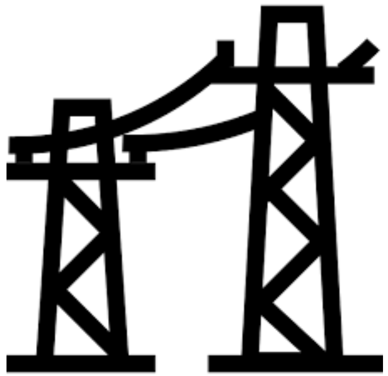
Optimizing for the future smart grid:
Efficient methods for nonconvex AC power flow problems

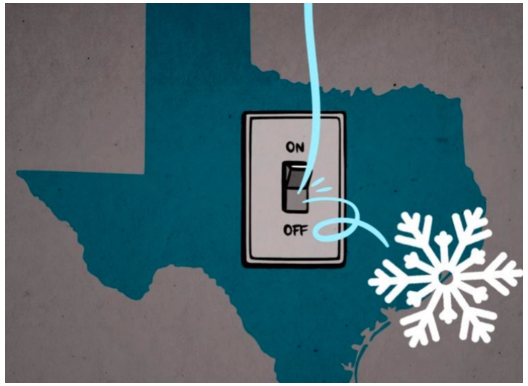
Elizabeth Glista



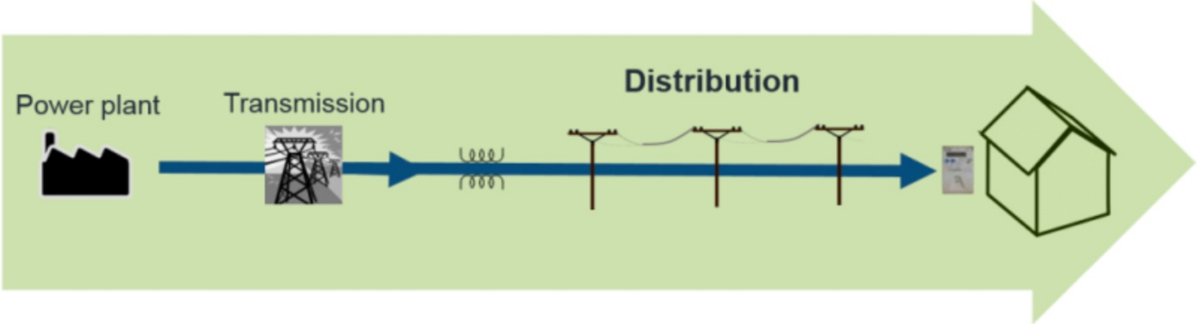
Berkeley Mechanical Engineering

April 28, 2023

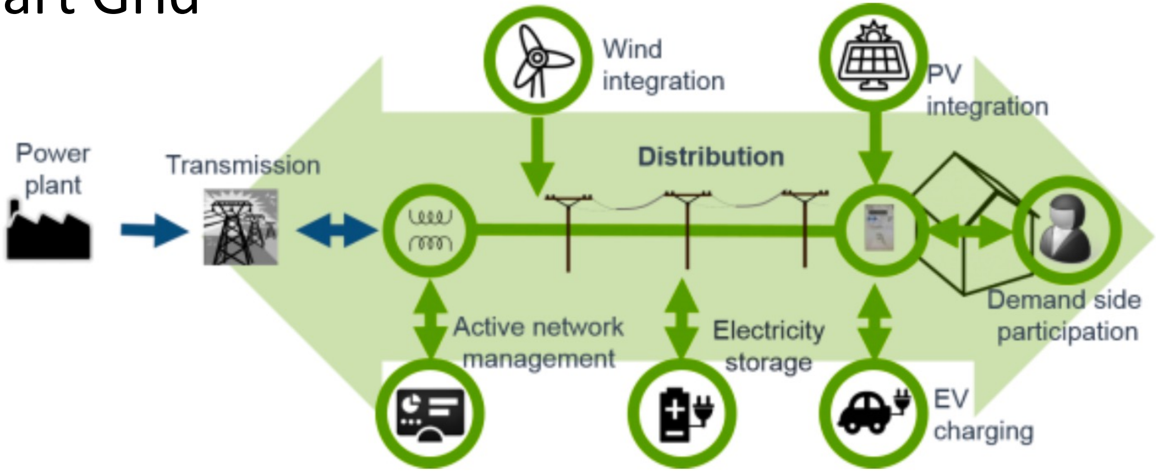
New challenges for the smart grid



Traditional Grid

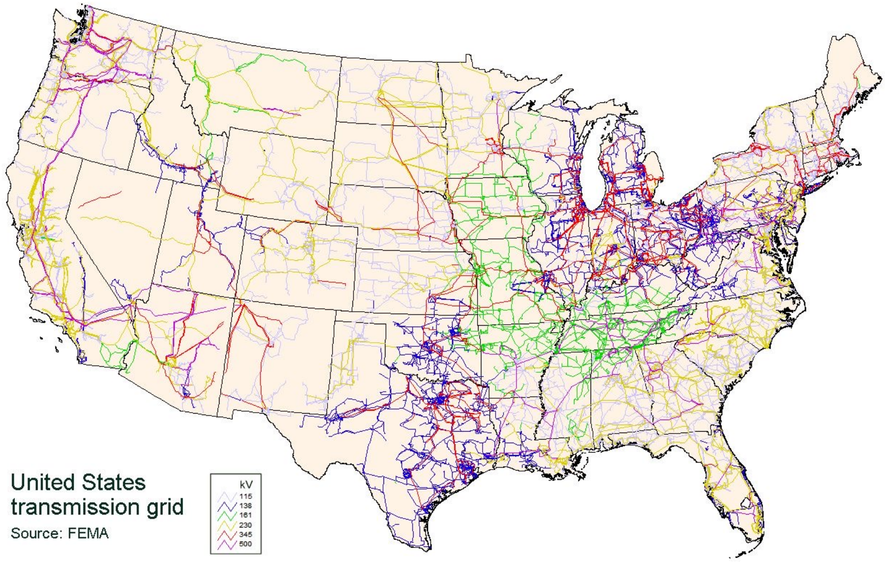


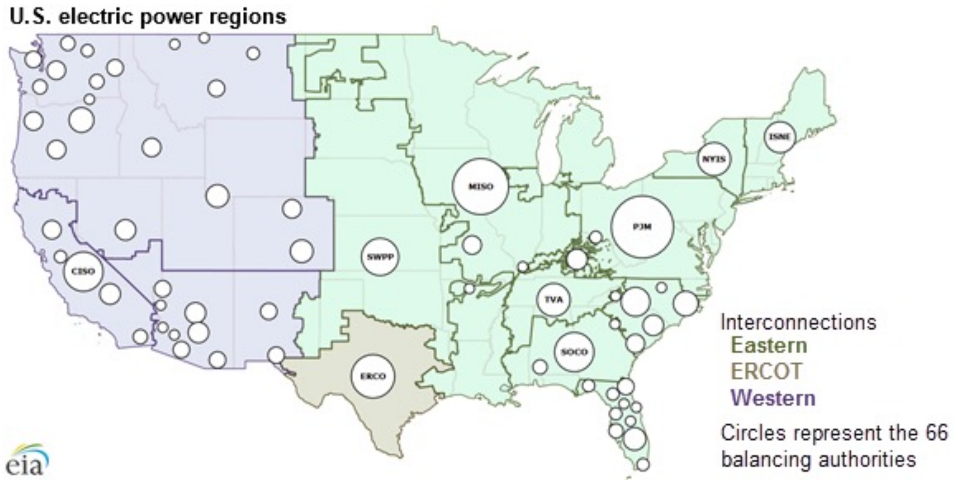
Smart Grid



New infrastructure
More data
Increased demand
More cyclic structure

Fundamental challenges in power system optimization





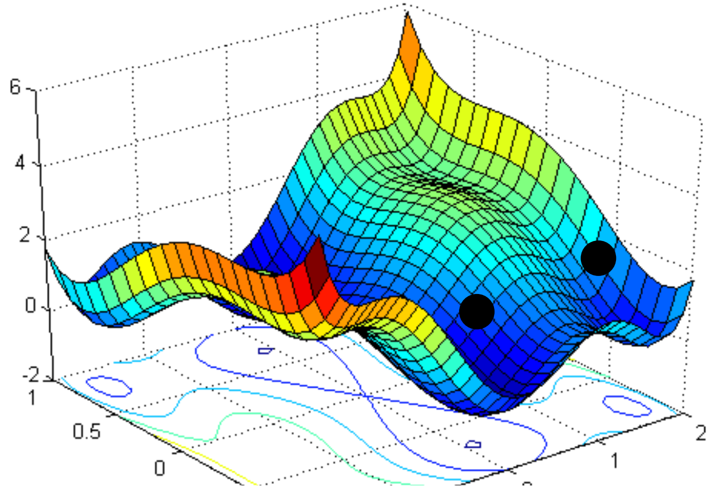
Scalability

- 145 million customers
- Over 7,300 power plants
- 160,000 miles high-voltage power lines

In practice

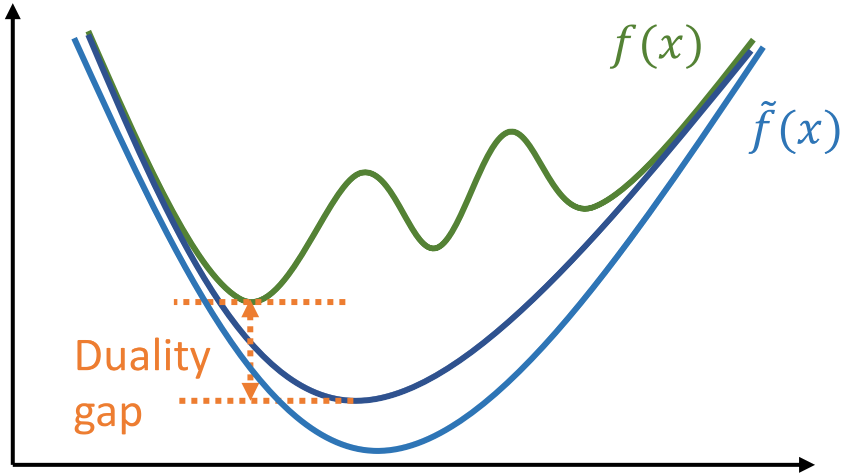
- Divided into smaller regions
- Poor local decisions → cascading failures in interconnected network

Fundamental challenges in power system optimization



Optimality

- Nonlinear nature of alternating current (AC) power flow → many problems are nonconvex
- Difference between local and global solutions is estimated at billions of $ annually in the US (source: FERC)



In practice

- Optimization stage: Linearize power flow equations (DC approximation)
- Use heuristics to generate feasible AC solution
- New interest in conic relaxations that have global guarantees

Goals of my PhD thesis work

- Take a cross-disciplinary approach to solve important problems in power systems optimization
  - Focus on both fundamental problems and new problems

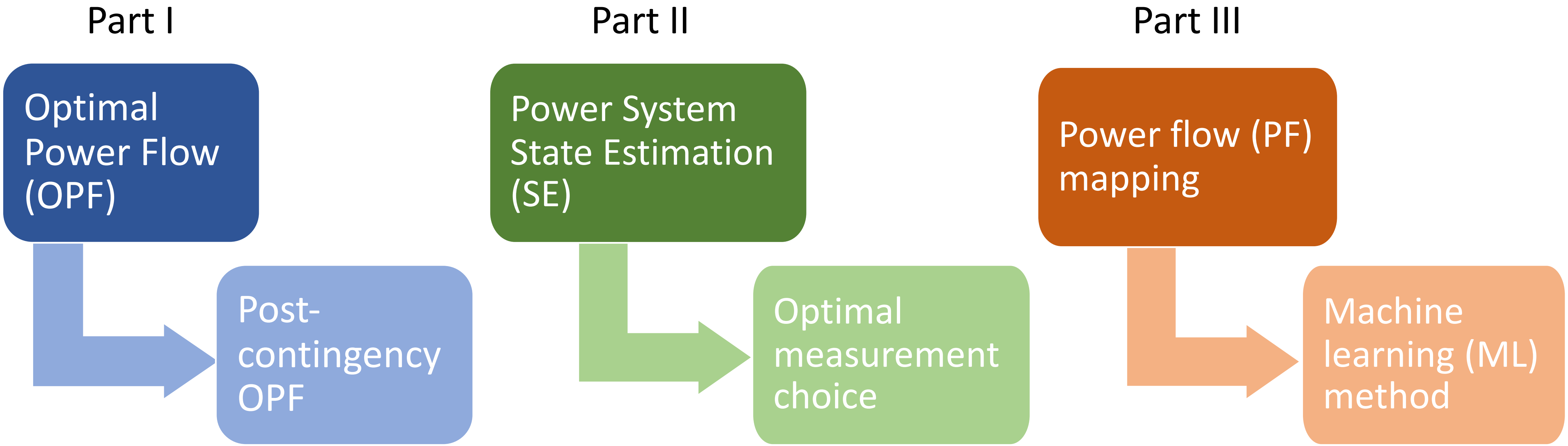
- Synthesize advanced methods from optimization and mathematics
  - Algebraic geometry, graph theory, numerical methods
  - Domain-specific understanding: the physics of power flow, sparse graph structure[1]

- Leverage novel theory to develop new algorithms

[1] S. Sojoudi and J. Lavaei, "Physics of power networks makes hard optimization problems easy to solve," 2012.

