Midterm Exam Solutions

7:00-9:00pm, 7 March

Read these instructions carefully

1. **Write your name and SID number on the front page, and your SID number on every page!**
2. This is a **closed book** exam, but you are allowed one single-sided cheat sheet and blank scratch paper. No phones, calculators or other electronic equipment.
3. The exam consists of 11 questions. The first 7 questions are multiple choice; the remaining 4 require written answers.
4. Approximate point totals for each question part are indicated in the margin. The maximum total number of points is 83.
5. **Multiple choice questions:** Answer these by **filling in** the circle adjacent to the correct answer. You should be able to answer all of these from memory, by inspection, or with a small calculation. There is **no** penalty for incorrect answers. There is **no** partial credit for these questions.
6. **Other questions:** Write your answers to these in the spaces provided below them. None of these questions requires a very long answer, so you should have enough space–if not you are writing too much. **Always show your working for these questions!**
7. The questions vary in difficulty: if you get stuck on some part of a question, leave it and go on to the next one. Point totals and space provided are not necessarily an indication of difficulty.

Your First Name: **Your Last Name:**

Your SID Number:

[exam starts on next page]

1. Let u be an arbitrary 0-1 vector of length n, and v be a uniformly random 0-1 vector of length n.

(a) The probability that $u + v = 0^n$ (with all arithmetic performed mod 2) is *3pts*

○ $\frac{1}{n}$; ● $\frac{1}{2^n}$; ○ $\frac{1}{2}$; ○ $\frac{n-1}{n}$; ○ not determined by the given data

(b) Assuming that $u \neq 0^n$, the probability that $u \cdot v = 0$ (dot product, mod 2) is *3pts*

○ $\frac{1}{n}$; ○ $\frac{1}{2^n}$; ● $\frac{1}{2}$; ○ $\frac{n-1}{n}$; ○ not determined by the given data

---

2. The vertices of a graph $G = (V, E)$ are colored independently and uniformly at random with one of five colors. Let X denote the number of edges that have the same color on both endpoints.

(a) The expectation of X is *3pts*

○ $\frac{|V|}{5}$; ● $\frac{|E|}{5}$; ○ $\frac{|E|^2}{25}$; ○ $\frac{|E|}{10}$; ○ not determined by the given data

(b) The variance of X is *3pts*

○ $\frac{|E|^2}{25}$; ○ $\frac{2\sqrt{|E|}}{5}$; ○ $\frac{4|E|^2}{25}$; ● $\frac{4|E|}{25}$; ○ not determined by the given data

---

3. You roll a fair, six-sided die until you roll six for the first time; let X denote the number of rolls (including the roll that came up six). You then roll the same die X more times; let Y denote the sum of the numbers obtained on these X rolls. The expectation of Y is *3pts*

● 21; ○ $\frac{7}{2}$; ○ 42; ○ $\frac{49}{4}$; ○ none of the above

---

4. Let G be a random graph in the $\mathcal{G}_{n,p}$ model. An *independent set* in G is a set of vertices that have no edges between them. The expected number of independent sets of size k in G is *3pts*

○ $n^k(1-p)^k$; ○ $\binom{n}{k}(1-p)^k$; ● $\binom{n}{k}(1-p)^{\binom{k}{2}}$; ○ $\binom{n}{k}p^{\binom{k}{2}}$; ○ none of the above

---

5. For $\alpha \geq 1$, an α-*minimum* cut in a graph G is a cut whose size is at most α times the size of a minimum cut. A generalization of Karger's min-cut algorithm discussed in class guarantees that, in any n-vertex graph G, every α-minimum cut will be output with probability at least $n^{-2\alpha}$. This implies that the number of distinct α-minimum cuts in G is *3pts*

○ at least $n^{2\alpha}$; ○ at least n^{α}; ○ at most $n^{-2\alpha}$; ○ at most n^{α}; ● at most $n^{2\alpha}$

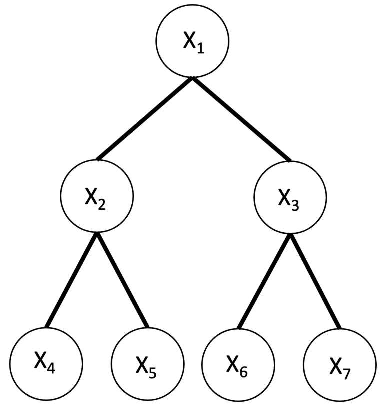

Figure 1: Binary tree for problem 6.

