Optimizing for the future smart grid: Efficient methods for nonconvex AC power flow problems

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New challenges for the smart grid

Traditional Grid

Smart Grid

- New infrastructure
- More data
- Increased demand
- More cyclic structure
Fundamental challenges in power system optimization

Scalability
- 145 million customers
- Over 7,300 power plants
- 160,000 miles high-voltage power lines

In practice
- Divided into smaller regions
- Poor local decisions → cascading failures in interconnected network
Fundamental challenges in power system optimization

**Optimality**
- Nonlinear nature of alternating current (AC) power flow → many problems are nonconvex
- Difference between local and global solutions is estimated at billions of $ annually in the US (source: FERC)

**In practice**
- Optimization stage: Linearize power flow equations (DC approximation)
- Use heuristics to generate feasible AC solution
- New interest in conic relaxations that have global guarantees
Goals of my PhD thesis work

- Take a cross-disciplinary approach to solve important problems in power systems optimization
  - Focus on both fundamental problems and new problems

- Synthesize advanced methods from optimization and mathematics
  - Algebraic geometry, graph theory, numerical methods
  - Domain-specific understanding: the physics of power flow, sparse graph structure

- Leverage novel theory to develop new algorithms

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Many problems in power systems planning and operation are based on fundamental problems.
Part I: Optimal Power Flow (OPF)

Two projects:
1) Finding the worst-case local minimum of OPF
2) Finding the global solution to a post-contingency OPF
Part I: Optimal Power Flow (OPF)

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Mentor: Prof. Somayeh Sojoudi
Goal: find minimum cost production of committed generating units

- While satisfying technological and physical constraints

Existing methods

- **Local:** Interior Point Methods (IPM), Sequential Quadratic Programming (SQP)
- **Global:** Convex relaxations (SDP, SOCP)$^{1,2,3}$

\[
\begin{align*}
\text{cost of real power generation} \\
(v \in \mathbb{C}^n, p_g \in \mathbb{R}^n, q_g \in \mathbb{R}^n)
\end{align*}
\]

\[
\begin{align*}
\min_x & \quad f(x) \\
\text{s.t.} & \quad h(x) = 0 \\
& \quad g(x) \leq 0
\end{align*}
\]

AC power flow equations

technological & physical constraints

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Finding the worst-case local minimum\textsuperscript{1}

**Goal:** Bound the worst-case performance of a generic local search solver


**Local search method**

- \( f(x) \) vs. \( x \)
- Example QCQP:
  \[
  \min_{x \in \mathbb{R}^n} \quad x^T M_0 x \\
  \text{s.t.} \quad x^T x = 1
  \]
The worst-case local minimum

**Idea:** Create an upper bound on the worst-case local minimum

Our method:

Construct a new “worst-case local min” problem

\[
\max_{x \in \mathbb{R}^n} x^T M_0 x + k
\]

s.t. \( x \in \{ \text{local minima of QCQP} \} \)

Feasible set given by first- and second-order optimality conditions

Take a convex relaxation of the “worst-case” problem into a semidefinite program (SDP)

\[
0 = \nabla_x L(x^*, \lambda^*) = 2M_0 x^* + 2 \sum_{i=1}^{p} \lambda_i^* M_i x^*
\]

\[(x^*)^T M_i x^* = a_i, \forall i = 1, ..., p\]

\[
M_0 + \sum_{i=1}^{p} \lambda_i^* M_i + c \sum_{i=1}^{p} M_i(x^*)(x^*)^T M_i \succeq 0
\]

for some \( c \) above a certain threshold, \( c > \bar{c} \)

Upper bound on problem

Show that tightness of SDP depends on choice of parameter \( c \)
If we take $c = 0$ in the SDP relaxation, the SDP relaxation is exact (thus its solution is the worst-case local minimum).

For $c > 0$, the SDP relaxation is not exact.

**Choice of parameter $c$:**

- Exact value of $c$ is not needed for the SDP relaxation.
- Selecting too large of a $c \implies$ larger optimality gap between the SDP relaxation & the original worst-case local min problem.