Underlying + new power system problems

Many problems in power systems planning and operation are based on fundamental problems



Part I: Optimal Power Flow (OPF)

Two projects:

1) Finding the worst-case local minimum of OPF
2) Finding the global solution to a post-contingency OPF

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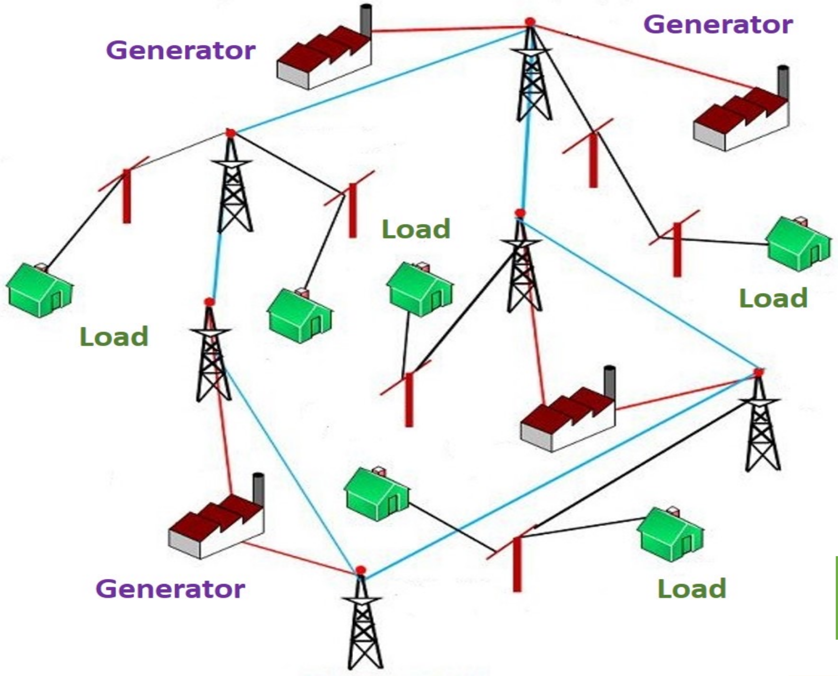
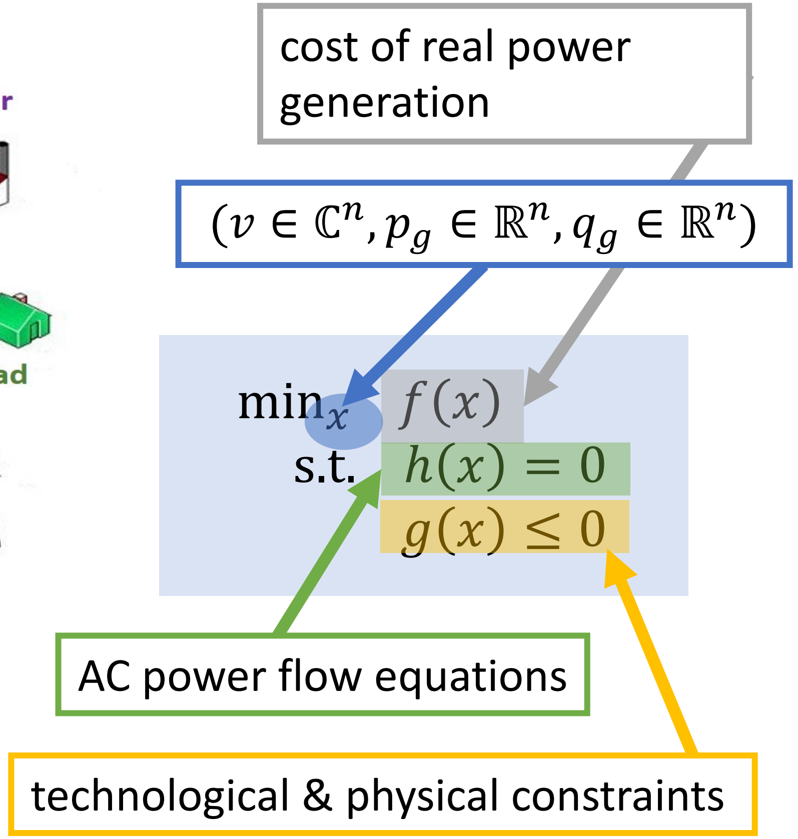
Mentor: Prof. Somayeh Sojoudi

Optimal Power Flow (OPF)

- Goal: find minimum cost production of committed generating units
  - While satisfying technological and physical constraints
- Existing methods
  - **Local:** Interior Point Methods (IPM), Sequential Quadratic Programming (SQP)
  - **Global:** Convex relaxations (SDP, SOCP)[1,2,3]

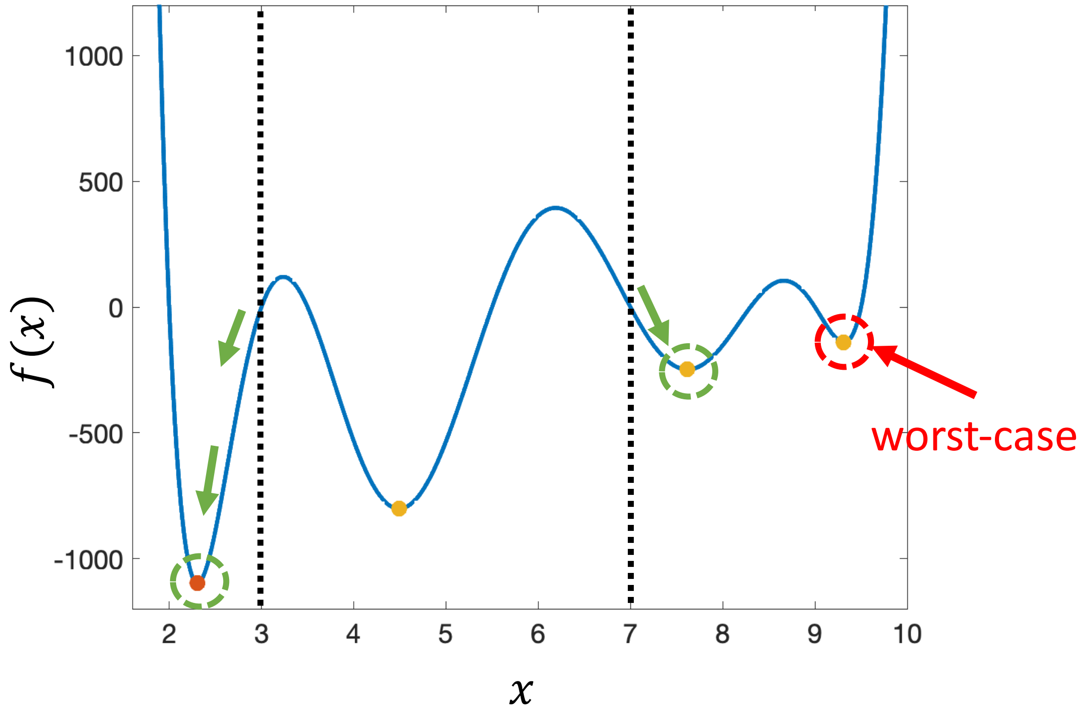




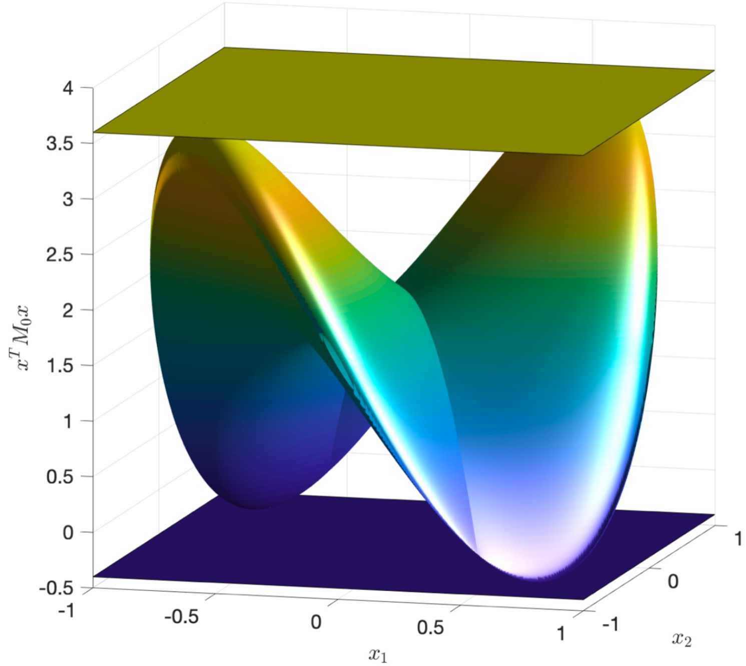
[1]X. Bai et al. "Semidefinite programming for optimal power flow problems," 2008.
[2]J. Lavaei and S. Low, "Zero Duality Gap in Optimal Power Flow Problem," 2012.
[3]W. Bukhsh et al. "Local Solutions of the Optimal Power Flow Problem," 2013.

Finding the worst-case local minimum[1]

Local search method



Goal: Bound the worst-case performance of a generic local search solver



Example QCQP

$$\min_{x \in \mathbb{R}^n} \; x^T M_0 x \quad \text{s.t.} \; x^T x = 1$$

[1]**E. Glista** and S. Sojoudi, "Convex model to evaluate worst-case performance of local search in the optimal power flow problem," *IEEE 59th Conference on Decision and Control*, 2020.

The worst-case local minimum

❖ **Idea:** Create an upper bound on the worse-case local minimum

Our method:

Construct a new "worst-case local min" problem

$$\max_{x \in \mathbb{R}^n} \quad x^T M_0 x + k$$
$$\text{s.t.} \quad x \in \{\text{local minima of QCQP}\}$$

Feasible set given by first- and second-order optimality conditions

$$0 = \nabla_x L(x^*, \lambda^*) = 2M_0 x^* + 2\sum_{i=1}^{p} \lambda_i^* M_i x^*$$

$$(x^*)^T M_i x^* = a_i, \forall i = 1, \ldots, p$$

$$M_0 + \sum_{i=1}^{p} \lambda_i^* M_i + c \sum_{i=1}^{p} M_i (x^*)(x^*)^T M_i \succcurlyeq 0$$

for some c above a certain threshold, $c > \bar{c}$

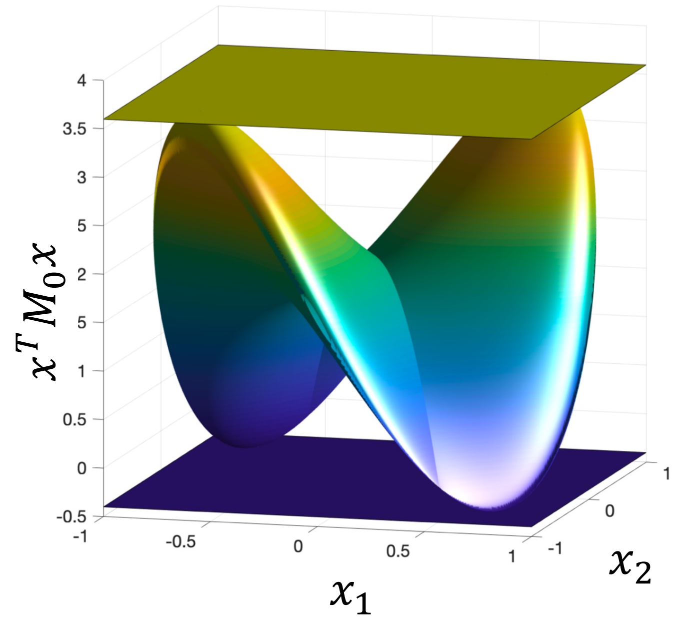
Take a convex relaxation of the "worse-case" problem into a semidefinite program (SDP)

Upper bound on problem

Show that tightness of SDP depends on choice of parameter *c*

Exactness of SDP relaxation

Example QCQP	$\min_{x\in\mathbb{R}^n} \; x^T M_0 x$ s.t. $\; x^T x = 1$



- If we take $c = 0$ in the SDP relaxation, the SDP relaxation is *exact* (thus its solution is the worst-case local minimum)
- For $c > 0$, the SDP relaxation is *not exact*

Choice of parameter c:

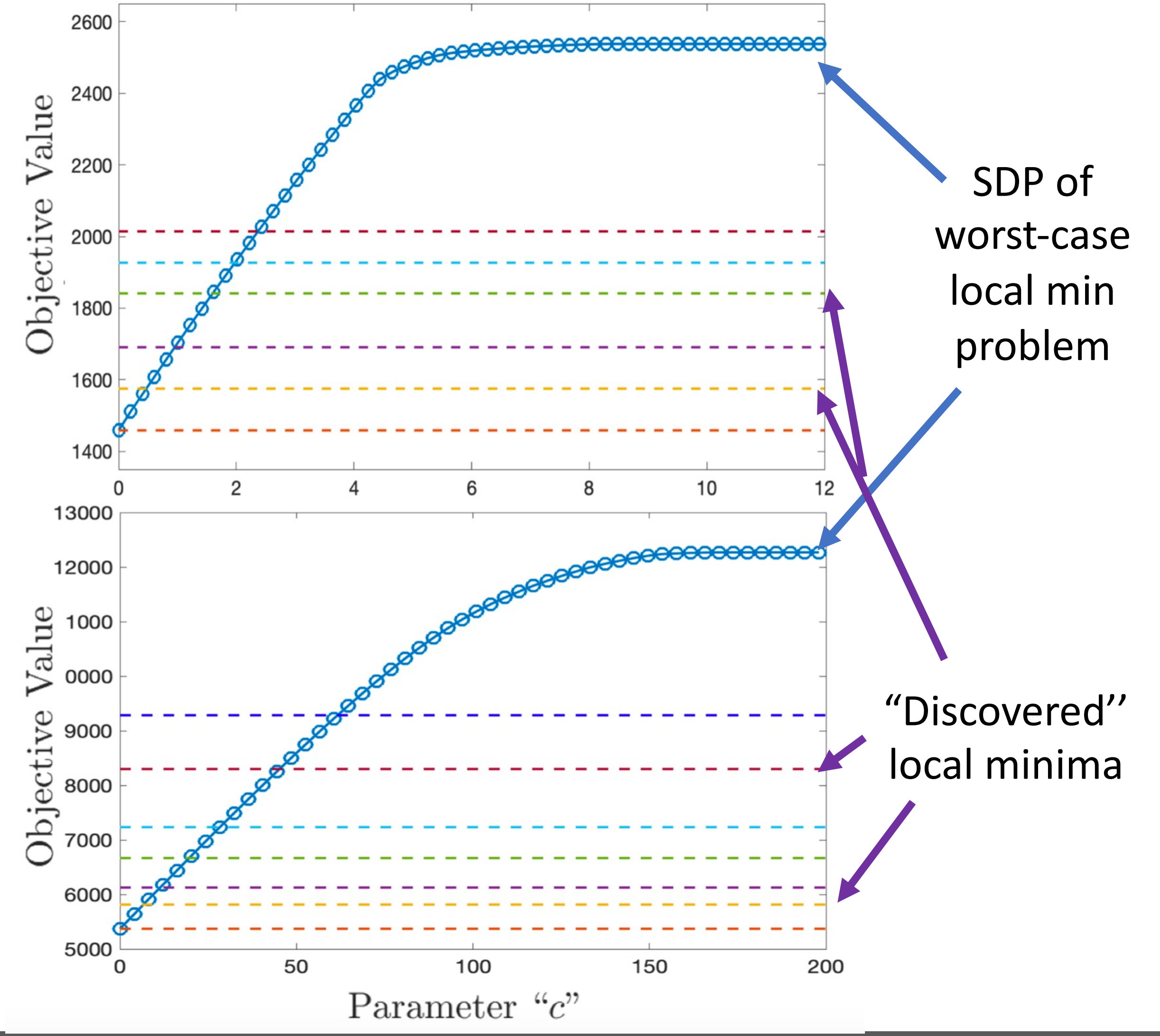
- Exact value of c is not needed for the SDP relaxation
- Selecting too large of a $c \implies$ larger *optimality gap* between the SDP relaxation & the original worst-case local min problem

- Also looked at introducing a penalty term to the objective to get tight SDP relaxation