6. Consider the following set of seven 0-1-valued random variables on a binary tree as shown in Figure 1. The root variable X_1 is uniform over $\{0, 1\}$. All other nodes have exactly one parent. Conditioned on the value of the parent node, each other random variable has the same value as their parent with probability $1 - p$ and the flipped value with probability p. Both children of a single parent are sampled independently.

(a) The probability $\Pr[X_4 = 1]$ is *3pts*

○ $(1-p)^2 + p^2$; ○ $(1-p)^2 + p^2 + 2p(1-p)$; ○ $\frac{(1-p)^2+p^2}{2}$; ● $\frac{1}{2}$; ○ $\frac{1}{4}$

(b) The probability $\Pr[X_2 = 1 | X_4 = 1 \cap X_5 = 1]$ is *3pts*

○ $\frac{p^2}{p^2+(1-p)^2}$; ● $\frac{(1-p)^2}{p^2+(1-p)^2}$; ○ $\frac{p(1-p)}{p^2+(1-p)^2}$; ○ $\frac{1}{2}$; ○ $\frac{1}{4}$

(c) The probability $\Pr[X_1 = 1 | \bigcap_{i=2}^{7} X_i = 1]$ is *3pts*

● $\frac{(1-p)^2}{p^2+(1-p)^2}$; ○ $\frac{(1-p)^6}{p^6+(1-p)^6}$; ○ $\frac{p^6}{p^6+(1-p)^6}$; ○ $\frac{1}{2}$; ○ $\frac{1}{4}$

---

7. Suppose we have a bag of M&Ms, which has 6 different colors and 10 candies for each color (60 candies total). Suppose we take 5 candies uniformly at random from the bag without replacement.

(a) The probability of getting exactly two different colors in the sample is *3pts*

○ $\sum_{i=0}^{5} \frac{\binom{6}{2}\binom{10}{i}\binom{10}{5-i}}{\binom{60}{5}}$; ● $\sum_{i=1}^{4} \frac{\binom{6}{2}\binom{10}{i}\binom{10}{5-i}}{\binom{60}{5}}$; ○ $\frac{1}{3^5}$; ○ $\prod_{i=1}^{5} \frac{20-i+1}{60-i+1}$

(b) The expected number of different colors in the sample is *3pts*

○ $6\Big(1 - (\frac{5}{6})^5\Big)$; ○ $5\Big(1 - \frac{\binom{50}{5}}{\binom{60}{5}}\Big)$; ○ $5\Big(1 - (\frac{5}{6})^5\Big)$; ● $6\Big(1 - \frac{\binom{50}{5}}{\binom{60}{5}}\Big)$

[continued overleaf]

8. Biased Permutations

Suppose we want to generate a random permutation of the integers $\{1, \ldots, n\}$, with a bias towards keeping elements in sorted order. This may be relevant for modeling the results of a sporting event, for example. We explore one scheme to achieve this.

(a) Argue that the following algorithm samples a *uniformly* random permutation of $\{1, \ldots, n\}$: *1pt*

- For each $i \in \{1, \ldots, n\}$ in parallel, flip a fair coin to generate a sequence of random bits until all integers i have a unique bit string
- Output the integers from smallest bit string to largest

We claim that, conditioned on the multiset of bit strings obtained, the assignment of the strings to the integers is uniformly random; this follows from the way the strings are generated. Thus, if all bit strings are distinct, we are assigning a set of n distinct values to the integers u.a.r. Sorting those values gives a total order, which must therefore be a random permutation. (This argument can be made more formal using the concept of "exchangeability" of random variables, but we did not expect a detailed justification.)

(b) Show that, with high probability, the number of coins we need to toss for each i is at most $c \log n$, for some constant c. Also, provide a lower bound on c. [HINT: Recall that, in the birthday problem, if there are m people and $n \gg m$ possible birthdays, the probability that no two people have the same birthday is approximately $\exp(-\frac{m^2}{2n})$.] *3pts*

Identifying bit strings of length t with possible birthdays and integers with people, the process is a birthday problem with n people and 2^t birthdays. From the hint, the probability that each person has a unique birthday is $\sim \exp(-\frac{n^2}{2^{t+1}})$. Now if we take $t = c \log_2 n$, this becomes $\exp(-\frac{1}{2}n^{2-c})$, which tends to 1 as $n \to \infty$ if we take the constant c to be any value greater than 2. Thus with high probability $c \log_2 n$ bits suffice for any $c > 2$.

