load. A typical household load profile is assumed at the buses, which peak at evening hours [19]. The number of EVs connected to the system is 11 and assumed to be constant for the purpose of this example (i.e. no P&P requests are made). Other parameters used in this example are given in Table II.

TABLE II  
OTHER SYSTEM PARAMETERS FOR EXAMPLE 1 IN PER UNIT (PU)

$T$	$\epsilon_{min}$	$\epsilon_{max}$	$c_{max}$	$v_0$
5 min	0.2 pu	1.0 pu	0.6 pu	1 pu
$v_{nom}$	$v_{min}$	$v_{max}$	$q_{min}$	$q_{max}$
1 pu	-0.1 pu	0.1 pu	-1.0 pu	1.0 pu

We consider two different cases. In Case 1 we choose  $T_2$  to be an identity matrix and in Case 2 it is chosen to be zero. All other weight matrices in (15) and (16) are chosen to be identity.

Without loss of generality, the simulation starting time is taken as  $t = 0$ . Results are shown in Figures 1-3. Figure 1 shows the reference trajectory  $v_{nom}$ , the calculated optimal periodic reference  $v^s$ , and the actual voltage trajectory  $v$  for the two cases. In both cases the optimal reference trajectory is tracked eventually with zero error. As expected, the optimal periodic reference in Case 2 is much closer to the nominal voltage compared to Case 1. As generation is not penalized, more generation input is expended to find a reachable trajectory that is closer to  $v_{nom}$  in Case 2. This is also evident from the required generation input curve in the two cases shown in Figure 2. From the optimization problem (15) it is clear that if the weight on generation is multiplied by a scalar less than 1, then a line in between these two curves is obtained.

Figure 1 also compares our control scheme with an uncoordinated charging scheme (i.e. charging starts as soon as

EVs are plugged in at  $t = 0$  at a constant charging power of  $C_{max}$ . In particular, voltage constraints are satisfied at all times by the designed controller, whereas uncoordinated charging violates constraints and causes a significant deviation of more than 30% from the nominal voltage.

All EVs are charged to their desired state of charge as exemplified for EV 1 in Figure 3, where the corresponding charging control is also shown.

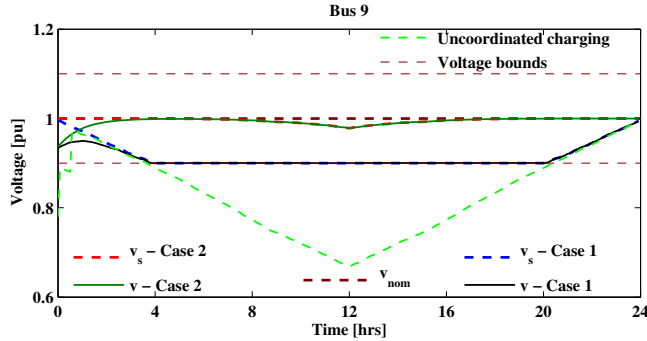


Fig. 1. Optimal periodic reference and voltage trajectory at bus 9 for Case 1 and 2, illustrating convergence of voltage trajectories. The optimal reference in Case 2 is closer to  $v_{nom}$  because there is no penalty on generation. Uncoordinated charging is not able to satisfy system constraints and causes a large voltage drop.

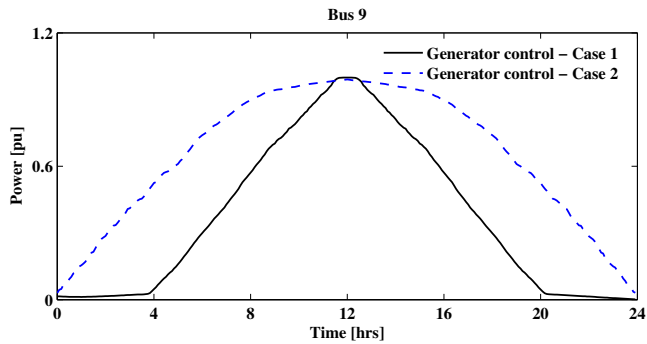


Fig. 2. Generation control effort required at bus 9 to track the optimal periodic reference. More generation control is used in Case 2 to find a closer reference  $v_s$  to  $v_{nom}$ .