Simulations on realistic networks

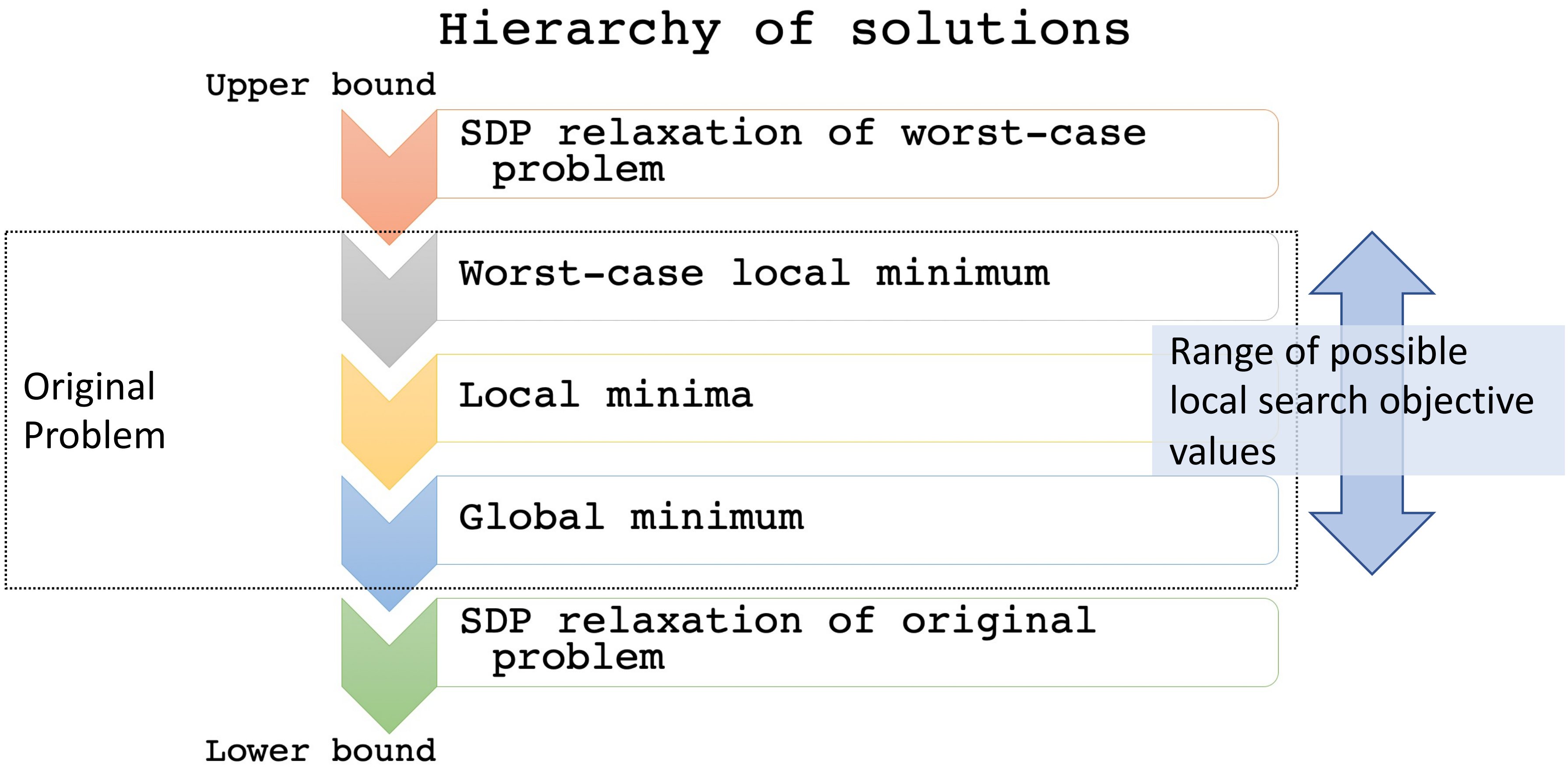
IEEE 9-bus network

IEEE 14-bus network



Solution timeframe:
~ 3-5 minutes

Relation to existing methods



Compare the **SDP worst-case upper bound** with the **SDP lower bound** to obtain bounds on the range of possible objective values obtained with local search

Worst-case local min: summary & conclusions

- Formulated a new problem to find the worst-case local minimum for a canonical QCQP (e.g. OPF)
  - Since this problem is still nonconvex, use an SDP relaxation to find an upper bound

- Find that the tightness of the upper bound depends on the choice of a parameter in the second-order necessary optimality condition

- Method provides a metric on how much SDP can outperform local search → evaluate the performance of the ***whole class of local search methods***

Part I: Optimal Power Flow (OPF)

Two projects:

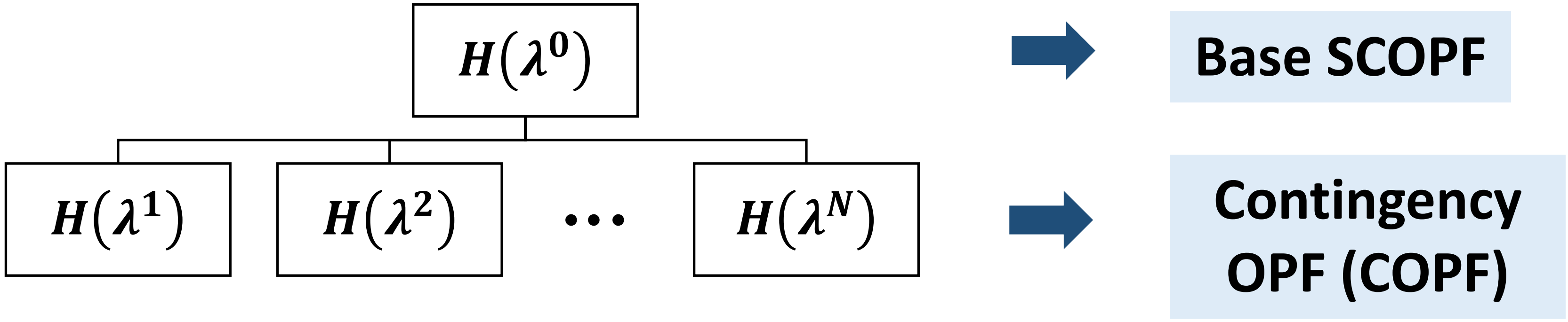
1) Finding the worst-case local minimum of OPF
2) Finding the global solution to a post-contingency OPF

Mentors: Prof. Somayeh Sojoudi, Prof. Javad Lavaei
Collaborator: SangWoo Park

Parametric OPF for post-contingency analysis[1]

$$\boldsymbol{H(\lambda)} \qquad \begin{aligned} \min_x \quad & f(x,\lambda) \\ \text{s.t.} \quad & h(x,\lambda) = 0 \\ & g(x,\lambda) \le 0 \end{aligned}$$

Want to efficiently solve coupled post-contingency OPF problems to global optimality given the solution to the base case



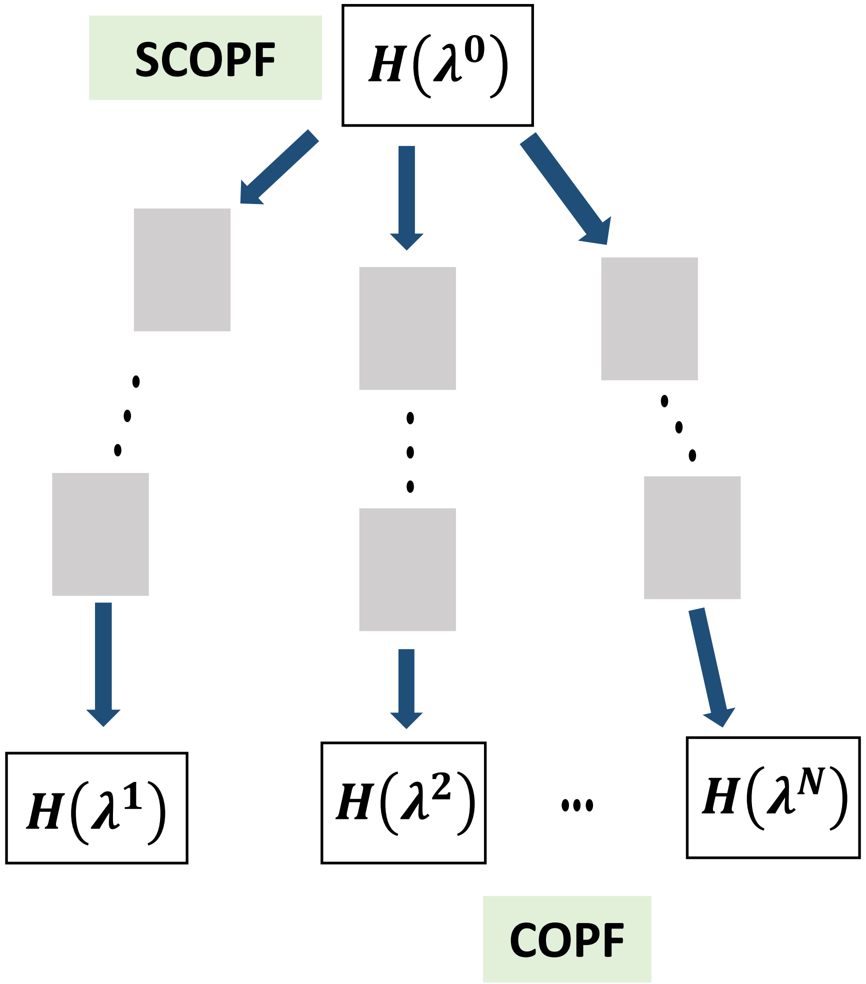
Base SCOPF problem approximates the contingencies but does not explicitly solve for them

[1]S. Park, **E. Glista**, J. Lavaei, and S. Sojoudi, "Homotopy method for finding the global solution of post-contingency optimal power flow," *American Control Conference*, 2020. *Won the Best Student Paper Award.*

Homotopy method to solve contingency-OPF

- **Goal:** Efficiently find the global solution to all the variations, given a global solution to the base problem

- **Approach:** Design sequences of intermediate problems that connect the base problem to each of its variations



Background on homotopy methods

- Homotopy and continuation methods have long been used in mathematics & engineering to solve systems of nonlinear equations
  - Power systems: continuation power flow[1]

"Easy" problem: $s(x) = 0$ → "Hard" problem: $f(x) = 0$

Define $H(x, \lambda) = \lambda \cdot s(x) + (1 - \lambda) \cdot f(x)$

- Discretize path from $\lambda_0 = 1$ to $\lambda_f = 0$
- Application to optimization is more recent[2,3]

$$H(\lambda) = \min_x \{\lambda \cdot s(x) + (1 - \lambda) \cdot f(x)\}$$

- Convergence to a global minimum is **not guaranteed** for nonconvex problems

[1] D. Mehta et al., "Numerical polynomial homotopy continuation method to locate all the power flow solutions," 2016.
[2] L.T. Watson and R.T. Haftka, "Modern homotopy methods in optimization," 1989.
[3] D.M. Dunlavy and D.P. O'Leary, "Homotopy optimization methods for global optimization," 2005.

Implementation of homotopy for contingency-OPF

Generator Outage

$$P^g(\lambda_1) = P^{g,o} \odot \lambda_1 + P^{g,f} \odot (\mathbf{1}_{|\mathcal{V}|} - \lambda_1)$$
$$Q^d(\lambda_2) = Q^{d,o} \odot \lambda_2 + Q^{d,f} \odot (\mathbf{1}_{|\mathcal{V}|} - \lambda_2)$$

power generated in base-case

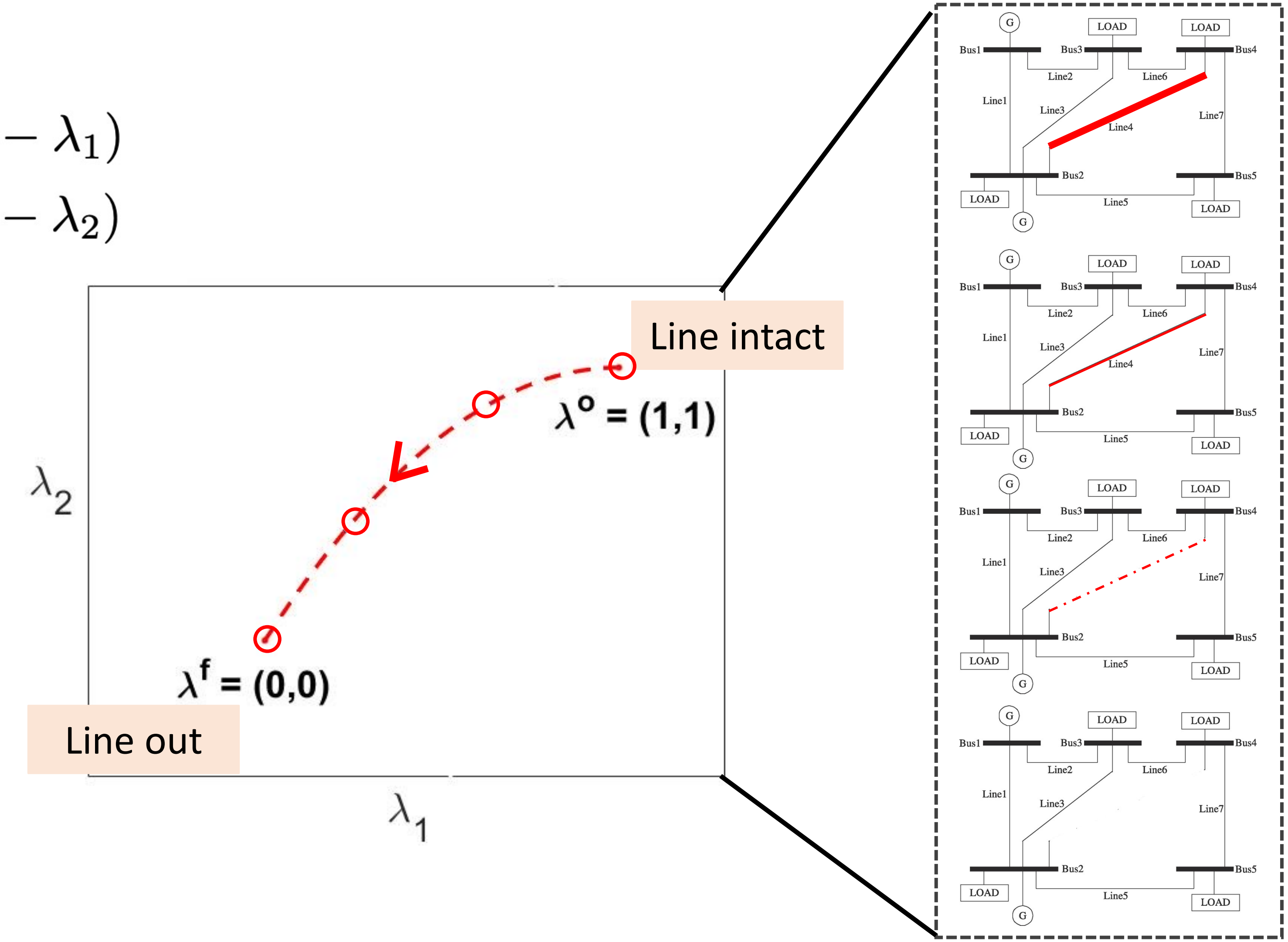
power generated in contingency (based on participation factors)