[continued overleaf]

(c) Now suppose that we *bias* the permutations by making the coin flips non-uniform, as follows: the random bits for integer i are 1 with probability $\frac{i}{n+1}$, and 0 with probability $1 - \frac{i}{n+1}$, for $i \in \{1, \ldots, n\}$. Show that the expected rank of the integer 1 in the resulting biased permutation is given by *5pts*

$$\mathrm{E}[\mathrm{rank}(1)] = 1 + \sum_{j=2}^{n} \frac{n+1-j}{n+1-j+jn}. \tag{1}$$

(Here we use the convention that the rank of the first element in the permutation is 1.) [HINT: Introduce indicator random variables X_j for the event that integer j comes before integer 1 in the permutation. You may need the formula for the sum of a geometric series: $\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}$ for $|r| < 1$.]

Following the hint, we may write the r.v. $\mathrm{rank}(1) = 1 + \sum_{j=2}^{n} X_j$, where X_j is the indicator of the event that integer j comes before 1 in the permutation. By linearity of expectation,

$$\mathrm{E}[\mathrm{rank}(1)] = 1 + \sum_{j=2}^{n} \mathrm{E}[X_j], \tag{2}$$

so it suffices to compute $\mathrm{E}[X_j] = \Pr[X_j = 1]$ for each j. Now $X_j = 1$ if and only if the first *non-equal* bits assigned to elements 1 and j are 1 and 0, respectively. Summing over the number k of *equal* bits assigned before this happens, we see that

$$\Pr[X_j = 1] = \sum_{k=0}^{\infty} p^k q = \tfrac{q}{1-p}, \tag{3}$$

where p is the probability that 1 and j are assigned the same bit, and q is the probability that 1 is assigned 1 and j is assigned 0. Now by examining the probabilities in the process, we see that

$$\begin{aligned} p &= \left\{\left(1 - \frac{1}{n+1}\right) \times \left(1 - \frac{j}{n+1}\right)\right\} + \left\{\frac{1}{n+1} \times \frac{j}{n+1}\right\} = \frac{n(n+1-j)+j}{(n+1)^2} = \frac{n^2+n-jn+j}{(n+1)^2}; \\ q &= \frac{1}{n+1} \times \left(1 - \frac{j}{n+1}\right) = \frac{n+1-j}{(n+1)^2}. \end{aligned}$$

Noting that $1 - p = \frac{n+1-j+jn}{(n+1)^2}$, and plugging these values into (3), we get that $\mathrm{E}[X_j] = \frac{n+1-j}{n+1-j+jn}$. Finally, plugging this into (2) gives the desired result.

(d) It turns out that the sum in equation (1) is $O(\log n)$. How does the expected rank of integer 1 in the biased scheme compare to its expected rank under a uniform permutation? *2pts*

Plugging in the given value $O(\log n)$ for the sum in (1), we get that $\mathrm{E}[\mathrm{rank}(1)] = O(\log n)$ for the biased permutation. On the other hand, the expected rank of 1 in a uniform permutation is clearly $\frac{n+1}{2}$, which is $\Theta(n)$ and thus much larger.

(e) Show that the number of coin flips in the biased scheme until integers 1 and 2 have distinct bit strings is at least $c'n$ with high probability, for some constant c' (which you **do not** need to specify). *3pts*

Re-using our calculation of $1-p$ from part (c), the probability that 1 and 2 are assigned *different* bits is $\frac{n+1-2+2n}{(n+1)^2} = \frac{3n-1}{(n+1)^2}$. Thus the number of flips until their bit strings differ is a geometric r.v. with parameter $\frac{3n-1}{(n+1)^2} < \frac{3}{n}$. The probability that such a r.v. is less than $c'n$ is (by a simple union bound) at most $c'n \times \frac{3}{n} = 3c'$. Now by taking $c' > 0$ a very small constant, we can make this as small as we like. [Note: Technically we usually use the phrase "with high probability" to mean a probability that tends to 1 as $n \to \infty$. Under this definition, we would have to choose c' so that $3c' \to 0$, meaning that c' itself has to tend to 0. Since our wording here was a bit loose, we did not deduct points for alternative interpretations here.]