*Example 2:* This example demonstrates how a distribution system can handle P&P requests using the methodology described in Section III-B. We simulate a 45-bus radial distribution system used in [6]. 85 EVs are already connected to the system. All other parameters are the same as in Table II except  $g_{min}$  and  $g_{max}$ , which are now  $-0.45pu$  and  $0.45pu$  respectively. We process 4 different P&P requests over 5 hours (starting at  $t = 10$ ). Details of the P&P requests are given in Table III. The load profile at bus 10 for these 5 hours is shown in Figure 4. Note that the load reaches its maximum value at  $t = 12$ .

TABLE III  
P&P REQUESTS FOR EXAMPLE 2

Type of request	At bus	Number of EVs	Request time	$d^*$	Request accepted at
Connection	10	2	10.8	1	Immediately
Connection	10	6	11.2	15	12.5
Disconnection	10	3	12.9	1	Immediately
Connection	22	8	13.3	1	Immediately

Results are shown in Figures 5-6. The first P&P request is

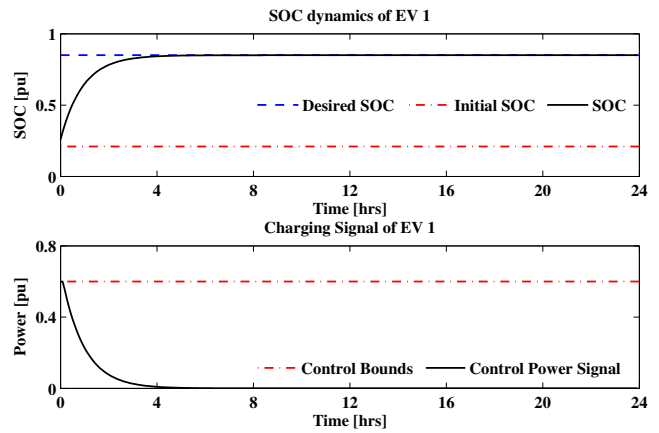


Fig. 3. SOC dynamics and charging control for EV 1 in Case 1. EV is charged to its desired SOC in 4 hours.

accepted immediately due to the mild load conditions at that time. However, for the second P&P request, due to the high load demand within the prediction horizon of the requested connection time, the steady-state computation determines that it will not be feasible to accommodate the request immediately and starts to prepare the system, regulating it to the best steady-state to allow this desired change. The shortest horizon in problem (18) is  $d^* = 15$  i.e. new EVs have to wait for 75 minutes before they start charging.

The disconnection request is accepted instantly because it is reducing strain on the distribution system and hence no feasibility issues arise. This is expected to be the case in most disconnection scenarios.

Interestingly, the third plug-in request is also accepted instantly despite the high load on the system and the large number of new EVs. This is due to the prediction capability of MPC. By using the load prediction, the controller determines that the system will have more capacity to accommodate new EVs as the load is going to reduce soon and hence they can be connected right away.

P&P requests modify the system and as a result the voltage trajectory deviates from its optimal reference trajectory, however it again tracks the reference trajectory eventually (see Figure 5). As in *Example 1*, our control scheme ensures that voltage constraints are satisfied at all times during and after the processing of P&P requests, whereas voltage bounds would not be satisfied by an uncoordinated charging. We next show the SOC dynamics and charging control of one of the two EVs connected at  $t = 10.8$  in Figure 6. To handle the upcoming P&P request of 6 EVs, the system charges this EV at the maximum possible charging power,  $C_{max}$ , so that new EVs can be accommodated on the same bus with the minimum possible waiting time (note that this is not generally the case without the P&P operation see for example Figure 3, where the control input is strictly decreasing). Due to this high charging power, the voltage at bus 10 deviates from its reference trajectory (see Figure 5). Due to the maximum load at  $t = 12$  (Figure 4), charging power is dropped for a short time period before it rises again to the optimal level allowed by the load profile of the system.