Line Outage

$$Y_{ij}(\lambda) = G^0_{ij}\lambda_1 + \mathbf{j}B^0_{ij}\lambda_2$$

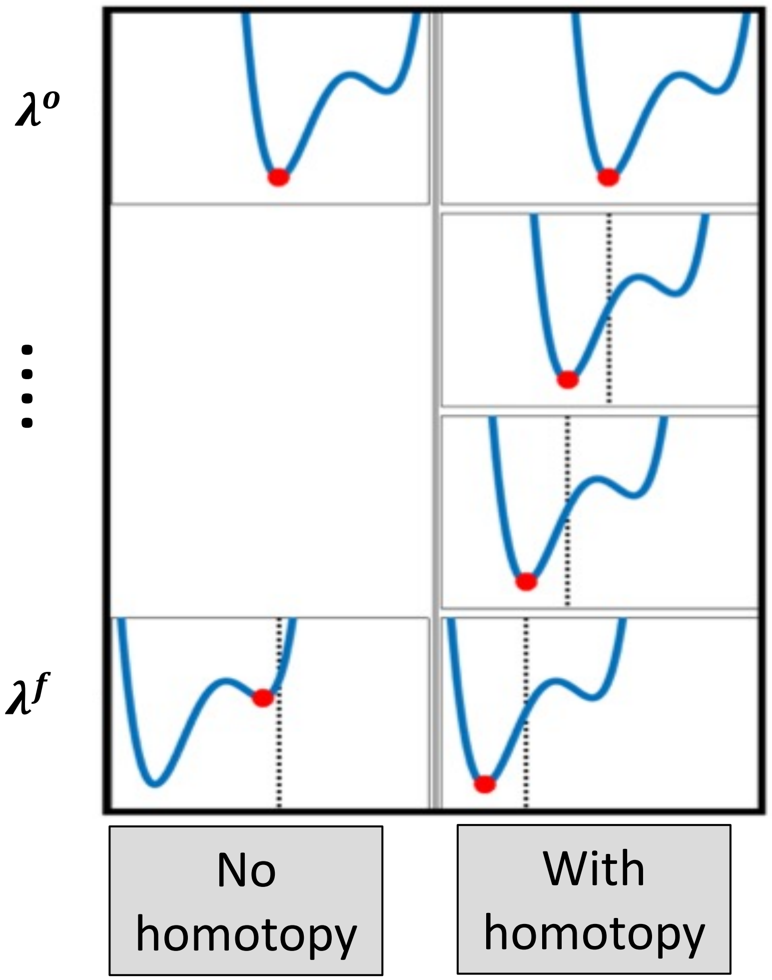
conductance

susceptance

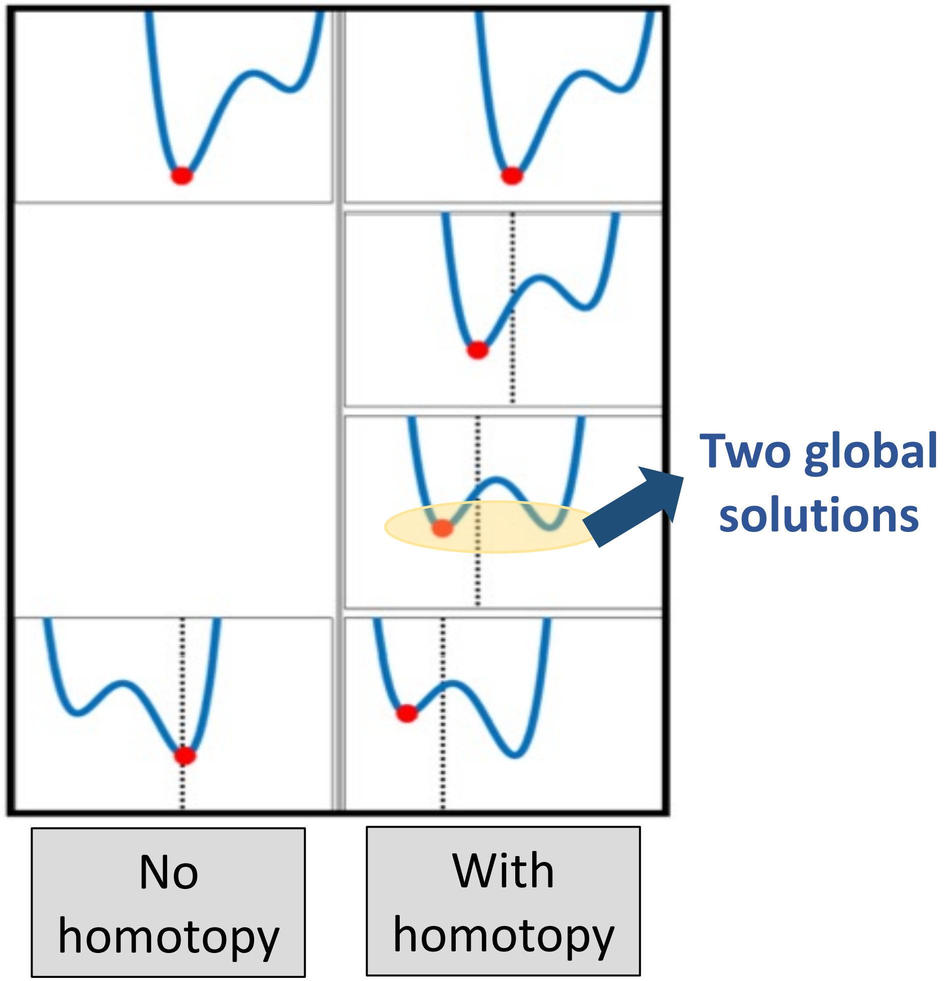


Some homotopy paths are more desirable

Scenario 1



Scenario 2

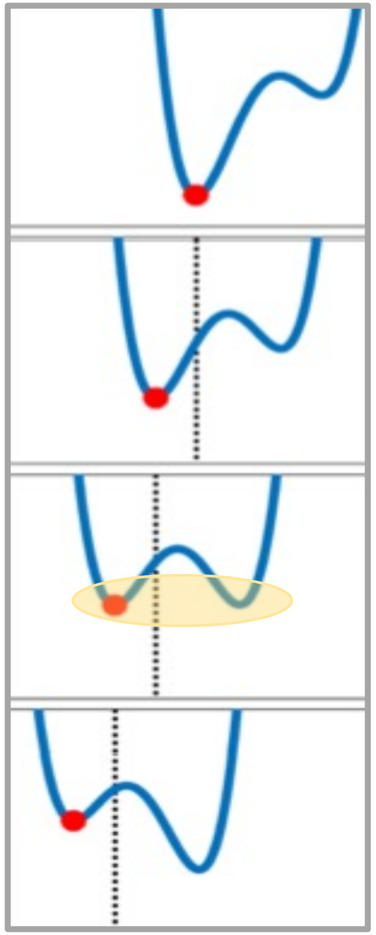


Theory to characterize desirable homotopy paths

- Some homotopy paths are more desirable than others

- If we can say that the global minimum is unique (& satisfies some conditions) for the homotopy OPF problems, then we can track the global minimum[1]
  - Assumes no degeneracy or infeasibility along the path

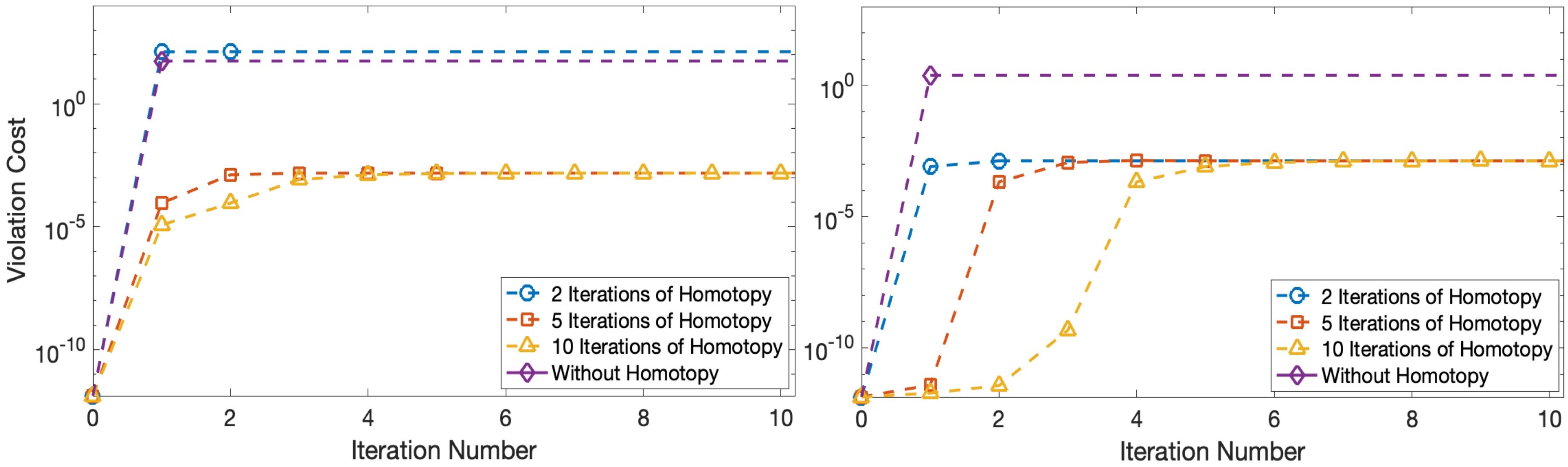
- Families of parametric optimization problems generically have a unique global solution satisfying conditions
  - Showed that this applies to contingency-OPF[1]



[1]S. Park, **E. Glista**, J. Lavaei, and S. Sojoudi, "An efficient homotopy method for solving the post-contingency optimal power flow to global optimality," *IEEE Access*. November 2022.

Homotopy method is effective in practice

3012-bus Polish network with different single line outages



Solution timeframe:
~ 2-5 minutes

Homotopy methods for COPF: conclusions

Need to solve many contingency-OPF problems to global optimality in a short period of time

Process:

- Defined a **homotopy method** that connects the Base OPF to COPF
- Each step of the homotopy problem is solved via **fast local-search algorithm**
- Characterized "good" **homotopy path** that will lead us to the global solution of COPF
- Applied to real-world networks

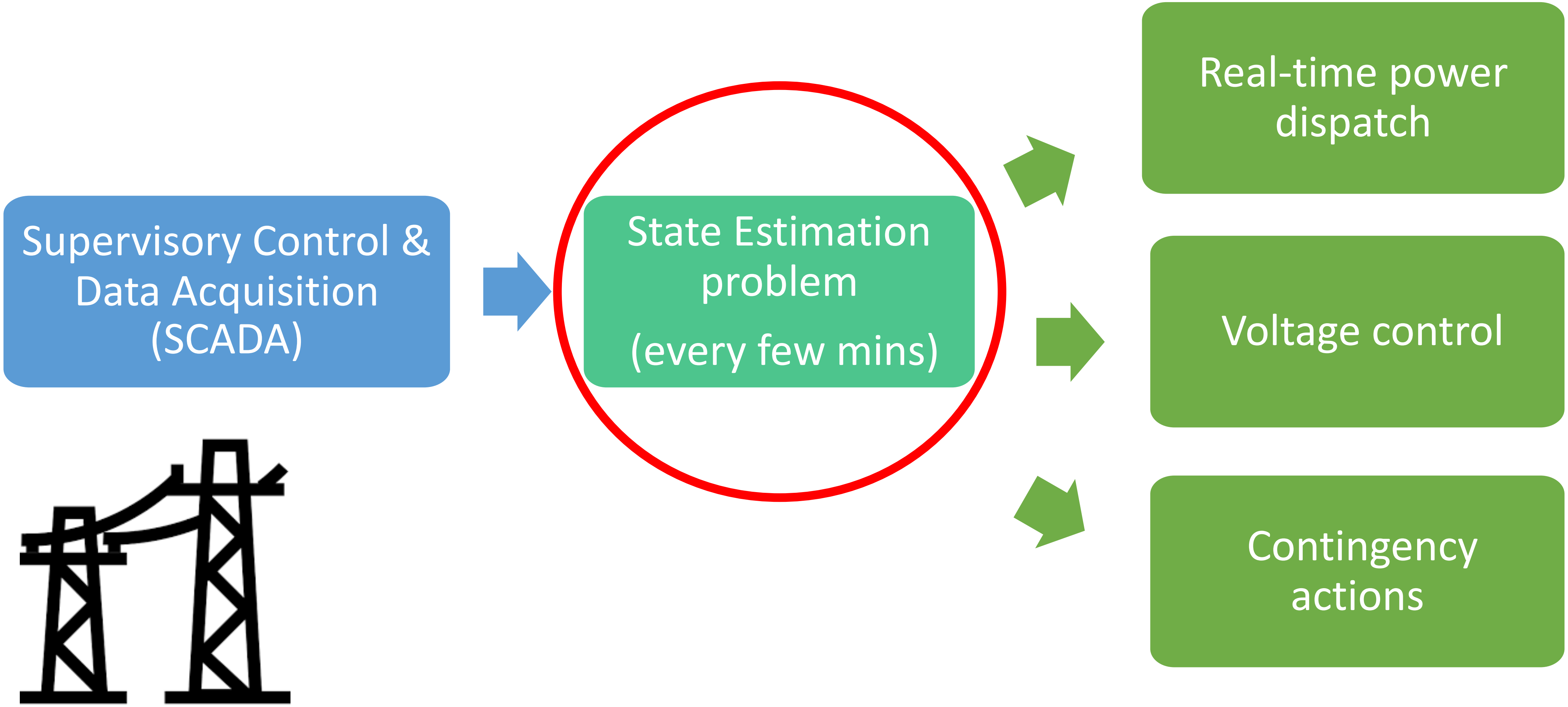
Demonstrated that the method results in ***significant violation cost reductions*** in about 10% of the hundreds of examined cases and no worse performance in the others

Part II: Power Systems State Estimation (SE)

Project: Optimizing sensor placement to ensure robustness in power system SE

Mentor: Prof. Somayeh Sojoudi

State estimation (SE) is critical for grid operation

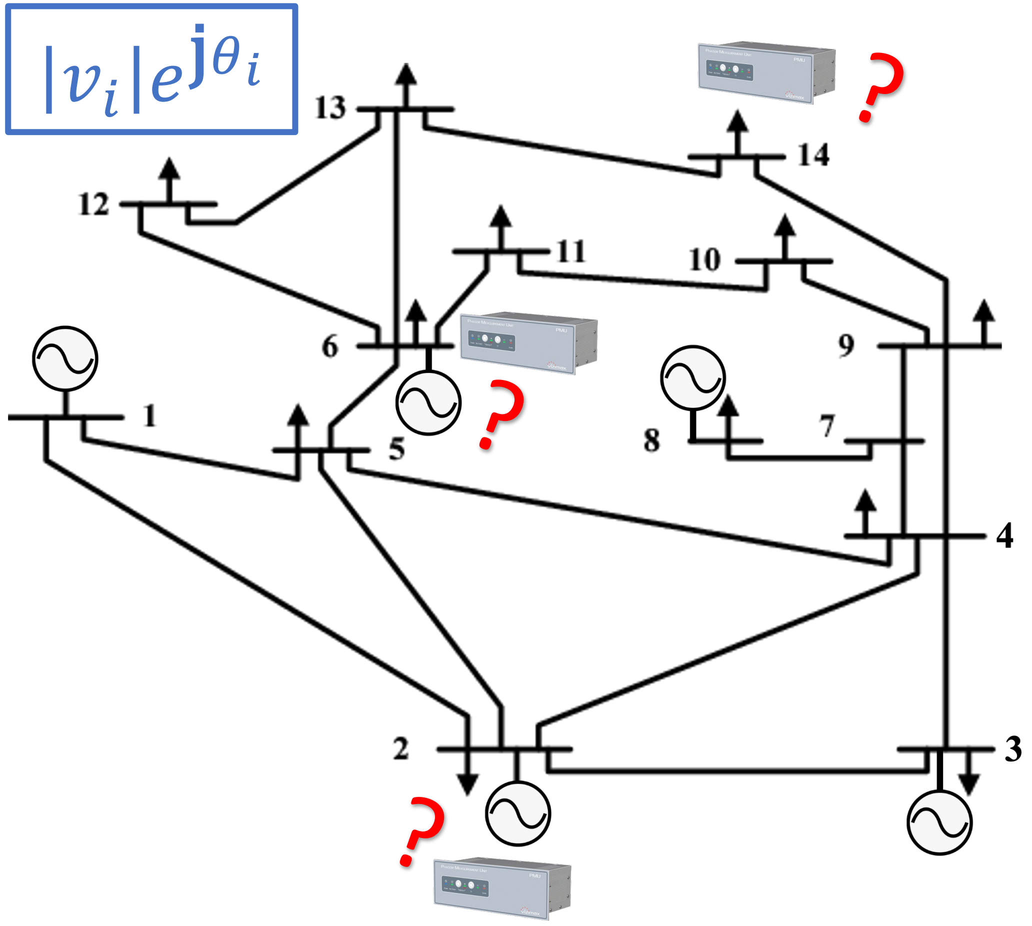


Robust SE and the optimization of sensor placement

- Determine real-time state of power network → power systems state estimation (SE)
- Noisy or corrupted/attacked data → robust power systems SE

❖ **Focus:** How to place sensors in a power network to optimize for robustness of power systems SE

➢ **Approach:** Formulate a mixed-integer linear program (MILP) for measurement choice that optimizes a robustness condition[1]



[1]**E. Glista** and S. Sojoudi, "A MILP for Optimal Measurement Choice in Robust Power Grid State Estimation," *2022 IEEE Power & Energy Society General Meeting. Won Best Conference Paper Award (Power Systems Modeling & Analysis).*

Builds on linearized SE model[1]

- Nonlinearity of AC power flow → SE is nonlinear, nonconvex → hard to solve!
- Two-stage model[1] that can be solved to global optimality with local search methods

Given: Power network given as $\mathcal{G} = (\mathcal{N}, \mathcal{L})$, set of buses $\mathcal{N}$ and set of lines $\mathcal{L}$, measurement set $\mathcal{M}$

Input: Noisy, corrupted measurements $\mathbf{y} \in \mathbb{R}^m$, where $m := |\mathcal{M}|$

→ **Stage 1:** Solve SE problem using linearized basis $\mathbf{x} \in \mathbb{R}^n$ to get an estimate $\hat{\mathbf{x}}$.