[continued overleaf]

9. Data Privacy

A social club wants to accurately estimate the average income of its n members by asking them to complete a one-question survey. However, they assume that members will be reluctant to share their exact incomes with the club. An intern, who recently took CS174 at UC Berkeley,7uh suggests the following scheme: each member independently picks an integer R uniformly at random from the range $[-r, r]$ (i.e., the set $\{-r, \ldots, 0, \ldots, r\}$) (for some r to be determined) and responds to the survey with the value $I + R$, where I is their actual income (in whole thousands of dollars). Note that the value submitted could possibly be negative. The club will then use the average of these submitted values as its estimate of the true average income.

For $1 \leq j \leq n$, define the random variable $X_j = I_j + R_j$, where I_j is the actual income of member j, and R_j the random integer chosen by j. Also, denote the club's estimate by $\hat{X} = \frac{1}{n}\sum_{j=1}^{n} X_j$, and the actual average income by $\mu = \frac{1}{n}\sum_{j=1}^{n} I_j$.

(a) Show that $\mathrm{E}[\hat{X}] = \mu$. *1pt*

By linearity of expectation we have $\mathrm{E}[\hat{X}] = \frac{1}{n}\sum_j \mathrm{E}[X_j] = \frac{1}{n}\sum_j(\mathrm{E}[I_j] + \mathrm{E}[R_j]) = \mu + 0 = \mu$, where we have used the facts that I_j is a constant and that $\mathrm{E}[R_j] = 0$.

(b) Show that $\mathrm{Var}[X_j] = \frac{r(r+1)}{3}$. [HINT: Recall that $\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$.] *3pts*

Note first that $\mathrm{Var}[X_j] = \mathrm{Var}[R_j]$ since I_j is a constant. Also, $\mathrm{Var}[R_j] = \mathrm{E}[R_j^2] - \mathrm{E}[R_j]^2 = \mathrm{E}[R_j^2]$. Then we have $\mathrm{Var}[R_j] = \mathrm{E}[R_j^2] = \frac{1}{2r+1}\sum_{i=-r}^{r} i^2 = \frac{2}{2r+1}\sum_{i=1}^{r} i^2 = \frac{2}{2r+1}\frac{r(r+1)(2r+1)}{6} = \frac{r(r+1)}{3}$.

(c) Calculate $\mathrm{Var}[\hat{X}]$. *2pts*

$\mathrm{Var}[\hat{X}] = \mathrm{Var}[\frac{1}{n}\sum_j X_j] = \frac{1}{n^2}\sum_j \mathrm{Var}[X_j]$, since the X_j are independent (because the R_j are). Plugging in the value of $\mathrm{Var}[X_j]$ from part (b), we get $\mathrm{Var}[\hat{X}] = \frac{1}{n^2} \cdot n \cdot \frac{r(r+1)}{3} = \frac{r(r+1)}{3n}$. [Note that the variance decreases with n and increases with r, as we would intuitively expect. Also, the variance is zero if $r = 0$.

[continued overleaf]

(d) Now suppose there are $n = 1000$ members and the club wants to ensure that its estimate satisfies $\Pr[|\hat{X} - \mu| \geq 2] \leq 0.001$. (I.e., the estimate should be within ± 2K of the actual average income with probability at least 0.999.) Use Chebyshev's inequality to show that taking $r = 3$ in the above scheme satisfies this requirement. *3pts*

By Chebyshev's inequality and part (b), $\Pr[|\hat{X} - \mu| \geq 2] \leq \frac{\mathrm{Var}[\hat{X}]}{4} = \frac{r(r+1)}{12n}$. We want this to be at most $\frac{1}{1000}$, which entails $r(r+1) \leq 12$. Clearly the value $r = 3$ satisfies this condition.

(e) The club feels that its members will not be willing to reveal their true incomes to within ± 3K. Show using a Hoeffding bound that the requirement in part (d) is in fact satisfied by taking $r = 16$, meaning that members only need to reveal their salaries within ± 16K. [HINTS: Recall that the Hoeffding bound for independent r.v.'s Z_i each in the range $[a_i, b_i]$, takes the form $\Pr[|\sum_i Z_i - M| \geq \lambda] \leq 2\exp\{-\frac{2\lambda^2}{\sum_i (b_i - a_i)^2}\}$, where $M = \sum_i \mathrm{E}[Z_i]$. You may use the fact that $2000/\ln(2000) > 256 = 16^2$.] *4pts*