- Also looked at introducing a penalty term to the objective to get tight SDP relaxation.
Simulations on realistic networks

IEEE 9-bus network

IEEE 14-bus network

SDP of worst-case local min problem

“Discovered” local minima

Solution timeframe: ~ 3-5 minutes
Relation to existing methods

Compare the **SDP worst-case upper bound** with the **SDP lower bound** to obtain bounds on the range of possible objective values obtained with local search.
Worst-case local min: summary & conclusions

• Formulated a new problem to find the worst-case local minimum for a canonical QCQP (e.g. OPF)
  • Since this problem is still nonconvex, use an SDP relaxation to find an upper bound

• Find that the tightness of the upper bound depends on the choice of a parameter in the second-order necessary optimality condition

• Method provides a metric on how much SDP can outperform local search → evaluate the performance of the whole class of local search methods
Part I: Optimal Power Flow (OPF)

Two projects:
1) Finding the worst-case local minimum of OPF
2) Finding the global solution to a post-contingency OPF

Mentors: Prof. Somayeh Sojoudi, Prof. Javad Lavaei
Collaborator: SangWoo Park
**Parametric OPF for post-contingency analysis**

\[
\begin{align*}
H(\lambda) & \quad \text{min}_x \quad f(x, \lambda) \\
\text{s.t.} & \quad h(x, \lambda) = 0 \\
& \quad g(x, \lambda) \leq 0
\end{align*}
\]

Want to efficiently solve coupled post-contingency OPF problems to global optimality given the solution to the base case.

Base SCOPF problem approximates the contingencies but does not explicitly solve for them.

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Goal: Efficiently find the global solution to all the variations, given a global solution to the base problem

Approach: Design sequences of intermediate problems that connect the base problem to each of its variations
Homotopy and continuation methods have long been used in mathematics & engineering to solve systems of nonlinear equations.

- Power systems: continuation power flow
  
  “Easy” problem: \( s(x) = 0 \)  
  “Hard” problem: \( f(x) = 0 \)

Define \( H(x, \lambda) = \lambda \cdot s(x) + (1 - \lambda) \cdot f(x) \)

- Discretize path from \( \lambda_0 = 1 \) to \( \lambda_f = 0 \)
- Application to optimization is more recent

\[
H(\lambda) = \min_{x} \{ \lambda \cdot s(x) + (1 - \lambda) \cdot f(x) \}
\]

- Convergence to a global minimum is not guaranteed for nonconvex problems

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Implementation of homotopy for contingency-OPF

Generator Outage

\[
P^g(\lambda_1) = P^{g,o}\lambda_1 + P^{g,f}\left(1_{|\mathcal{V}|} - \lambda_1\right)
\]

\[
Q^d(\lambda_2) = Q^{d,o}\lambda_2 + Q^{d,f}\left(1_{|\mathcal{V}|} - \lambda_2\right)
\]

Line Outage

\[
Y_{i,j}(\lambda) = G_{i,j}^0\lambda_1 + jB_{i,j}^0\lambda_2
\]

\[
\lambda^o = (1,1)
\]

\[
\lambda^f = (0,0)
\]
Some homotopy paths are more desirable

Scenario 1

Scenario 2

No homotopy  With homotopy

No homotopy  With homotopy

Two global solutions
Some homotopy paths are more desirable than others

If we can say that the global minimum is unique (& satisfies some conditions) for the homotopy OPF problems, then we can track the global minimum

- Assumes no degeneracy or infeasibility along the path

Families of parametric optimization problems generically have a unique global solution satisfying conditions

- Showed that this applies to contingency-OPF

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Homotopy method is effective in practice

3012-bus Polish network with different single line outages

Solution timeframe: ~2-5 minutes
Homotopy methods for COPF: conclusions

Need to solve many contingency-OPF problems to global optimality in a short period of time

Process:
- Defined a homotopy method that connects the Base OPF to COPF
- Each step of the homotopy problem is solved via fast local-search algorithm
- Characterized “good” homotopy path that will lead us to the global solution of COPF
- Applied to real-world networks

Demonstrated that the method results in significant violation cost reductions in about 10% of the hundreds of examined cases and no worse performance in the others
Part II: Power Systems State Estimation (SE)

*Project:* Optimizing sensor placement to ensure robustness in power system SE

*Mentor:* Prof. Somayeh Sojoudi
State estimation (SE) is critical for grid operation

