1 Phase-variable representation - 10 points

For the transfer function below, write the state equations and the output equation for the phase-variable representation.

\[
\frac{s^2 + 2s + 1}{s^4 + 2s^3 + 4s^2 + 4s + 5}
\]

2 Transfer function - 10 pts

Find the transfer function \( G(s) = \frac{Y(s)}{R(s)} \) for the following system represented in state space \( \dot{x} = Ax + Bu, y = Cx \).

\[
A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}
\]

3 Time Response - 20 pts

Find the output response, \( c(t) \), for each of the systems. Also find the time constant, rise time, and settling time for each case.

(a) \( G(s) = \frac{2}{s + 2} \)  
(b) \( G(s) = \frac{6}{s + 6} \)
4  Time response - 20 pts

For each of the transfer functions shown below, find the locations of the poles and zeros, plot them on the s-plane, and then write an expression for the general form of the step response without solving for the inverse Laplace transform. State the nature of each response (overdamped, underdamped, and so on).

(a) \[ G(s) = \frac{3}{(s + 5)(s + 2)} \]

(b) \[ G(s) = \frac{3}{s^2 + 5s + 60} \]

5  Second-order system - 20 pts

For each pair of second-order system specifications that follow, find the location of the second-order pair of poles.

(a) %OS = 10%; \( T_s = 0.5 \) second

(b) \( T_s = 6 \) seconds; \( T_p = 4 \) seconds

6  Time response - 20 pts

For the system, do the following:

(a) Find the transfer function \( G(s) = X(s)/F(s) \)

(b) Find \( \xi, \omega_n, \%OS, T_s, T_p, \) and \( T_r \).