1 State space representation - 40 points

Find the state-space representation of the network shown in the following figure if the output is $v_o(t)$.

2 Phase-variable representation - 30 points

For each transfer function, write the state equations and the output equation for the phase-variable representation.

(a) \[ \frac{s^2 + 3s + 1}{s^4 + 3s^3 + 5s^2 + 6s + 1} \]

(b) \[ \frac{s^4 + 5s^3 + 8s^2 + 2s + 3}{s^5 + 5s^4 + s^2} \]
3 Transfer function - 30 points

Find the transfer function $G(s) = Y(s)/R(s)$ for each of the following systems represented in state space $\dot{x} = Ax + Bu, y = Cx$.

(a) 
\[
A = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
-2 & -5 & -4
\end{bmatrix},
B = \begin{bmatrix} 0 \\ 0 \\ 8 \end{bmatrix},
C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}
\]

(b) 
\[
A = \begin{bmatrix}
5 & 8 & -4 \\
7 & 0 & 1 \\
-6 & -5 & -2
\end{bmatrix},
B = \begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix},
C = \begin{bmatrix} 4 & 1 & 3 \end{bmatrix}
\]