CS172 Computability & Complexity, Spring 2021

Homework 10

Out: 16 Apr. Due: 23 Apr.

Instructions: *Submit your solutions in pdf format on Gradescope by* **5pm on Friday, April 23***. Solutions may be written either in LaTeX (with either machine-drawn or hand-drawn diagrams) or* **legibly** *by hand. (The LaTeX source for this homework is provided in case you want to use it as a template.) Please be sure to begin the solution for each problem on a new page, and to tag each of your solutions to the correct problem! Per course policy, no late solutions will be accepted. Take time to write* **clear** *and* **concise** *answers; confused and long-winded solutions may be penalized. You are encouraged to form small groups (two to four people) to work through the homework, but you* **must** *write up all your solutions on your own. Depending on grading resources, we reserve the right to grade a random subset of the problems and check off the rest; so you are advised to attempt all the problems.*

1. Consider the problem of deciding whether a Boolean formula in 2-CNF (i.e., conjunctive normal form with two literals per clause) is unsatisfiable. This problem corresponds to the language

$$\text{2-U}\textsc{n}\text{S}\textsc{at} = \{\langle\phi\rangle : \phi \text{ is in 2-CNF and is not satisfiable}\}.$$

We know from CS170 that there exists a polynomial time algorithm for deciding whether such a ϕ is satisfiable, so we know that 2-UnSat $\in \mathrm{P}$.

(a) Show that 2-UnSat belongs to NL.
[Note: Be careful here! You do not have enough space to write down a complete assignment!]

(b) By giving a reduction from Path, show that 2-UnSat is NL-complete (w.r.t. log-space reduction).

(c) Deduce that 2-Sat is also NL-complete.

2. Consider the problem Strong-Con, defined as follows:

$$\text{S}\textsc{trong}\text{-C}\textsc{on} := \{\langle G\rangle : G \text{ is a strongly connected directed graph}\}.$$

(Recall that a directed graph G is strongly connected if, for every pair of vertices u, v, there is a path from u to v and from v to u in G.) Show that Strong-Con is NL-complete.

3. This problem concerns the following two languages:

$$\begin{aligned} \text{A}\textsc{ll}_{\text{DFA}} &= \{\langle D\rangle : D \text{ is a DFA that accepts all input strings}\} \\ \text{A}\textsc{ll}_{\text{NFA}} &= \{\langle N\rangle : N \text{ is an NFA that accepts all input strings}\} \end{aligned}$$

(a) Show that $\text{A}\textsc{ll}_{\text{DFA}}$ is NL-complete.

(b) Show that $\text{A}\textsc{ll}_{\text{NFA}}$ is PSpace-complete. [Hint: For hardness, try a direct reduction from any language in PSpace to $\overline{\text{A}\textsc{ll}_{\text{NFA}}}$. Consider a computation sequence of a PSpace TM M on input w, and construct an NFA that "guesses" a place where the sequence goes wrong. (This is somewhat reminiscent of the proof we saw earlier in the class that $\text{A}\textsc{ll}_{\text{CFG}}$ is undecidable.)]

(c) Parts (a) and (b) indicate that there is a huge difference in computational complexity between $\text{A}\textsc{ll}_{\text{DFA}}$ and $\text{A}\textsc{ll}_{\text{NFA}}$. Why does this not contradict the fact that DFAs and NFAs are equivalent, in the sense that both recognize the regular languages?