→ **Stage 2:** Recover an estimate of underlying voltage vector $\hat{\mathbf{v}}$ from $\hat{\mathbf{x}}$.

[1]M. Jin et al., "Scalable and robust state estimation from abundant but untrusted data," *IEEE Transactions on Smart Grid*, vol. 11, no. 3, pp. 1880–1894, 2020.

Linearized SE model & mutual incoherence

Linearized model

$$\mathbf{y} = A\mathbf{x} + \mathbf{w} + \mathbf{b}$$

measurements — $\mathbf{y}$; sensing matrix — A; unknown state — $\mathbf{x}$; random noise — $\mathbf{w}$; bad data — $\mathbf{b}$

$$\hat{x} = \underset{\mathbf{x},\mathbf{b}}{\operatorname{argmin}} \frac{1}{2|\mathcal{M}|} \|\mathbf{y} - A\mathbf{x} - \mathbf{b}\|_2^2 + \lambda \|\mathbf{b}\|_1 \quad \text{(SE)}$$

estimated state — $\hat{x}$; measurement set — $\mathcal{M}$; regularization parameter > 0 — λ

Global state recovery = impossible

Local recovery = ?

- Mutual coherence is a measure of the cross-correlation of the columns of a matrix
- "**Mutual incoherence**" measures alignment of two submatrices in A, one related to clean data, one to corrupted data

Bad data support
$\mathcal{B} := \text{supp}(\mathbf{b})$

Clean data support
$\mathcal{B}^c = \mathcal{M} \setminus \mathcal{B}$

Mutual incoherence
$\rho(\mathcal{B}) = \|A_{\mathcal{B}^c}^{T\dagger} A_{\mathcal{B}}^{T}\|_\infty$

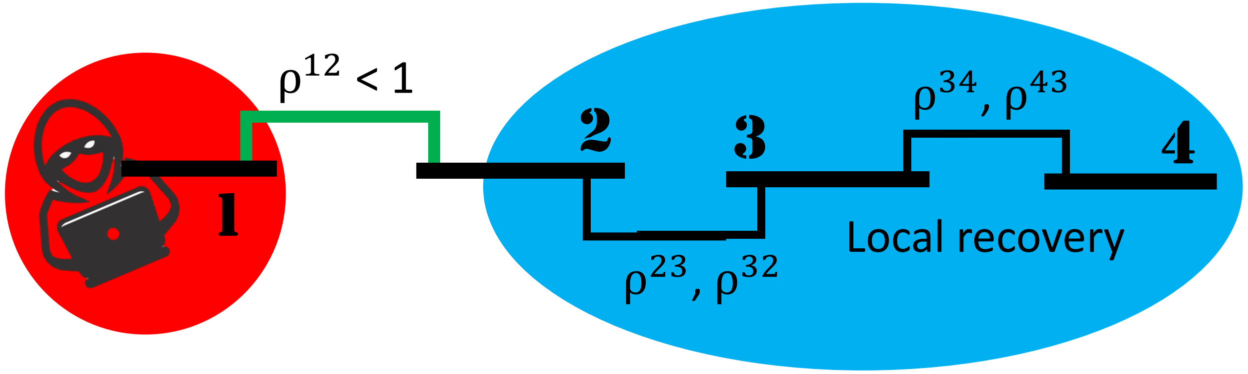
If $\rho(\mathcal{B}) < 1$, then Stage 1 recovers $\hat{\mathbf{x}}$ with small error from $\mathbf{x}^*$ with high probability[1]

[1]M. Jin et al., "Scalable and robust state estimation from abundant but untrusted data," *IEEE Transactions on Smart Grid*, vol. 11, no. 3, pp. 1880–1894, 2020.

Local certification of mutual incoherence

Instead of considering the bad data support (unknown), we consider a local condition

Consider an attack on a single line $(i, j) \in \mathcal{L}$

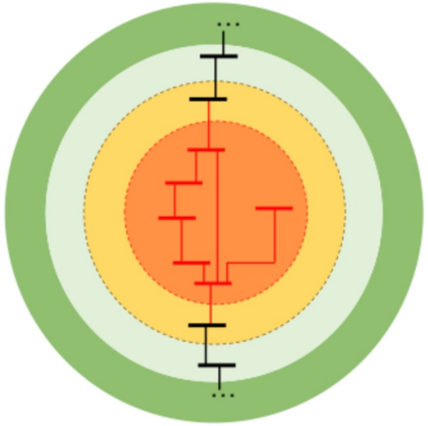


Local mutual incoherence condition

$$\rho^{ij} := \left\| A^{T\dagger}_{\mathcal{M}^{ij}_{\mathrm{ib}}, \mathcal{X}^{ij}_{\mathrm{b}}} A^{T}_{\mathcal{M}^{ij}_{\mathrm{db}}, \mathcal{X}^{ij}_{\mathrm{b}}} \right\|_{\infty} < 1$$

Partition of measurements & state variables

$$A = \begin{bmatrix} A_{\mathcal{M}_{\mathrm{a}}, \mathcal{X}_{\mathrm{a}}} & 0 & 0 \\ A_{\mathcal{M}_{\mathrm{db}}, \mathcal{X}_{\mathrm{a}}} & A_{\mathcal{M}_{\mathrm{db}}, \mathcal{X}_{\mathrm{b}}} & 0 \\ 0 & A_{\mathcal{M}_{\mathrm{ib}}, \mathcal{X}_{\mathrm{b}}} & 0 \\ 0 & A_{\mathcal{M}_{\mathrm{s}}, \mathcal{X}_{\mathrm{b}}} & A_{\mathcal{M}_{\mathrm{s}}, \mathcal{X}_{\mathrm{s}}} \end{bmatrix}$$

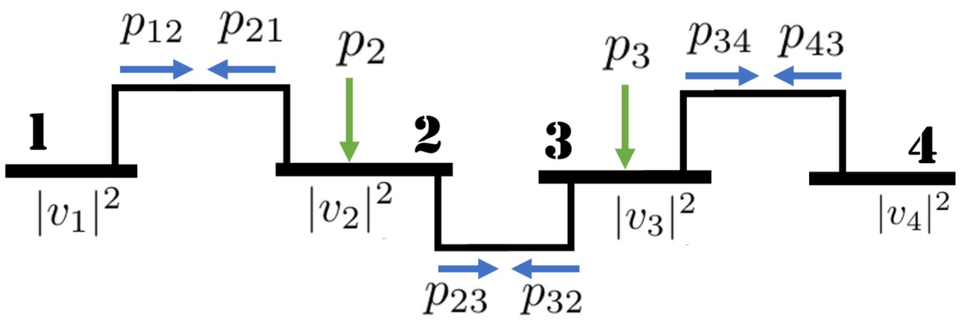


A different partition is defined for each attacked $(i, j) \in \mathcal{L}$

Measurement choice as a MILP

❖ **Idea:** Optimize the choice of measurements in $\mathcal{M}$ such that $\rho^{ij} < 1$ for all $(i,j) \in \mathcal{L}$

Our method:



1. Consider all possible measurements
   - network topology
   - available sensors
2. Partition measurements
   - attacked region
   - boundary regions
   - safe region
3. Introduce binary measurement choice variable $\phi \in \{0,1\}^m$
   - ϕ couples optimization problems over each $\rho^{ij}, \forall (i,j) \in \mathcal{L}$
4. Formulate an optimization problem over ϕ to minimize β where $\rho^{ij} \leq \beta$ for all $(i,j) \in \mathcal{L}$
5. Relax nonlinear constraints and prove exact

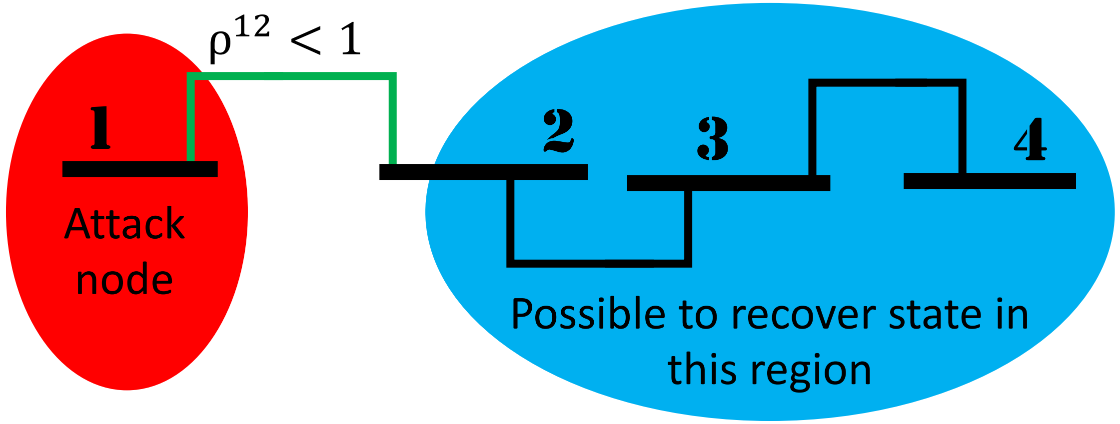
Simulations on IEEE test cases

Network	Results of problem that minimizes β where $\rho^{ij} \leq \beta$ for all $(i,j) \in \mathcal{L}$			Results of problem that minimizes the number of violations of $\rho^{ij} < 1$		
	Fraction of meas.	$\beta = \max \rho^{ij}$	Solve time (s)	Fraction of meas.	Lines where $\rho^{ij} < 1$	Solve time (s)
case5	29 / 39	1.26	0.69	30 / 39	6 / 12	1.89
case9	42 / 57	1.48	1.12	36 / 57	12 / 18	1.49
case14	95 / 120	1.61	9.33	92 / 120	18 / 40	120.5
case30	193 / 248	1.61	39.3	190 / 248	37 / 82	831.1

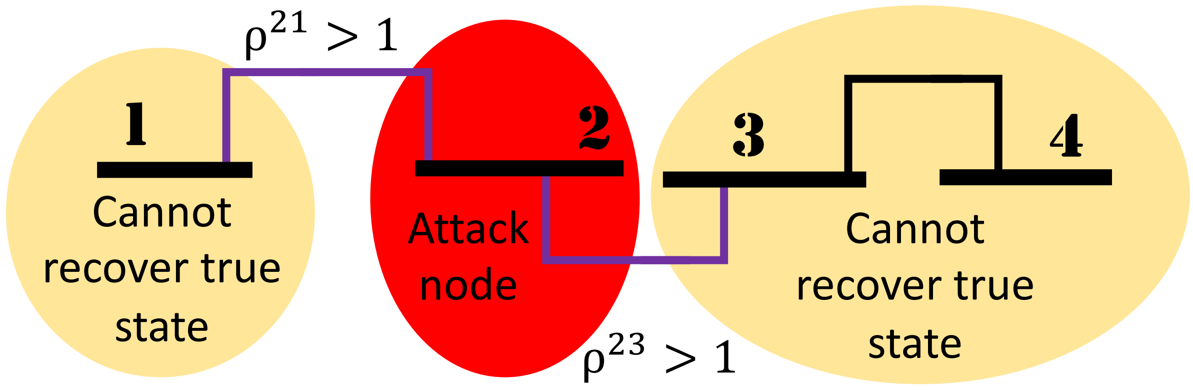
No choice of measurements such that ***all*** lines are robust in case of attack!