Observe first that $\Pr[|\hat{X} - \mu| \geq 2] = \Pr[|\sum_j X_j - n\mu| \geq 2n] = \Pr[|\sum_j R_j - 0| \geq 2n]$, where $0 = \sum_j \mathrm{E}[R_j]$. Since the R_j are independent and take values in the range $[-r, r]$, we can use the Hoeffding bound to deduce that

$$\Pr[|\hat{X} - \mu| \geq 2] = \Pr[|\textstyle\sum_j R_j - 0| \geq 2n] \leq 2\exp\{-\frac{8n^2}{4r^2 n}\} = 2\exp\{-\frac{2n}{r^2}\}.$$

Now in order for this to be at most $\frac{1}{1000}$, we require that $r^2 \leq 2000/\ln(2000)$. Since $2000/\ln(2000) > 16^2$, we conclude that taking $r = 16$ satisfies the condition.

[continued overleaf]

10. Safety Monitoring

The number of radioactive emissions per second from a certain piece of material in equilibrium follows a Poisson distribution with parameter $\lambda_0 = 10$. The material becomes unstable if the emission rate per second jumps to $\lambda_1 = 14$. (You may assume that these are the only two possible emission rates.) You have a monitoring device that measures the number of emissions each second. Your goal is to sound an alarm when the material becomes unstable. To do this, you decide to sound the alarm when you observe at least $12t$ emissions over a period of t seconds, where the value of t is to be computed below.

In this problem, you will use the following form of the Chernoff bound for a Poisson random variable Z with parameter λ:

$$\Pr[Z \geq \lambda + x] \leq \exp\left\{-\frac{x^2}{2(\lambda + x)}\right\}; \qquad \Pr[Z \leq \lambda - x] \leq \exp\left\{-\frac{x^2}{2(\lambda + x)}\right\}.$$

(a) Let X denote the number of emissions observed in t seconds when the current Poisson parameter is λ. What is the distribution of X? *1pt*

Poisson with parameter λt (sum of t independent Poisson(λ) r.v.'s).

(b) Suppose that the material becomes unstable. Compute a value of t that ensures you will sound the alarm after at most t seconds with probability at least $1 - \varepsilon$. [HINT: Use the second (lower tail) Chernoff bound above, with suitable values for λ and x (both of which will be multiples of t).] *3pts*

Since the material is unstable, the number of emissions X in t seconds is a Poisson$(14t)$ r.v. We want to compute the tail probability

$$\Pr[X < 12t] = \Pr[X < (\lambda_1 - 2)t] \leq \exp\{-\tfrac{4t^2}{2(16t)}\} = \exp(-t/8),$$

where we have used the given Chernoff bound on the lower tail with $\lambda = \lambda_1 t = 14t$ and $x = 2t$. In order to make this at most the desired value ε, we require $t \geq 8\ln(\varepsilon^{-1})$.

[continued overleaf]

(c) Suppose that the material does not become unstable. Compute a value of t that ensures you will not sound the alarm after t seconds with probability at least $1-\varepsilon$. [HINT: Use the first (upper tail) Chernoff bound, again with suitable (different) values for λ and x.] *3pts*

Now let Y be the number of emissions in t seconds for the stable material, which is a Poisson r.v. with parameter $\lambda_0 t$. In this scenario we use the Chernoff bound on the upper tail, with $\lambda = \lambda_0 t = 10t$ and $x = 2t$, to get

$$\Pr[Y > 12t] = \Pr[Y > (\lambda_0 + 2)t] \leq \exp\{-\tfrac{4t^2}{2(12t)}\} = \exp(-t/6).$$

In order to make this at most the desired value ε, we require $t \geq 6\ln(\varepsilon^{-1})$.

(d) Deduce from parts (b) and (c) a value of t to use in your test that ensures the probability of any kind of error (i.e., failing to sound the alarm when you should, or sounding it when you should not) is at most 2ε. *1pt*

By a union bound, it suffices to make both probabilities in parts (b) and (c) at most ε. This is ensured by taking the larger of the two required values of t, namely $t \geq 8\ln(\varepsilon^{-1})$.

[continued overleaf]

11. Poisson Approximation

Suppose 12 balls are thrown randomly into 3 bins. Let $\mathcal{E}$ be the event that the bin loads are $(6, 4, 2)$ respectively.