- Supervisory Control & Data Acquisition (SCADA)
- State Estimation problem (every few mins)
- Real-time power dispatch
- Voltage control
- Contingency actions
Robust SE and the optimization of sensor placement

- Determine real-time state of power network → power systems state estimation (SE)
- Noisy or corrupted/attacked data → robust power systems SE

**Focus:** How to place sensors in a power network to optimize for robustness of power systems SE

**Approach:** Formulate a mixed-integer linear program (MILP) for measurement choice that optimizes a robustness condition

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Builds on linearized SE model\(^1\)

- Nonlinearity of AC power flow $\rightarrow$ SE is nonlinear, nonconvex $\rightarrow$ hard to solve!
- Two-stage model\(^1\) that can be solved to global optimality with local search methods

**Given:** Power network given as $\mathcal{G} = (\mathcal{N}, \mathcal{L})$, set of buses $\mathcal{N}$ and set of lines $\mathcal{L}$, measurement set $\mathcal{M}$

**Input:** Noisy, corrupted measurements $\mathbf{y} \in \mathbb{R}^m$, where $m = |\mathcal{M}|$

**Stage 1:** Solve SE problem using linearized basis $\mathbf{x} \in \mathbb{R}^n$ to get an estimate $\hat{\mathbf{x}}$.

**Stage 2:** Recover an estimate of underlying voltage vector $\hat{\mathbf{v}}$ from $\hat{\mathbf{x}}$.

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Linearized SE model & mutual incoherence

\[ \hat{x} = \arg\min_{x, b} \frac{1}{2|\mathcal{M}|} \| y - Ax - b \|_2^2 + \lambda \| b \|_1 \quad (SE) \]

- Mutual coherence is a measure of the cross-correlation of the columns of a matrix
- "Mutual incoherence" measures alignment of two submatrices in \( A \), one related to clean data, one to corrupted data

\[ \rho(\mathcal{B}) = \frac{\| A_{\mathcal{B}}^T A_{\mathcal{B}}^T \|_\infty}{\| A_{\mathcal{B}} \|_2} \]

If \( \rho(\mathcal{B}) < 1 \), then Stage 1 recovers \( \hat{x} \) with small error from \( x^* \) with high probability

\[ 1 \]

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Local certification of mutual incoherence

Instead of considering the bad data support (unknown), we consider a local condition.

Consider an attack on a single line \((i, j) \in \mathcal{L}\).

Local mutual incoherence condition

\[
\rho^{ij} := \left\| A_{M_i}^{T+} A_{M_j}^{T} A_{b_i}^{T} A_{b_j}^{T} \right\|_\infty < 1
\]
Measurement choice as a MILP

**Idea:** Optimize the choice of measurements in $\mathcal{M}$ such that $\rho^{ij} < 1$ for all $(i, j) \in \mathcal{L}$

Our method:

- **Consider all possible measurements**
  - network topology
  - available sensors

- **Partition measurements**
  - attacked region
  - boundary regions
  - safe region

- **Introduce binary measurement choice variable $\phi \in \{0,1\}^m$**

- **Formulate an optimization problem over $\phi$ to minimize $\beta$ where $\rho^{ij} \leq \beta$ for all $(i, j) \in \mathcal{L}$**

- **Relax nonlinear constraints and prove exact**

### Example Diagram

- Node 1: $|v_1|^2$ with $p_{12}$ and $p_{21}$
- Node 2: $|v_2|^2$ with $p_2$ and $p_{23}$
- Node 3: $|v_3|^2$ with $p_3$, $p_{34}$, and $p_{43}$
- Node 4: $|v_4|^2$
Simulations on IEEE test cases

<table>
<thead>
<tr>
<th>Network</th>
<th>Fraction of meas.</th>
<th>$\beta = \max \rho^{ij}$</th>
<th>Solve time (s)</th>
<th>Fraction of meas.</th>
<th>Lines where $\rho^{ij} &lt; 1$</th>
<th>Solve time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>case5</td>
<td>29 / 39</td>
<td>1.26</td>
<td>0.69</td>
<td>30 / 39</td>
<td>6 / 12</td>
<td>1.89</td>
</tr>
<tr>
<td>case9</td>
<td>42 / 57</td>
<td>1.48</td>
<td>1.12</td>
<td>36 / 57</td>
<td>12 / 18</td>
<td>1.49</td>
</tr>
<tr>
<td>case14</td>
<td>95 / 120</td>
<td>1.61</td>
<td>9.33</td>
<td>92 / 120</td>
<td>18 / 40</td>
<td>120.5</td>
</tr>
<tr>
<td>case30</td>
<td>193 / 248</td>
<td>1.61</td>
<td>39.3</td>
<td>190 / 248</td>
<td>37 / 82</td>
<td>831.1</td>
</tr>
</tbody>
</table>

No choice of measurements such that all lines are robust in case of attack!

