CS172 Computability & Complexity, Spring 2021

Homework 3

Out: 5 Feb. Due: 12 Feb.

Instructions: *Submit your solutions in pdf format on Gradescope by* **5pm on Friday, February 12**. *Solutions may be written either in LaTeX (with either machine-drawn or hand-drawn diagrams) or* **legibly** *by hand. (The LaTeX source for this homework is provided in case you want to use it as a template.) Please be sure to begin the solution for each problem on a new page, and to tag each of your solutions to the correct problem! Per course policy, no late solutions will be accepted. Take time to write* **clear** *and* **concise** *answers; confused and long-winded solutions may be penalized. You are encouraged to form small groups (two to four people) to work through the homework, but you* **must** *write up all your solutions on your own. Depending on grading resources, we reserve the right to grade a random subset of the problems and check off the rest; so you are advised to attempt all the problems.*

1. In class we claimed that NFAs can be exponentially more efficient than DFAs, in the sense that they may use exponentially fewer states to recognize the same language. In this problem we will verify this claim using as a concrete example the languages L_k over the alphabet $\{0,1\}$ defined by

$$L_k = \{x = x_1x_2 \ldots x_n \in \{0,1\}^* : n \geq k \wedge x_{n-k+1} = 0\},$$

i.e., L_k is the set of 0-1 strings whose kth letter from the end is 0.

(a) For each $k \geq 1$, design an NFA with $k+1$ states that accepts L_k.

(b) For each $k \geq 1$, design a DFA with 2^k states that accepts L_k.

(c) Show that the number of states in part (b) is optimal by proving that *any* DFA accepting L_k must have at least 2^k states. [HINT: Find a set of 2^k distinguishable strings in L_k.]

2. Consider the regular language $L = 0^*10^*$ consisting of all 0-1 strings that contain exactly one 1.

(a) Find a set of three distinguishable strings w.r.t. L.

(b) Use the set you found in part (a) to construct a minimal DFA that accepts L, as in the proof of the Myhill-Nerode theorem. Explain how you derived your construction.

3. Which of the following languages are regular? If the language is regular, exhibit a finite automaton or a regular expression for it. If not, prove that the language has infinitely many distinguishable strings.

(a) The set of all strings over the alphabet $\{(,)\}$ that consist of correctly nested pairs of parentheses. (E.g., the string '(()())()' belongs to this language, but the strings '())(' and '(()' do not.)

(b) The language $\{0^i1^j : i,j \geq 0 \text{ and } i \neq j\}$ over the alphabet $\{0,1\}$.

(c) The set of all words over the alphabet $\{a,b\}$ in which the number of occurrences of "abb" and of "bba" are the same. [Note: The string $abba$ contains one occurrence of each.]

(d) The set of all words over the alphabet $\{a,b\}$ in which the number of occurrences of "aaa" and of "bbb" are the same. [Note: The string $bbbaaaabbb$ contains two occurrences of each.]

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4. Let PRIMES denote the language $\{1^p : p \text{ is prime}\}$ over the unary alphabet $\{1\}$. For any two distinct primes p, q, show that 1^p and 1^q are distinguishable w.r.t. PRIMES. Hence deduce that PRIMES is not regular. [HINT: Try a proof by contradiction. Assuming w.l.o.g. that $p > q$, consider distinguishing strings of the form $1^{k(p-q)}$ for various values of k.]

5. Which of the following statements are true? If the statement is true, provide a proof; if it is false, provide a simple counterexample. In your counterexamples, you may use without proof any examples of regular and non-regular languages that we have seen in the course.

(a) If the language L contains a regular language L', then L is regular.

(b) If L is a regular language containing at least 100 strings, then the language L_{100} consisting of the lexicographically first 100 strings of L is also regular.

(c) If L_1 and L_2 are not regular, then $L_1 \cap L_2$ is not regular.

(d) If $L_1, L_2, L_3 \ldots$ are all regular, then the language $\bigcup_{i=1}^{\infty} L_i$ is also regular.

6. Apply the minimization procedure discussed in class and in Note 1 to construct a minimal DFA that is equivalent to the following DFA. Show clearly the steps you used to arrive at your answer.