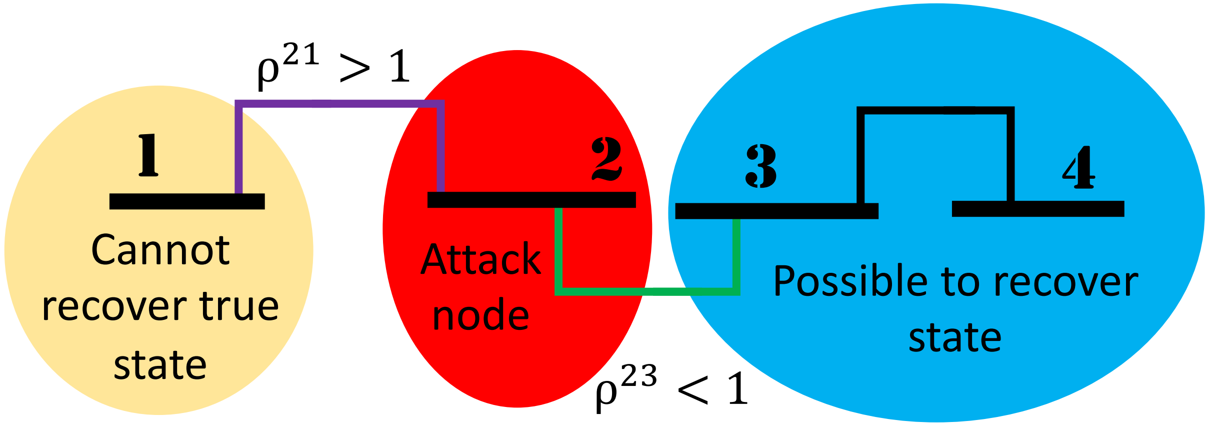
However, we can find subsets of measurements that are more optimal than others in terms of SE robustness

Measurement choice: conclusions



Measurement Set A





Measurement Set B

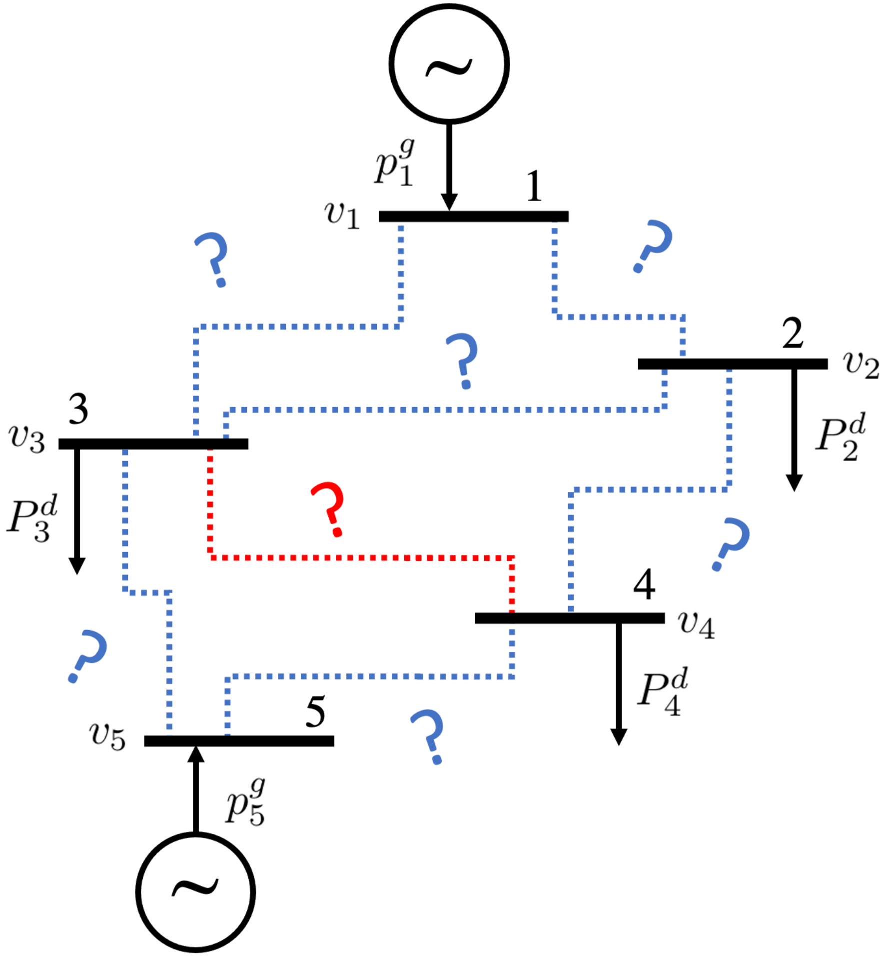
Having more lines satisfy the mutual incoherence condition ***guarantees*** a reduction in the impact of the attack on power system SE for nodes far from the attacked region

Part III: Power Flow (PF) Mapping Problem

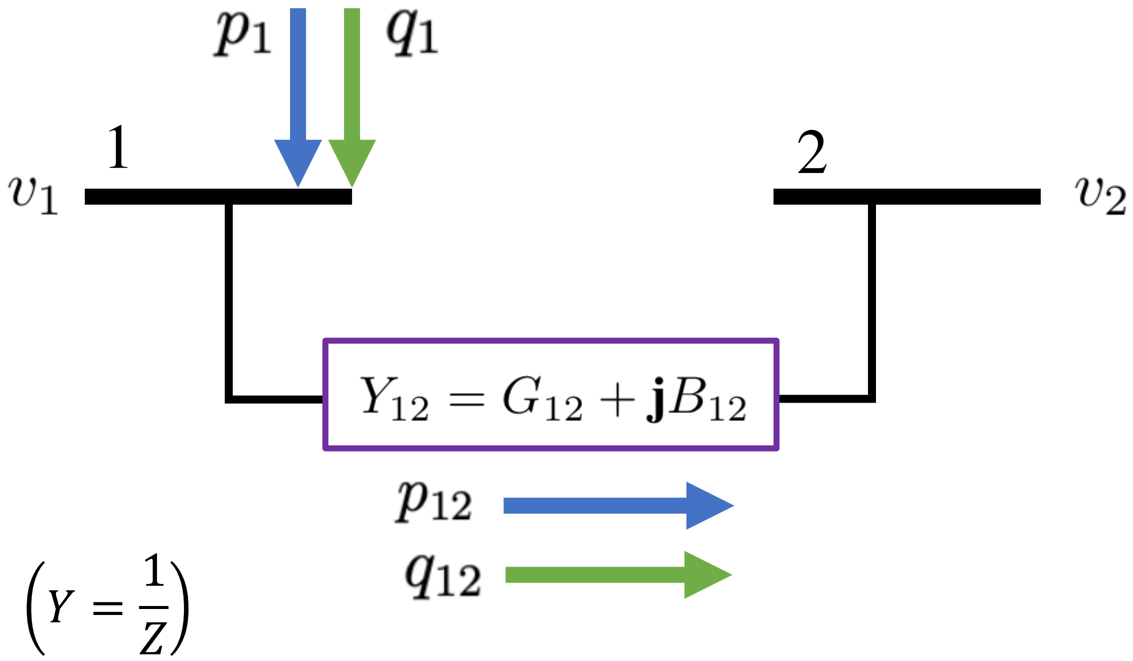
Project: Learning the power system topology using a data-driven, physics-informed optimization

Mentor: Prof. Somayeh Sojoudi

Uncertain topology → problems for most PF methods



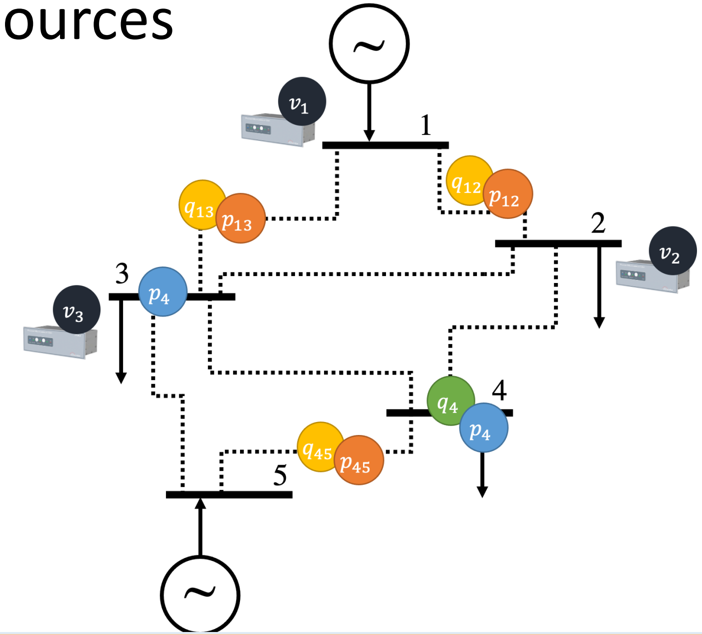
Uncertain topology in WB5 network

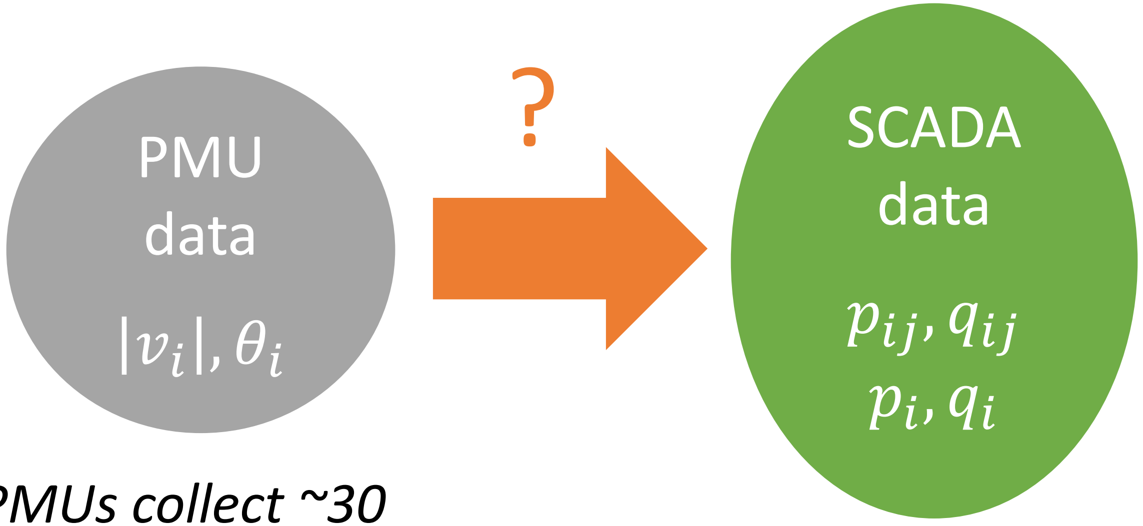


Uncertain topology = parameter uncertainty

Exploiting large datasets to learn topology

Data collection from SCADA and PMU sources





PMUs collect ~30 to 60 samples/sec

SCADA systems collect ~1 sample every 4 sec

- **Previous Approaches:** Neural networks (NN), maximum likelihood estimation, support vector regression (SVR)[1,2] → overfitting + ignore physics!

[1]J. Yu, Y. Weng, and R. Rajagopal, "Robust mapping rule estimation for power flow analysis in distribution grids," in 2017 *NAPS*.
[2]J. Yuan and Y. Weng, "Support matrix regression for learning power flow in distribution grid with unobservability," *IEEE Transactions on Power Systems*, vol. 37, no. 2, pp. 11510-1161, 2022.

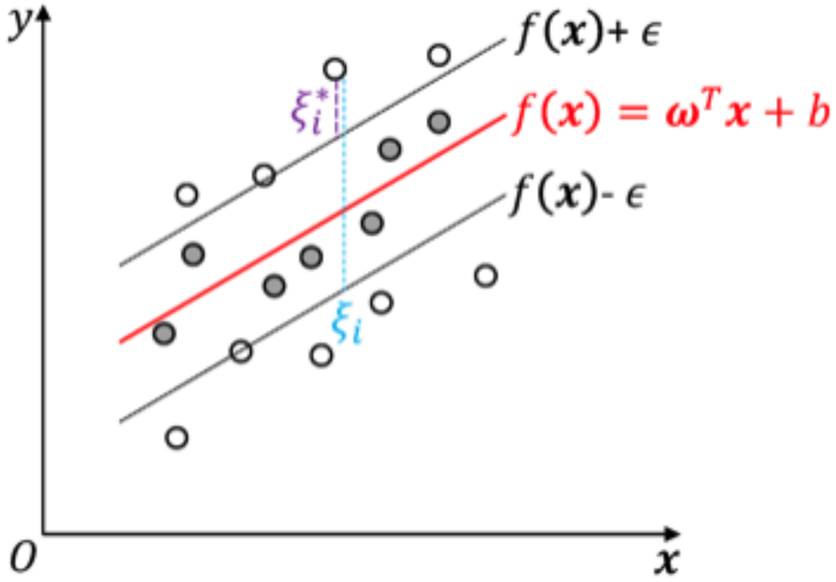
Data-driven approach to learn topology

- **Goal:** Recover the underlying power system topology from system data
  - "Topology" = network connectivity & line parameters
  - Robust in the presence of outliers and noise in data

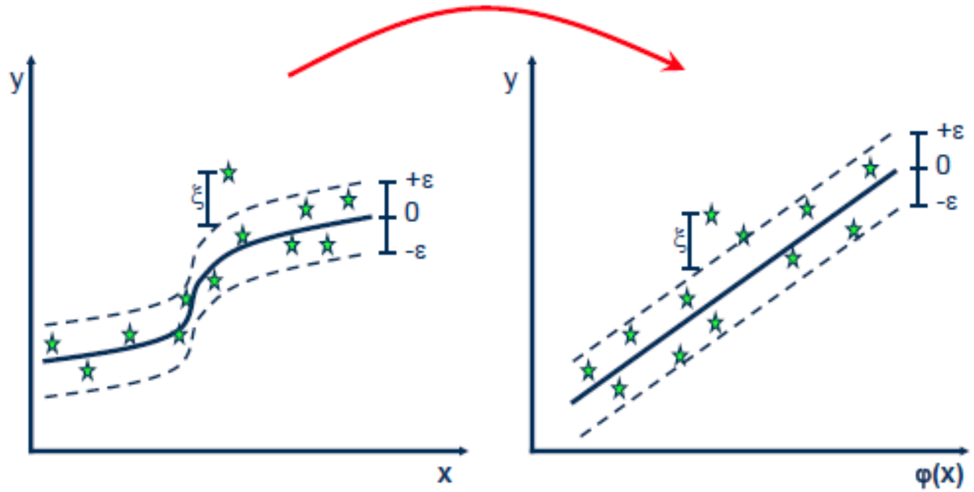
- **Our Approach:** Design a constrained support vector regression (SVR) problem
  - Approach allows *exact* representation of the true AC power system & its inherent sparsity
  - Can efficiently solve SVR optimization problem with off-the-shelf quadratic program (QP) solvers or tailored algorithm

[1]**E. Glista** and S. Sojoudi, "Leveraging the physics of AC power flow in support vector regression to identify power system topology," *submitted to the 2023 Conference on Decision and Control (CDC),* 2023.

