Section 1

1. **(Counting) (Book Exercise 1.1)** We flip a fair coin ten times. Find the probability of the following events.

   (a) The number of heads and the number of tails are equal.

   (b) There are more heads than tails.

   (c) The i-th flip and the $(11-i)$-th flip are the same for $i = 1, \ldots, 5$.

   (d) We flip at least four consecutive heads.

2. **(Counting) (Book Exercise 1.4)** We are playing a tournament in which we stop as soon as one of us wins n games. We are evenly matched, so each of us wins any game with probability 1/2, independently of other games. What is the probability that the loser has won k games when the match is over?

3. **(Independence)** Suppose we roll an unbiased six-sided die 3 times. Let E_{ij} denote the event that the ith and the jth rolls produce the same number for $i, j \in \{1, 2, 3\}, i < j$. Show that the events $\{E_{ij}\}$ are pairwise independent but not independent as a family.

4. **(Conditional Probability) (Book Exercise 1.14)** I am playing in a racquetball tournament, and I am up against a player I have watched but never played before. I consider three possibilities for my prior model: we are equally talented, and each of us is equally likely to win each game; I am slightly better, and therefore I win each game independently with probability 0.6; or he is slightly better, and thus he wins each game independently with probability 0.6. Before we play, I think that each of these three possibilities is equally likely. In our match we play until one player wins three games. I win the second game, but he wins the first, third, and fourth. After this match, in my posterior model, with what probability should I believe that my opponent is slightly better than I am?