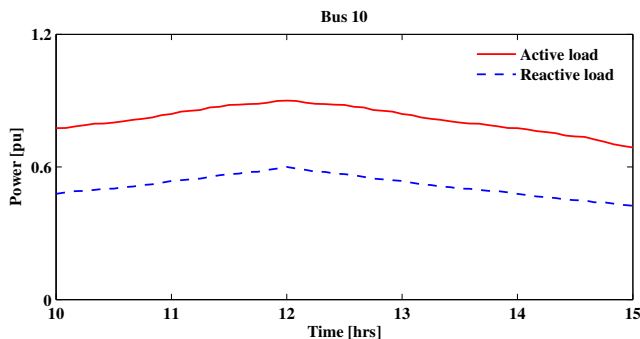


Fig. 4. Load profile at bus 10. Load is maximum at  $t = 12$ .

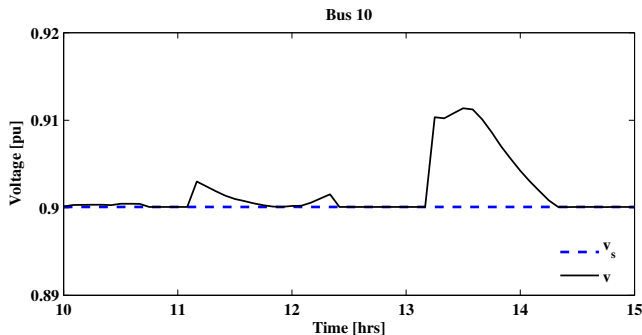


Fig. 5. Optimal periodic reference and voltage trajectory at bus 10. Voltage deviates from the reference trajectory to accommodate new EVs but again tracks the reference eventually.

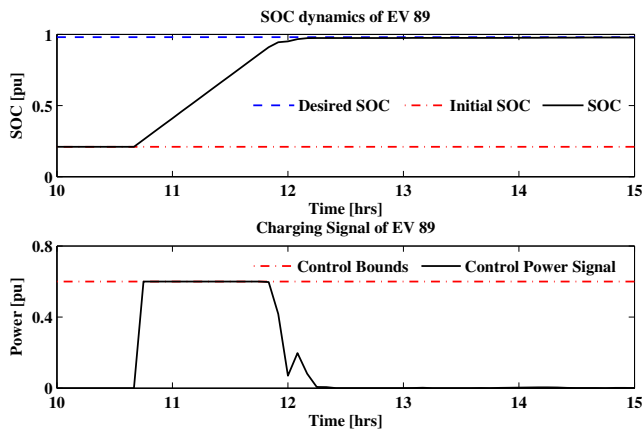


Fig. 6. SOC dynamics and charging control of the EV connected at  $t = 10.8$ . EV is charged at the maximum possible charging power in order to accommodate the new EVs - connected at  $t = 11.25$  - as soon as possible.

## V. CONCLUSION

In this paper, we have proposed a predictive controller that is capable of handling P&P requests of EVs. The proposed controller minimizes the waiting time for connection/disconnection of EVs from the system. An MPC approach for tracking periodic references is proposed to integrate EV charging with generator control. A reachable voltage reference, achieving an optimal tradeoff between minimizing voltage fluctuations and required generation control, is calculated, which is exponentially tracked under the assumption of a periodic load with a given forecast. The exponential convergence of the state of charge of EVs to their

desired values under the proposed controller has been shown. The performance of the proposed method was demonstrated for the control of two radial distribution system, an illustrative example with 9 and a large example with 45 buses.

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