Background on SVR



Idea: Find linear estimator that maximizes data proximity to plane



Kernel trick for nonlinear mappings

Power flow (PF) mapping as constrained SVR

❖ **Idea:** Create new SVR formulation with constraints that represent network sparsity

Our method:

Formulate PF mapping *exactly* as quadratic kernel

$K(\boldsymbol{x}_1, \boldsymbol{x}_2) = (\langle \boldsymbol{x}_1, \boldsymbol{x}_2 \rangle)^2 = \phi(\boldsymbol{x}_1)^T \phi(\boldsymbol{x}_2)$ for carefully chosen state $x \in \mathbb{R}^{2n}$ where n is the number of buses equipped with PMUs

→ Power flow mapping: $p_{ij} = \left\langle \mu_{p_{ij}}, \phi(\boldsymbol{x}) \right\rangle, q_{ij} = \left\langle \mu_{q_{ij}}, \phi(\boldsymbol{x}) \right\rangle$

Define SVR problem with multiple types of SCADA measurements

State equation model: $\boldsymbol{y}_t = W\phi(\boldsymbol{x}_t)$ for time steps $t \in \{1, \dots, T\}$

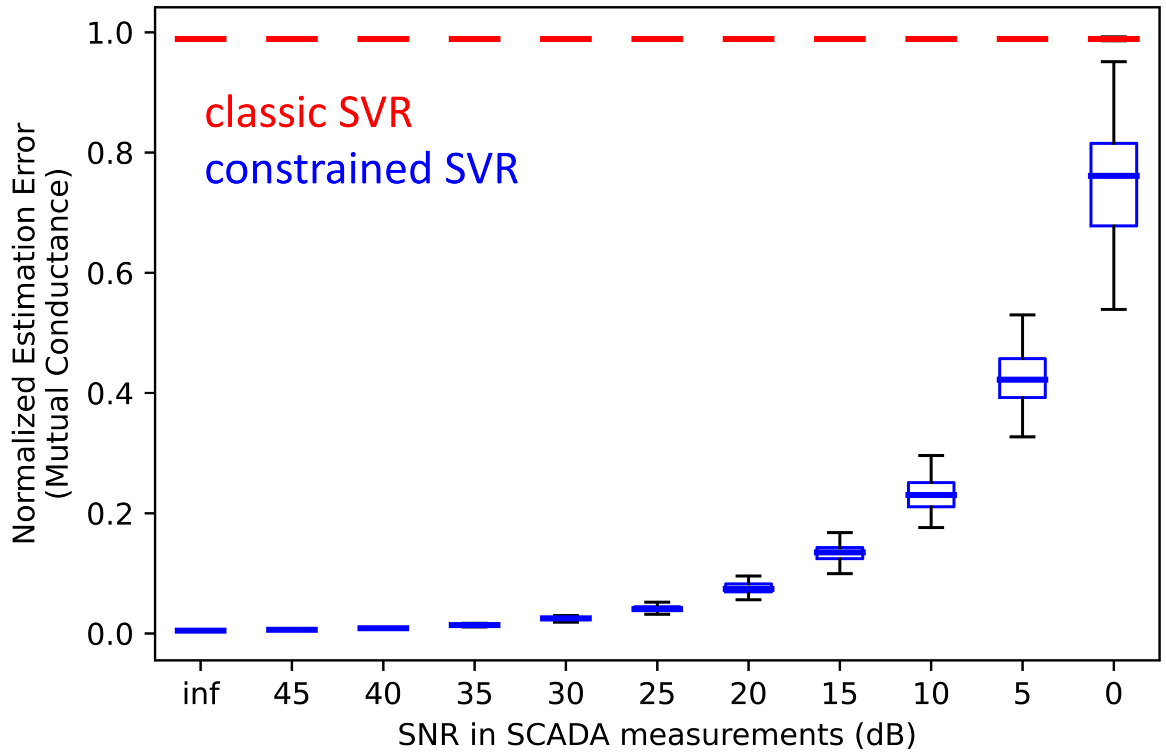
Define sparsity pattern for power network → add as constraint to SVR

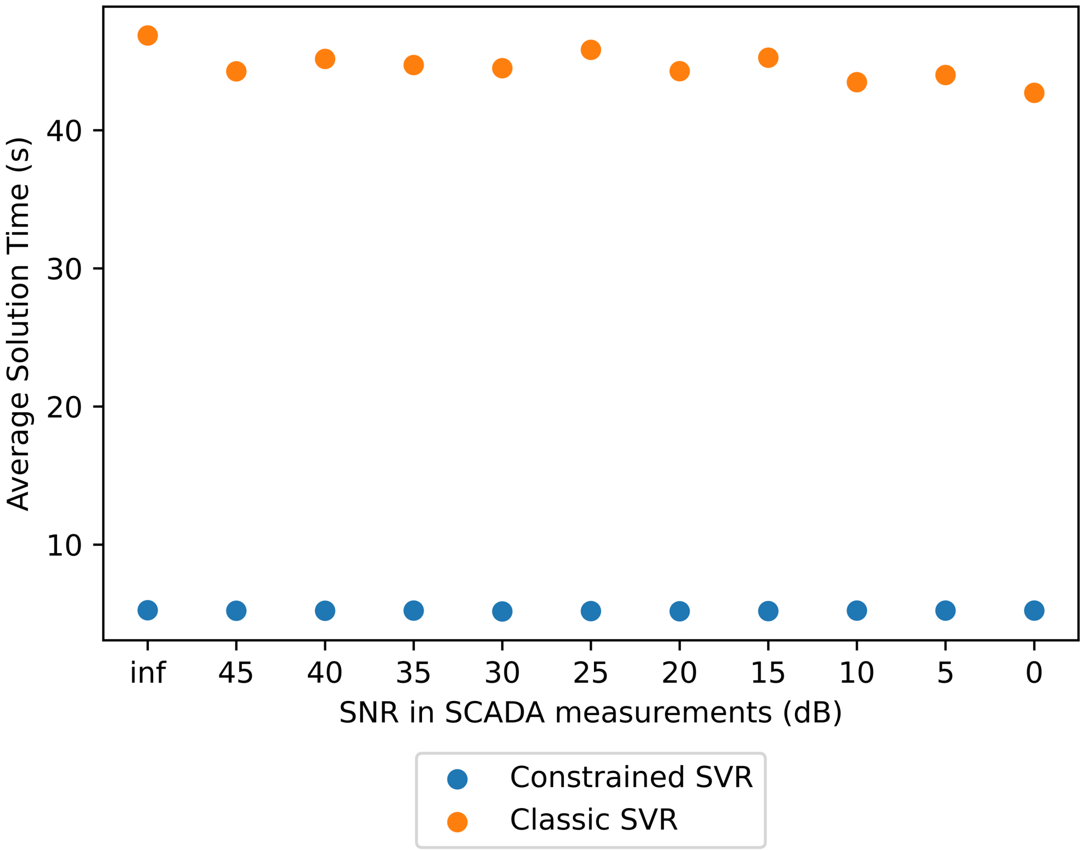
Controls the structure of W

Show that the constrained SVR and its dual are both convex quadratic programs (QPs)

Simulations with SCADA + PMU errors

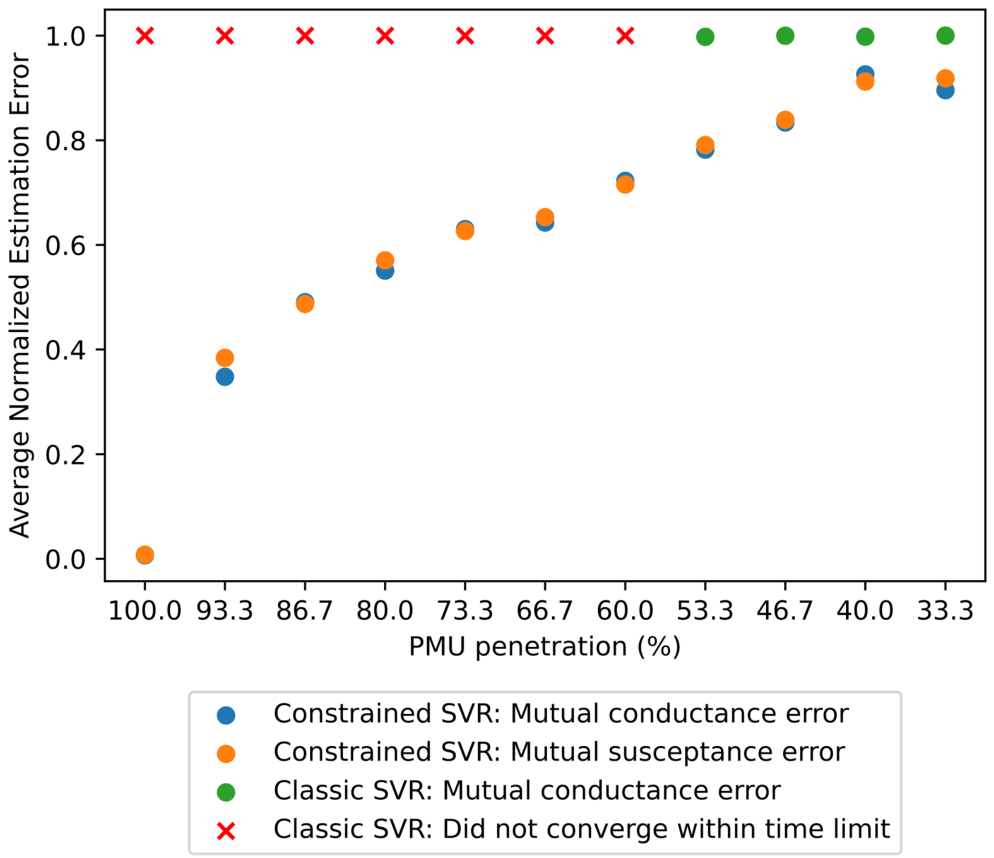
14-bus IEEE network

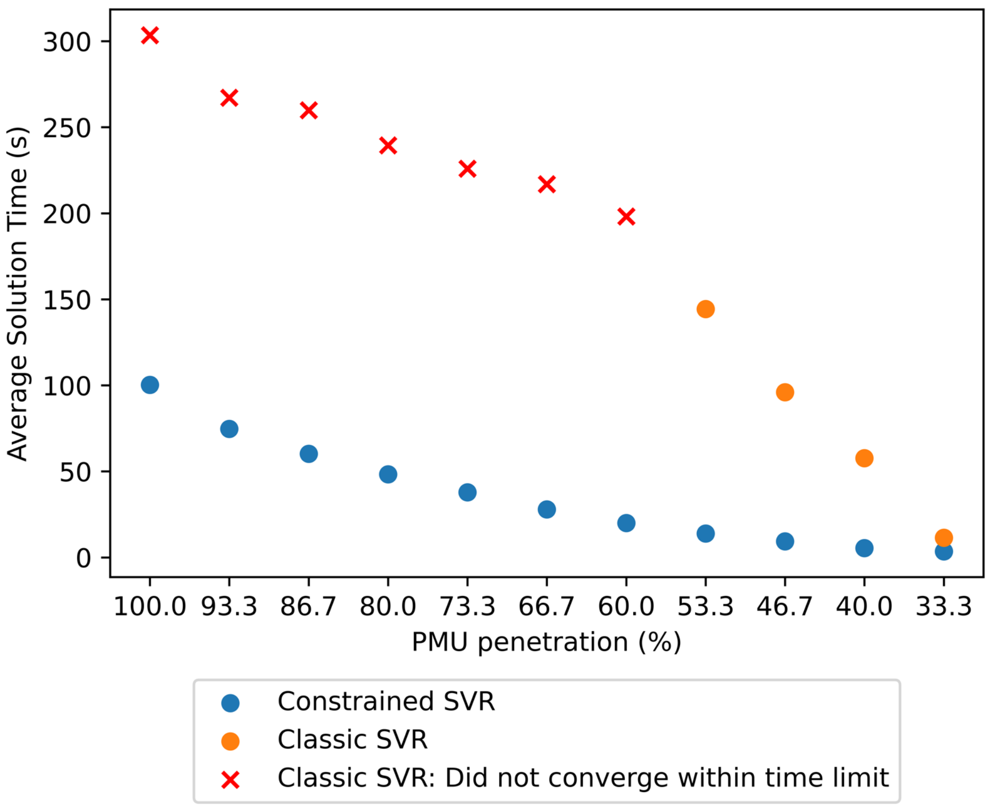




Simulations with varying PMU penetration

30-bus IEEE network





Summary & conclusions

- Proposed a new constrained SVR method that can exactly learn the true power network topology in the case without noise
  - Method has high accuracy in the cases with measurement noise and/or outliers and varying levels of PMU penetration
  - Performs much better than state-of-the-art methods in terms of line parameter recovery and solution time

Acknowledgements & collaborations

Research advisor:

Prof. Somayeh Sojoudi

Mechanical Engineering, EECS (joint)

Collaborator:

Prof. Javad Lavaei

Industrial Engineering & Operations Research

Collaborator:

SangWoo Park

Industrial Engineering & Operations Research

Mentor:

Jean-Paul Watson

Lawrence Livermore National Laboratory (LLNL)

Acknowledgements & collaborations

Current & previous members of
Prof. Sojoudi's research group

Thank you!

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The worst-case local min

Worst-case local min

$$\max_{x\in\mathbb{R}^n} \quad x^T M_0 x + k$$
$$\text{s.t. } \; x \in \{\text{local minima of OPF}\}$$

Lagrange multiplier $\lambda \in \mathbb{R}^p$, (x^*, λ^*) corresponding to local minima:

First-order conditions	$0 = \nabla_x L(x^*, \lambda^*) = 2M_0 x^* + 2\sum_{i=1}^{p} \lambda_i^* M_i x^*$ $(x^*)^T M_i x^* = a_i \qquad \forall i = 1, \dots, p$
Second-order necessary condition	$y^T\left(\nabla_{xx}^2 L(x^*, \lambda^*)\right) y \geq 0$ for all y such that $y^T M_i x^* = 0, \forall i = 1, \dots, p$ where $\nabla_{xx}^2 L(x^*, \lambda^*) = 2M_0 + 2\sum_{i=1}^{p} \lambda_i^* M_i$

The worst-case local min

Worst-case local min

$$\max_{x\in\mathbb{R}^n} \; x^T M_0 x + k \quad \text{s.t. } x \in \{\text{local minima of OPF}\}$$

Lagrange multiplier $\lambda \in \mathbb{R}^p$, (x^*, λ^*) corresponding to local minima:

First-order conditions

$$0 = \nabla_x L(x^*, \lambda^*) = 2M_0 x^* + 2\sum_{i=1}^{p} \lambda_i^* M_i x^*$$

$$(x^*)^T M_i x^* = a_i \qquad \forall i = 1, \dots, p$$

Alternative second-order necessary condition

$$M_0 + \sum_{i=1}^{p} \lambda_i^* M_i + c \sum_{i=1}^{p} M_i (x^*)(x^*)^T M_i \succcurlyeq 0$$

for some c above a certain threshold, $c > \bar{c}$

The worst-case local minimum

Worst-case local min

$$\max_{x\in\mathbb{R}^n,\lambda\in\mathbb{R}^p} \quad x^T M_0 x + k$$

$$\text{s.t.} \quad x^T M_i x = a_i \qquad \forall i = 1, \dots, p$$

$$\left(M_0 + \textstyle\sum_{i=1}^{p} \lambda_i M_i\right) x = 0$$

$$M_0 + \sum_{i=1}^{p} \lambda_i M_i + c \sum_{i=1}^{p} M_i x x^T M_i \succcurlyeq 0$$

for some c above a certain threshold, $c > \bar{c}$

- Nonconvex
- Any upper bound on the problem will also upper bound the worst-case local minimum
- Use a relaxation of the problem into a semidefinite program (SDP)

SDP relaxation of the worst-case local min

Define a matrix $W \in \mathbb{S}^{n+p+1}$ as:

$$W = \begin{bmatrix} 1 \\ x \\ \lambda \end{bmatrix} \begin{bmatrix} 1 & x^T & \lambda^T \end{bmatrix} = \begin{bmatrix} 1 & x^T & \lambda^T \\ x & xx^T & x\lambda^T \\ \lambda & \lambda x^T & \lambda\lambda^T \end{bmatrix}$$

$W \succcurlyeq 0$

$W_{11} = 1$

~~$\text{rank}(W) = 1$~~

SDP of worst-case local min

$$\max_{W \in \mathbb{S}^{n+p+1}, W \succcurlyeq 0, W_{11}=1} \quad \text{trace}\{M_0 W_{22}\} + k$$

$$\text{s.t.} \quad \text{trace}\{M_i W_{22}\} = a_i \qquad \forall i = 1, \dots, p$$