However, we can find subsets of measurements that are more optimal than others in terms of SE robustness.
Measurement choice: conclusions

Having more lines satisfy the mutual incoherence condition **guarantees** a reduction in the impact of the attack on power system SE for nodes far from the attacked region.
Part III: Power Flow (PF) Mapping Problem

*Project*: Learning the power system topology using a data-driven, physics-informed optimization

*Mentor*: Prof. Somayeh Sojoudi
Uncertain topology $\rightarrow$ problems for most PF methods

Uncertain topology in WB5 network

\[ Y = \frac{1}{Z} \]

Uncertain topology = parameter uncertainty
Exploiting large datasets to learn topology

Data collection from SCADA and PMU sources

PMU data
|v_i|, \theta_i

PMUs collect \(\sim30\) to 60 samples/sec

SCADA data
\(p_{ij}, q_{ij}\)
\(p_i, q_i\)

SCADA systems collect
\(~1\) sample every 4 sec

Previous Approaches: Neural networks (NN), maximum likelihood estimation, support vector regression (SVR)\(^1,2\) → overfitting + ignore physics!

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\(^1\) J. Yu, Y. Weng, and R. Rajagopal, “Robust mapping rule estimation for power flow analysis in distribution grids,” in 2017 NAPS.

Data-driven approach to learn topology

- **Goal:** Recover the underlying power system topology from system data
  - “Topology” = network connectivity & line parameters
  - Robust in the presence of outliers and noise in data

- **Our Approach:** Design a constrained support vector regression (SVR) problem
  - Approach allows exact representation of the true AC power system & its inherent sparsity
  - Can efficiently solve SVR optimization problem with off-the-shelf quadratic program (QP) solvers or tailored algorithm

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Background on SVR

Idea: Find linear estimator that maximizes data proximity to plane

Kernel trick for nonlinear mappings
Power flow (PF) mapping as constrained SVR

**Idea:** Create new SVR formulation with constraints that represent network sparsity

Our method:

- Formulate PF mapping *exactly* as quadratic kernel
  
  \[ K(x_1, x_2) = ((x_1, x_2))^2 = \phi(x_1)^T \phi(x_2) \]

  for carefully chosen state \( x \in \mathbb{R}^{2n} \)

  where \( n \) is the number of buses equipped with PMUs

- Define SVR problem with multiple types of SCADA measurements

- Define sparsity pattern for power network \( \rightarrow \) add as constraint to SVR

  \[ K(x_1, x_2) = ((x_1, x_2))^2 = \phi(x_1)^T \phi(x_2) \]

  \[ p_{ij} = \langle \mu_{p_{ij}}, \phi(x) \rangle, \quad q_{ij} = \langle \mu_{q_{ij}}, \phi(x) \rangle \]

- State equation model:
  \[ y_t = W \phi(x_t) \]
  
  for time steps \( t \in \{1, ..., T\} \)

- Show that the constrained SVR and its dual are both convex quadratic programs (QPs)

- Controls the structure of \( W \)
Simulations with SCADA + PMU errors

14-bus IEEE network

- Classic SVR
- Constrained SVR

Normalized Estimation Error (Mutual Conductance)

Average Solution Time (s)
Simulations with varying PMU penetration

30-bus IEEE network

![Graphs showing performance metrics vs. PMU penetration.](image-url)
Summary & conclusions

- Proposed a new constrained SVR method that can exactly learn the true power network topology in the case without noise
  - Method has high accuracy in the cases with measurement noise and/or outliers and varying levels of PMU penetration
  - Performs much better than state-of-the-art methods in terms of line parameter recovery and solution time

Improved power grid reliability, security, and efficiency

New infrastructure
More data
Increased demand

New methods & algorithms

Efficient methods for nonconvex AC power flow problems
Acknowledgements & collaborations

Research advisor: Prof. Somayeh Sojoudi
Mechanical Engineering, EECS (joint)

Collaborator: Prof. Javad Lavaei
Industrial Engineering & Operations Research

Collaborator: SangWoo Park
Industrial Engineering & Operations Research

Mentor: Jean-Paul Watson
Lawrence Livermore National Laboratory (LLNL)
Acknowledgements & collaborations

Current & previous members of Prof. Sojoudi’s research group
Thank you!