(a) Compute the probability of $\mathcal{E}$ under the Poisson approximation. State clearly the properties of the approximation that you are using. *4pts*

Under the Poisson approximation, the bin loads are each *independent* Poisson(4) r.v.'s, where $4 = \frac{12}{3}$ is the average bin load. Thus we have

$$\Pr_{\text{approx}}[\mathcal{E}] = \mathrm{e}^{-4}\frac{4^6}{6!} \times \mathrm{e}^{-4}\frac{4^4}{4!} \times \mathrm{e}^{-4}\frac{4^2}{2!} = \mathrm{e}^{-12}\frac{4^{12}}{6!4!2!}.$$

(b) Compute the exact probability of $\mathcal{E}$. You should leave your answer in terms of factorials and powers. *4pts*

We can compute the exact probability of $\mathcal{E}$ by counting the number of outcomes of the ball tosses that result in the given loads. This count is $\frac{12!}{6!4!2!}$, since if we think of placing the balls by picking a random permutation of all 12 balls and putting the first 6 in bin 1, the next 4 in bin 2 and the last 2 in bin 3, then we count each of the given bin load vectors exactly 6!4!2! times (the order of balls in each bin is irrelevant). Since there are 3^{12} total placements of the balls, we get

$$\Pr_{\text{exact}}[\mathcal{E}] = \frac{12!}{3^{12}6!4!2!}.$$

(c) The ratio of your answer to part (a) to your answer to part (b) is equal to the probability that a Poisson r.v. with some parameter λ takes on some value k. What are the values of λ and k? Explain your answer by referring to properties of the Poisson approximation you learned in class. [NOTE: You do not have to do a calculation to answer this part!] *2pts*

According to the properties of the Poisson approximation learned in class, we have

$$\Pr_{\text{exact}}[\mathcal{E}] = \Pr_{\text{approx}}[\mathcal{E} \mid \mathcal{E}_0] = \frac{\Pr_{\text{approx}}[\mathcal{E}]}{\Pr_{\text{approx}}[\mathcal{E}_0]}, \tag{4}$$

where $\mathcal{E}_0$ is the event (in the independent Poisson model) that the total number of balls in the bins is 12. (Note that $\Pr_{\text{approx}}[\mathcal{E} \mid \mathcal{E}_0] = \Pr_{\text{approx}}[\mathcal{E}]/\Pr_{\text{approx}}[\mathcal{E}_0]$ since $\mathcal{E} \subset \mathcal{E}_0$.) Therefore the ratio of the probabilities in parts (a) and (b) should be exactly $\Pr_{\text{approx}}[\mathcal{E}_0]$, which in turn is the probability that a Poisson r.v. with parameter 12 takes the value 12. [Note that when you divide the numerical values of the solutions to parts (a) and (b) you do indeed get exactly $\mathrm{e}^{-12}\frac{12^{12}}{12!}$, as expected—but you were not supposed to do this.]

(d) Now let $\mathcal{F}$ be the event that the first bin receives exactly six balls. Would the ratio of the probability of $\mathcal{F}$ under the Poisson approximation to the exact probability of $\mathcal{F}$ be the same as in part (c)? Briefly justify your answer. *2pts*

No. The reason is that equation (4) in part (c) still holds with $\mathcal{E}$ replaced by $\mathcal{F}$ (and $\mathcal{E}_0$ as before). However, now it is *not* the case that $\Pr_{\text{approx}}[\mathcal{F} \mid \mathcal{E}_0] = \Pr_{\text{approx}}[\mathcal{F}]/\Pr_{\text{approx}}[\mathcal{E}_0]$, because $\mathcal{F} \not\subseteq \mathcal{E}_0$. Rather, we have

$$\Pr_{\text{exact}}[\mathcal{F}] = \Pr_{\text{approx}}[\mathcal{F} \mid \mathcal{E}_0] = \frac{\Pr_{\text{approx}}[\mathcal{F} \cap \mathcal{E}_0]}{\Pr_{\text{approx}}[\mathcal{E}_0]} = \frac{\Pr_{\text{approx}}[\mathcal{F}] \times \Pr_{\text{approx}}[\mathcal{E}_0 \mid \mathcal{F}]}{\Pr_{\text{approx}}[\mathcal{E}_0]},$$

so the ratio is now $\Pr_{\text{approx}}[\mathcal{E}_0]/\Pr_{\text{approx}}[\mathcal{E}_0 \mid \mathcal{F}]$, which contains an extra factor of $1/\Pr_{\text{approx}}[\mathcal{E}_0 \mid \mathcal{F}]$. [The probability on the denominator is just the probability that bins 2 and 3 together receive 6 balls, given that bin 1 has received 6 balls, which in turn is the probability that a Poisson(8) r.v. takes value 6.]

[The End]