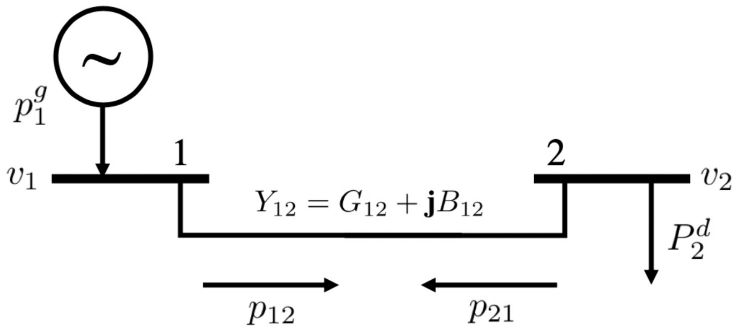
$$M_0 W_{21} + \sum_{i=1}^{p} M_i (W_{23})_i = 0$$

$$M_0 + \sum_{i=1}^{p} M_i (W_{31})_i + c \sum_{i=1}^{p} M_i W_{22} M_i \succcurlyeq 0$$

$$\text{trace}\{M_0 W_{22}\} + \sum_{i}^{p} a_i (W_{31})_i = 0$$

for some c above a certain threshold, $c > \bar{c}$

Simulations on realistic networks[1]



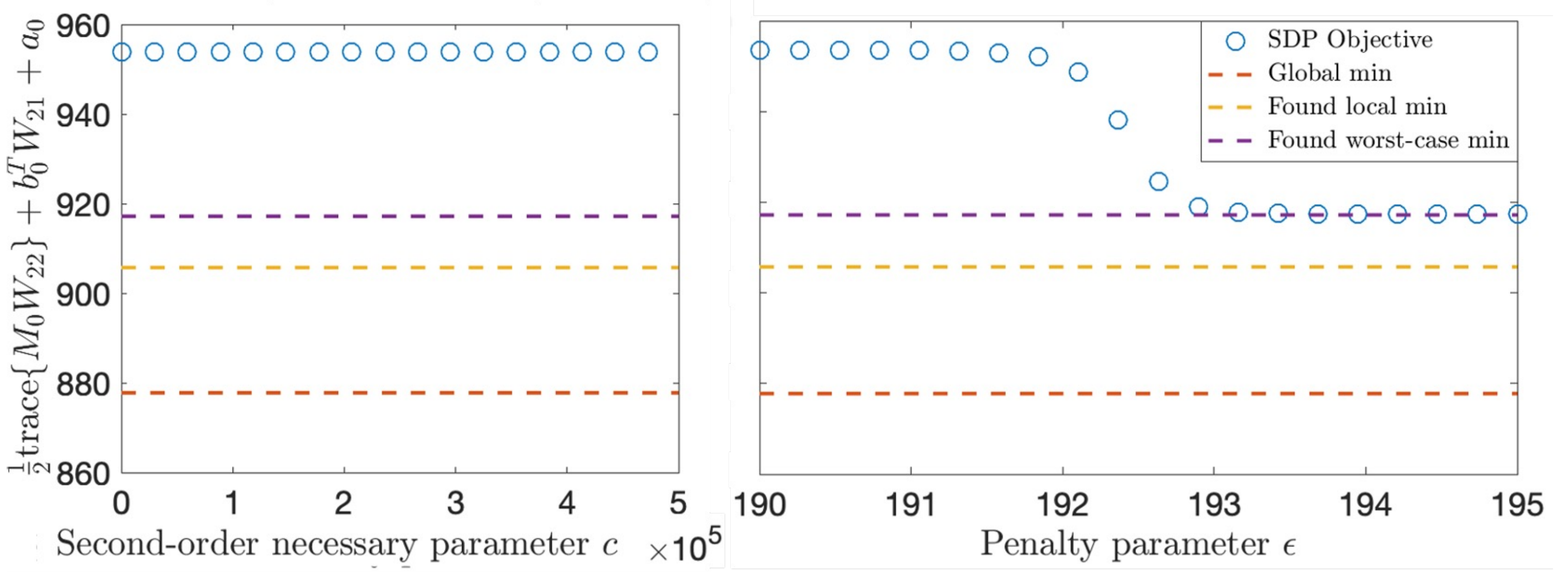
Power generation costs:

$$f_i(p_i^g) = c_{i2}(p_i^g)^2 + c_{i1}p_i^g + c_{i0}$$

$$x = [\text{Re}\{v\}^T \ \ \text{Im}\{v\}^T \ \ (p^g)^T \ \ (q^g)^T]^T$$

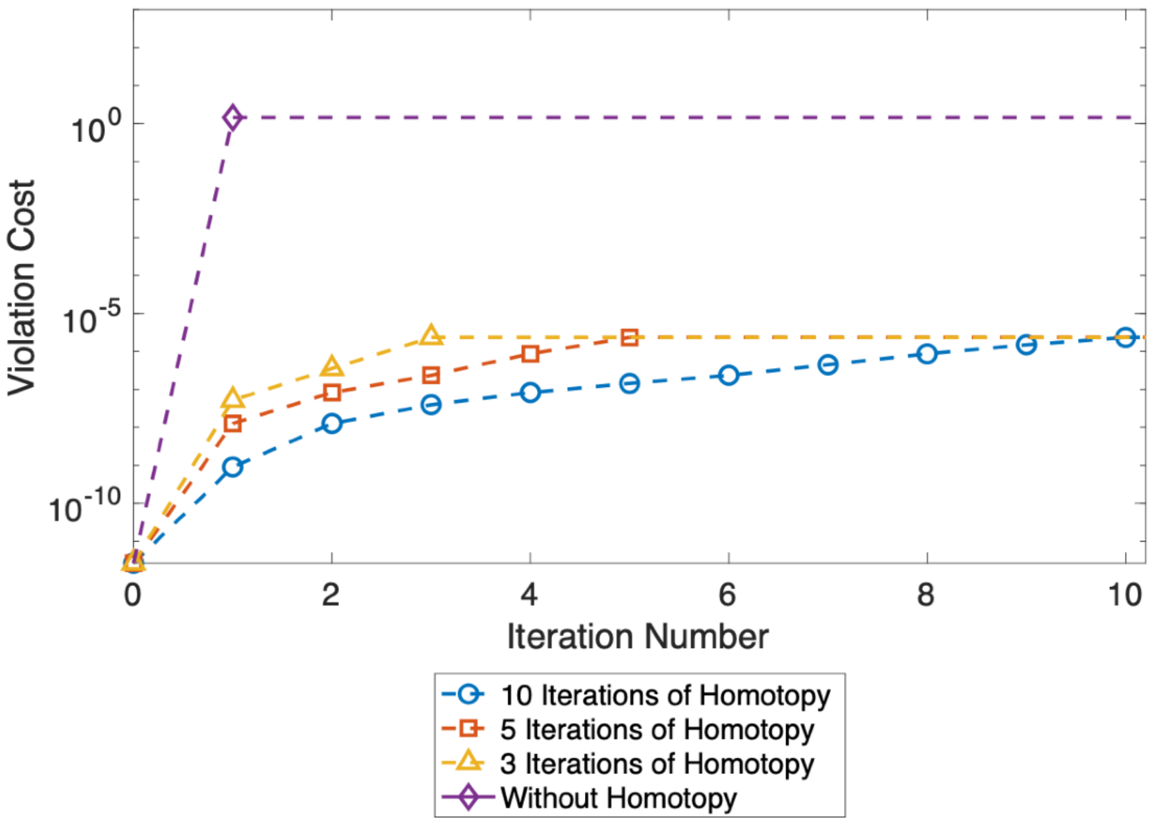
Add a penalty: $\epsilon \cdot (\text{trace}\{W\} - \text{trace}\{W_{22}^{pq}\})$

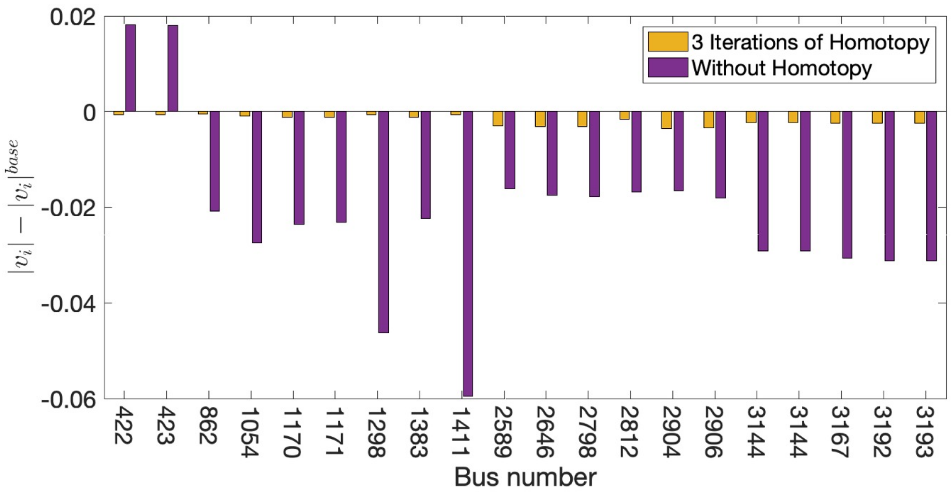
WB2 2-bus network

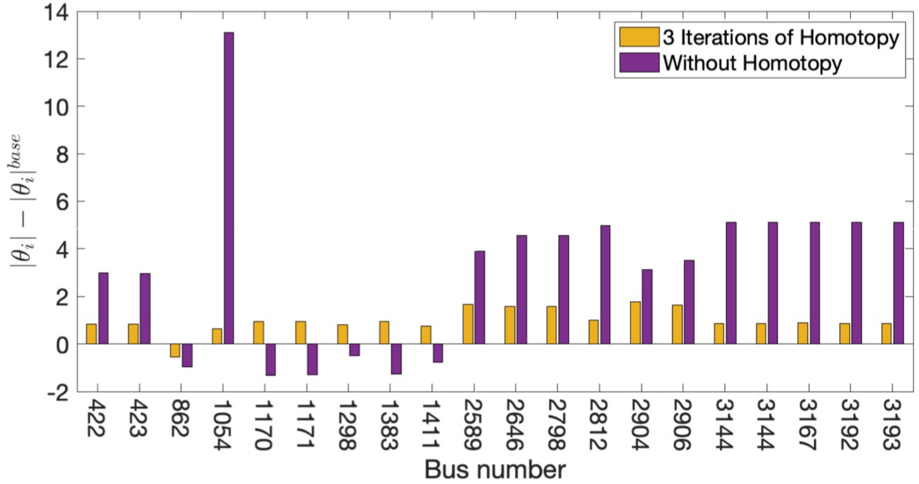


[1] **E. Glista** and S. Sojoudi, "A Semidefinite Program to Bound the Worst-case Solution of Local Search Methods in Optimal Power Flow," 2020.

3375-bus Polish network with single generator outage







Measurement choice problem v1

$$\min_{\substack{\beta\in\mathbb{R},\phi\in\{0,1\}^m \\ Z^{ij},Y^{ij},\forall(i,j)\in\mathcal{L}}} \quad \beta$$

Objective: minimize maximum ρ^{ij}

$$\text{s.t.} \quad \underline{M} \le \sum_{i=1}^{m} \phi_i \le \overline{M}$$

Constraints on number of measurements

$$\sum_{i=1}^{|\Phi_x|} \Phi_x[i] \ge 1, \qquad \forall x \in \mathcal{X}$$

Constraints on measurement dependencies (substitute for rank constraint)

$\forall (i,j) \in \mathcal{L}$:

$$\sum_{r=1}^{m_{\mathrm{ib}}^{ij}} R_{kr}^{ij} Z_{rl}^{ij} = S_{kl}^{ij} \phi[\tilde{\mathcal{M}}_{\mathrm{db}}^{ij}(l)], \quad \forall k \in [n_{\mathrm{b}}^{ij}], \forall l \in [m_{\mathrm{db}}^{ij}]$$

$$Y_{rl}^{ij} \ge \max\{-Z_{rl}^{ij}, Z_{rl}^{ij}\}, \quad \forall r \in [m_{\mathrm{ib}}^{ij}], \forall l \in [m_{\mathrm{db}}^{ij}]$$

$$Y_{rl}^{ij} \le C\phi[\tilde{\mathcal{M}}_{\mathrm{ib}}^{ij}(r)], \quad \forall r \in [m_{\mathrm{ib}}^{ij}], \forall l \in [m_{\mathrm{db}}^{ij}]$$

$$\sum_{l=1}^{m_{\mathrm{db}}^{ij}} Y_{rl}^{ij} \le \beta, \quad \forall r \in [m_{\mathrm{ib}}^{ij}]$$

Definition of $\rho^{ij} := \left\| A^{T\dagger}_{\mathcal{M}_{\mathrm{ib}}^{ij},\mathcal{X}_{\mathrm{b}}^{ij}} A^{T}_{\mathcal{M}_{\mathrm{db}}^{ij},\mathcal{X}_{\mathrm{b}}^{ij}} \right\|_\infty$

$$\rho^{ij} = \|Z^{ij}\|_\infty \qquad R^{ij} := A^{T}_{\tilde{\mathcal{M}}_{\mathrm{ib}}^{ij},\mathcal{X}_{\mathrm{b}}^{ij}}$$

$$Y^{ij} = |Z^{ij}| \qquad S^{ij} := A^{T}_{\tilde{\mathcal{M}}_{\mathrm{db}}^{ij},\mathcal{X}_{\mathrm{b}}^{ij}}$$

Measurement choice problem v2

$$\min_{\substack{\phi \in \{0,1\}^m \\ Z^{ij}, Y^{ij}, \beta^{ij}, \forall (i,j) \in \mathcal{L}}} \sum_{(i,j)\in\mathcal{L}} \mathbf{1}\{\beta^{ij} \geq 1\}$$

Objective: minimize number of violations of ρ^{ij} >1

$$\text{s.t.} \quad \underline{M} \leq \sum_{i=1}^{m} \phi_i \leq \overline{M}$$

$$\sum_{i=1}^{|\Phi_x|} \Phi_x[i] \geq 1, \qquad \forall x \in \mathcal{X}$$

$$\forall (i,j) \in \mathcal{L}:$$

$$\sum_{r=1}^{m_{\text{ib}}^{ij}} R_{kr}^{ij} Z_{rl}^{ij} = S_{kl}^{ij} \phi[\tilde{\mathcal{M}}_{\text{db}}^{ij}(l)], \quad \forall k \in [n_{\text{b}}^{ij}], \forall l \in [m_{\text{db}}^{ij}]$$

$$Y_{rl}^{ij} \geq \max\{-Z_{rl}^{ij}, Z_{rl}^{ij}\}, \quad \forall r \in [m_{\text{ib}}^{ij}], \forall l \in [m_{\text{db}}^{ij}]$$

$$Y_{rl}^{ij} \leq C\phi[\tilde{\mathcal{M}}_{\text{ib}}^{ij}(r)], \quad \forall r \in [m_{\text{ib}}^{ij}], \forall l \in [m_{\text{db}}^{ij}]$$

$$\sum_{l=1}^{m_{\text{db}}^{ij}} Y_{rl}^{ij} \leq \beta^{ij}, \quad \forall r \in [m_{\text{ib}}^{ij}]$$

β^{ij} represents ρ^{ij}

Sensor placement for robust SE: summary & conclusions

- Novel framework to ***formally optimize the placement of sensors*** in a power network in order to satisfy a condition for SE robustness

- Method:
  - Leveraged a linearized SE framework and the concept of local partitioning
  - Defined a MILP that optimizes the local mutual incoherence metric for each line in the network

- Can be used to place new sensors in an existing legacy power network in order to improve SE robustness

- Could be used to classify the measurements that are most susceptible to error propagation