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The worst-case local minimum

\[
\begin{align*}
\max_{x \in \mathbb{R}^n} & \quad x^T M_0 x + k \\
\text{s.t.} & \quad x \in \{\text{local minima of OPF}\}
\end{align*}
\]

Lagrange multiplier \( \lambda \in \mathbb{R}^p \), \((x^*, \lambda^*)\) corresponding to local minima:

**First-order conditions**

\[ 0 = \nabla_x L(x^*, \lambda^*) = 2M_0 x^* + 2 \sum_{i=1}^{p} \lambda_i^* M_i x^* \]

\[ (x^*)^T M_i x^* = a_i \quad \forall i = 1, \ldots, p \]

**Second-order necessary condition**

\[ y^T (\nabla_{xx}^2 L(x^*, \lambda^*)) y \geq 0 \]

for all \( y \) such that \( y^T M_i x^* = 0 \), \( \forall i = 1, \ldots, p \)

where \( \nabla_{xx}^2 L(x^*, \lambda^*) = 2M_0 + 2 \sum_{i=1}^{p} \lambda_i^* M_i \)
The worst-case local minimum

\[ \max_{x \in \mathbb{R}^n} \quad x^T M_0 x + k \]
\[ \text{s.t. } x \in \{ \text{local minima of OPF} \} \]

Lagrange multiplier \( \lambda \in \mathbb{R}^p \), \((x^*, \lambda^*)\) corresponding to local minima:

**First-order conditions**

\[ 0 = \nabla_x L(x^*, \lambda^*) = 2M_0 x^* + 2 \sum_{i=1}^{p} \lambda_i^* M_i x^* \]
\[ (x^*)^T M_i x^* = a_i \quad \forall i = 1, \ldots, p \]

**Alternative second-order necessary condition**

\[ M_0 + \sum_{i=1}^{p} \lambda_i^* M_i + c \sum_{i=1}^{p} M_i (x^*) (x^*)^T M_i \geq 0 \]

for some \( c \) above a certain threshold, \( c > \bar{c} \)
The worst-case local minimum

\[
\max_{x \in \mathbb{R}^n, \lambda \in \mathbb{R}^p} \quad x^T M_0 x + k \\
\text{s.t.} \quad x^T M_i x = a_i \quad \forall i = 1, \ldots, p \\
(M_0 + \sum_{i=1}^p \lambda_i M_i)x = 0 \\
M_0 + \sum_{i=1}^p \lambda_i M_i + c \sum_{i=1}^p M_i x x^T M_i \succeq 0
\]

for some \( c \) above a certain threshold, \( c > \bar{c} \)

- Nonconvex
- Any upper bound on the problem will also upper bound the worst-case local minimum
- Use a relaxation of the problem into a semidefinite program (SDP)
SDP relaxation of the worst-case local min

Define a matrix $W \in \mathbb{S}^{n+p+1}$ as:

$$W = \begin{bmatrix} 1 & x^T & \lambda^T \\ x & x^T & \lambda^T \\ \lambda^T & \lambda x^T & \lambda \lambda^T \end{bmatrix}$$

$$W \succeq 0 \quad W_{11} = 1 \quad \text{rank}(W) = 1$$

$$
\begin{align*}
\max_{W \in \mathbb{S}^{n+p+1}, W \succ 0, W_{11}=1} & \quad \text{trace}\{M_0 W_{22}\} + k \\
\text{s.t.} & \quad \text{trace}\{M_i W_{22}\} = a_i \quad \forall i = 1, \ldots, p \\
& \quad M_0 W_{21} + \sum_{i=1}^{p} M_i (W_{23})_i = 0 \\
& \quad M_0 + \sum_{i=1}^{p} M_i (W_{31})_i + c \sum_{i=1}^{p} M_i W_{22} M_i \succeq 0 \\
& \quad \text{trace}\{M_0 W_{22}\} + \sum_{i}^{p} a_i (W_{31})_i = 0
\end{align*}
$$

for some $c$ above a certain threshold, $c > \bar{c}$
Simulations on realistic networks

