CS172 Computability & Complexity, Spring 2021

Homework 5

Out: 26 Feb. Due: 5 Mar.

Instructions: *Submit your solutions in pdf format on Gradescope by* **5pm on Friday, March 5**. *Solutions may be written either in LaTeX (with either machine-drawn or hand-drawn diagrams) or* **legibly** *by hand. (The LaTeX source for this homework is provided in case you want to use it as a template.) Please be sure to begin the solution for each problem on a new page, and to tag each of your solutions to the correct problem! Per course policy, no late solutions will be accepted. Take time to write* **clear** *and* **concise** *answers; confused and long-winded solutions may be penalized. You are encouraged to form small groups (two to four people) to work through the homework, but you* **must** *write up all your solutions on your own. Depending on grading resources, we reserve the right to grade a random subset of the problems and check off the rest; so you are advised to attempt all the problems.*

1. Design a (one-tape) Turing machine M, with input alphabet $\{0, 1, \#\}$, which accepts the language $\{b_i\#b_{i+1} : i \geq 1\}$, where b_i is the standard binary encoding of integer i (with no leading zeros). You should briefly document the design of your machine by explaining the role of each state and showing a state transition diagram. Use mnemonic names for the states.

Check your design on the following inputs: (a) 1010#1011; (b) 1111#10000; (c) 1010#1111; (d) 10#1#1. For each of these inputs, hand-turn your machine and write down a few representative intermediate configurations that illustrate its operation.

[NOTES: You will probably want to introduce some extra symbols into the tape alphabet to help keep track of the computation. Your TM need not preserve the original input string, and may leave the head in any location. You may find it helpful to make the machine mark the left-hand end of the tape with a special symbol. You should not spend a lot of time trying to minimize the number of states used, but you should be able to come up with a machine with around 10 states.]

2. For each of the following variants of the standard Turing machine model, either prove that the variant is equivalent to the standard model or give a rigorous argument that explains why it is not.

(a) "Arbitrary jumps": on each transition, the TM may move its head an arbitrary distance in either direction. I.e., the transition function is a function from $Q \times \Gamma$ to $Q \times \Gamma \times (\{L, R\} \times \mathbb{N})$, where $\mathbb{N}$ is the set of natural numbers; for example, the transition $\delta(q, a) = (q', a', (L, n))$ means that the head moves n places to the left. (If this would cause it to fall off the end of the tape, the head moves to the left end of the tape.) Note that the transition function is fixed for any given such TM.

(b) "Left reset": the TM may move its head either one square to the right or all the way back to the left-hand end of the tape (but no regular left moves are allowed). I.e., the transition function is a function from $Q \times \Gamma$ to $Q \times \Gamma \times \{R, \perp\}$, where $\perp$ stands for "reset head to left end of tape".

(c) "One-way": the TM may move its head one square to the right or remain stationary, but may not move its head to the left. I.e., the transition function is a function from $Q \times \Gamma$ to $Q \times \Gamma \times \{R, S\}$, where S stands for "stationary".

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3. Recall from class that limiting the number of registers of a RAM to some fixed constant does not affect its computational power, since any language accepted by a RAM is also accepted by a RAM with only three registers. This problem investigates what happens when we limit the *size* of the registers, i.e., we allow them to contain only integers in some fixed range $[-N, \ldots, N]$, where N is a constant.

(a) Consider first a RAM in which both the number and the size of the registers are limited. Can any language that is accepted by a standard RAM also be accepted by such a RAM? Justify your answer carefully: woolly reasoning will receive little credit.

(b) Now suppose we allow the RAM to have infinitely many registers, as in the standard model, and we limit only the size of the registers. Can any language that is accepted by a standard RAM also be accepted by such a RAM? Justify your answer carefully: woolly reasoning will receive little credit.