Section 9

1. **(Pairwise Independence) (MU Exercise 15.2)**
   (a) Let X, Y be numbers chosen independently and uniformly at random from $\{0, ..., n\}$. Let Z be their sum modulo $n + 1$. Show that X, Y, Z are pairwise independent but not independent.
   (b) Extend this example to give a collection of random variables that are k-wise independent but not $(k + 1)$-wise independent.

2. **(Hashing) (MU Exercise 15.3)** For any family of hash functions from a finite set U to a finite set V, show that, when h is chosen at random from that family of hash functions, there exists a pair of elements x and y such that:

$$\Pr(h(x) = h(y)) \geq \frac{1}{|V|} - \frac{1}{|U|} \tag{1}$$

   This result should not depend on how the function h is chosen from the family.

3. **(Pairwise Independence) (MU Exercise 15.6)** Our analysis of Bucket sort in Section 5.2.2 assumed that n elements were chosen independently and uniformly at random from the range $[0, 2^k)$. Suppose instead that n elements are chosen uniformly from the range $[0, 2^k)$ in such a way that they are only pairwise independent. Show that, under these conditions, Bucket sort still requires linear expected time.