Power generation costs:

\[ f_i(p_i^g) = c_{i2}(p_i^g)^2 + c_{i1}p_i^g + c_{i0} \]

\[ x = [\text{Re}\{v\}^T \text{Im}\{v\}^T (p^g)^T (q^g)^T]^T \]

Add a penalty: \( \epsilon \cdot (\text{trace}\{W\} - \text{trace}\{W_{22}^{pq}\}) \)

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3375-bus Polish network with single generator outage
Measurement choice problem v1

Objective: minimize maximum $\rho^{ij}$

**Constraints on number of measurements**

\[
\sum_{i=1}^{M} \phi_i \leq \bar{M}
\]

**Constraints on measurement dependencies** (substitute for rank constraint)

\[
\begin{align*}
\sum_{i=1}^{m} R_{kr} Z_{rl} &= S_{kl}^T \phi [\tilde{M}_{db} (l)], & \forall k &\in [n_b], \forall l &\in [m_{db}] \\
Y_{rl}^{ij} &\geq \max \{-Z_{rl}^{ij}, Z_{rl}^{ij}\}, & \forall r &\in [m_{ib}], \forall l &\in [m_{db}] \\
Y_{rl}^{ij} &\leq C [\tilde{M}_{ib} (r)], & \forall r &\in [m_{ib}], \forall l &\in [m_{db}] \\
\sum_{l=1}^{m_{db}} Y_{rl}^{ij} &\leq \beta, & \forall r &\in [m_{ib}]
\end{align*}
\]

**Definition of $\rho^{ij}$**

\[
\rho^{ij} = \| Z^{ij} \|_{\infty}
\]

\[
R^{ij} := A_{M_{ib} \times M_{db} \times X}^{T ij} A_{M_{db} \times X}^{ij}
\]

\[
S^{ij} := A_{M_{db} \times X}^{T ij} A_{M_{db} \times X}^{ij}
\]

\[
Y^{ij} = |Z^{ij}| 
\]
Measurement choice problem v2

\[
\min_{\phi \in \{0,1\}^m} \sum_{(i,j) \in \mathcal{L}} 1\{\beta^{ij} \geq 1\}
\]

Objective: minimize number of violations of \(\rho^{ij} > 1\)

s.t. \(M \leq \sum_{i=1}^m \phi_i \leq M\)

\[\sum_{i=1}^{\Phi_x} \Phi_x[i] \geq 1, \quad \forall x \in \mathcal{X}\]

\[\forall (i,j) \in \mathcal{L} : \]

\[\sum_{r=1}^{m_{ib}} R_{kr}^{ij} Z_{rl}^{ij} = S_{kl}^{ij} \Phi[\tilde{M}_{db}(l)], \quad \forall k \in [n^{ij}], \forall l \in [m_{db}^{ij}]\]

\[Y_{rl}^{ij} \geq \max\{-Z_{rl}^{ij}, Z_{rl}^{ij}\}, \quad \forall r \in [m_{ib}^{ij}], \forall l \in [m_{db}^{ij}]\]

\[Y_{rl}^{ij} \leq C \Phi[\tilde{M}_{ib}(r)], \quad \forall r \in [m_{ib}^{ij}], \forall l \in [m_{db}^{ij}]\]

\[\sum_{l=1}^{m_{db}} Y_{rl}^{ij} \leq \beta^{ij}, \quad \forall r \in [m_{ib}^{ij}]\]

\(\beta^{ij}\) represents \(\rho^{ij}\)
Sensor placement for robust SE: summary & conclusions

- Novel framework to **formally optimize the placement of sensors** in a power network in order to satisfy a condition for SE robustness

- Method:
  - Leveraged a linearized SE framework and the concept of local partitioning
  - Defined a MILP that optimizes the local mutual incoherence metric for each line in the network

- Can be used to place new sensors in an existing legacy power network in order to improve SE robustness

- Could be used to classify the measurements that are most susceptible to error propagation