Section 2

1. **(Conditional Probability) (Book Exercise 1.16)** Consider the following game, played with three standard six-sided dice. If the player ends with all three dice showing the same number, she wins. The player starts by rolling all three dice. After this first roll, the player can select any one, two, or all of the three dice and re-roll them. After this second roll, the player can again select any of the three dice and re-roll them one final time. For questions (a)–(d), assume that the player uses the following optimal strategy: if all three dice match, the player stops and wins; if two dice match, the player re-rolls the die that does not match; and if no dice match, the player re-rolls them all.

   (a) Find the probability that all three dice show the same number on the first roll.

   (b) Find the probability that exactly two of the three dice show the same number on the first roll.

   (c) Find the probability that the player wins, conditioned on exactly two of the three dice showing the same number on the first roll.

   (d) By considering all possible sequences of rolls, find the probability that the player wins the game.

2. **(Expectation)** Suppose you have 10 pairs of socks, with each pair being a different color. You put them all in the washing machine. The washing machine eats four socks at random. What is the expected number of complete pairs that make it out alive?

3. **(Binomial Random Variables) (Book Exercise 2.25)** If X is a $B(n, 1/2)$ random variable with $n \geq 1$, show that the probability that X is even is $1/2$.

4. **(Expectation) (Book Exercise 2.3)** Give some examples of functions f and random variables X where $E[f(X)] < f(E[X])$, $E[f(X)] = f(E[X])$, and $E[f(X)] > f(E[X])$. Go beyond solutions involving $f(X) = X$ or $X = c$ with c a deterministic constant.