

# Design Studies Inspired by Perry’s “Sea & Sky.”

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## Abstract

“Sea & Sky” is a large metal sculpture by Charles O. Perry (1929-2011) that depicts the sweep of an equilateral triangle along an elegant 3D space curve. This report documents the effort by which I tried to create an accurate CAD model from just a few pictures of that sculpture. This sculpture then served as inspiration for several derivative sculptures that fall into the same family of shapes – all of which are generated by a smooth sweep of a triangular cross-section. These explorations then lead to a new investigation of the kind of shapes that could be built with a very small set of “standard” building blocks in the shape of quarter-toroids bending through 90°.

## Introduction

I let myself get inspired by the abstract geometrical sculptures by artists such as Brent Collins, Charles O. Perry, Keizo Ushio, Chris Ohler, George Hart, Eva Hild, or Helaman Ferguson. I try to find an underlying generative logic in such sculptures and then capture it in a short computer program. I give these programs many open parameters, and by varying those parameters, I can then create many more sculptures that all seem to belong to the same family of shapes. I typically realize these derivative sculptures as small maquettes on a 3D-printer. In a few instances, a close collaboration with the artist who inspired me has led to additional large sculptures realized by the original artist.

In the summer of 2022, one of my undergraduate research students forwarded me a picture of a sculpture (Fig.1a) by Charles O. Perry, called “Sea & Sky” [10] that, until then, had escaped my attention. I immediately was fascinated by this simple, yet elegant shape and wanted to make an accurate computer model of it. However, based on that one single view, I could not really figure out what was going on. Fortunately, Perry’s homepage [3] lists most of his sculptures, and the index of his “SOLID Sculptures” [4] provides another view (Fig.1b) [5] and a pointer [6] to its location in Kirishima, Japan.



(a)

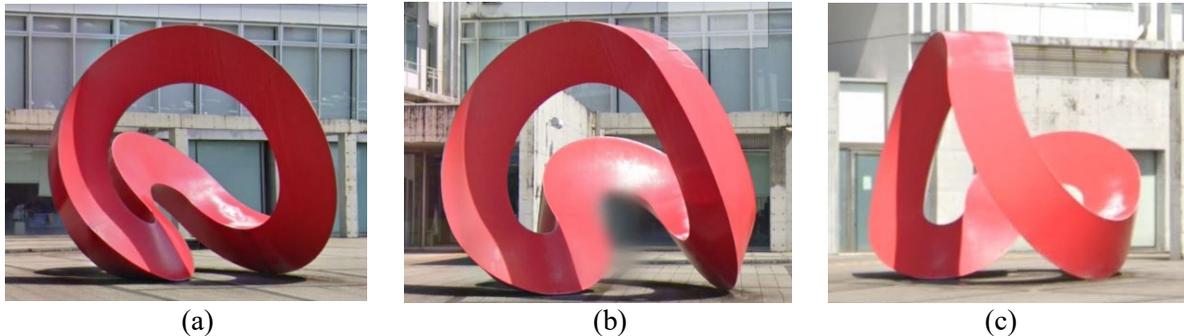


(b)

**Figure 1:** Two views of Perry’s “Sea & Sky” sculpture available on the web.

## Understanding Perry's Sculpture

Using Google's Street View allowed me to obtain three more views of this sculpture from different directions (Figs.2a,b,c). Now it became quite clear that this is a curvy sweep with a constant cross-section in the shape of an equilateral triangle. By carefully tracing one of the ribbon-like prism faces around the whole loop of the sculpture, it was possible to establish that the overall twist of this 3-sided prismatic beam is 120 degrees. This kind of "Möbius-twisted" prism is a theme that occurs repeatedly in Perry's "SOLID" sculptures [4]. This gave me confidence that I was on the right track to understand the geometry of this sculpture. Through the same kind of face-tracking, I could also obtain more detailed information about the amount of local twist between some significant places along the beam, for instance, where the curvy prism touches the supporting platform.

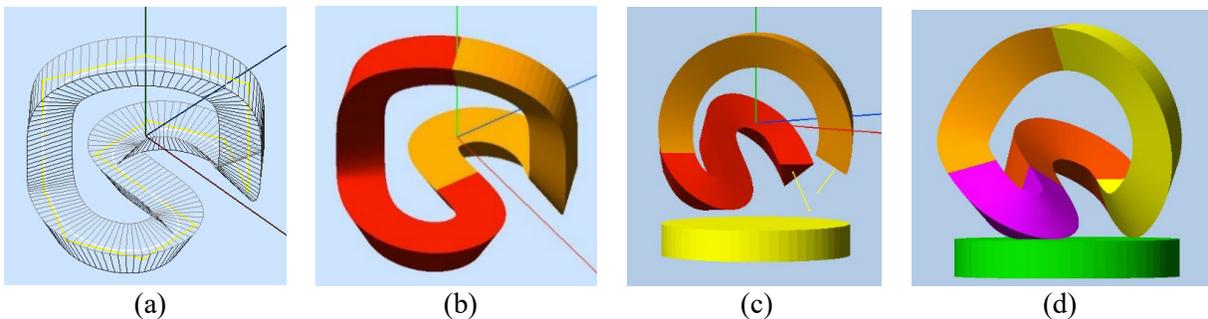


**Figure 2:** Additional views of Perry's "Sea & Sky" sculpture from Google Street View.

## Initial Modeling Effort

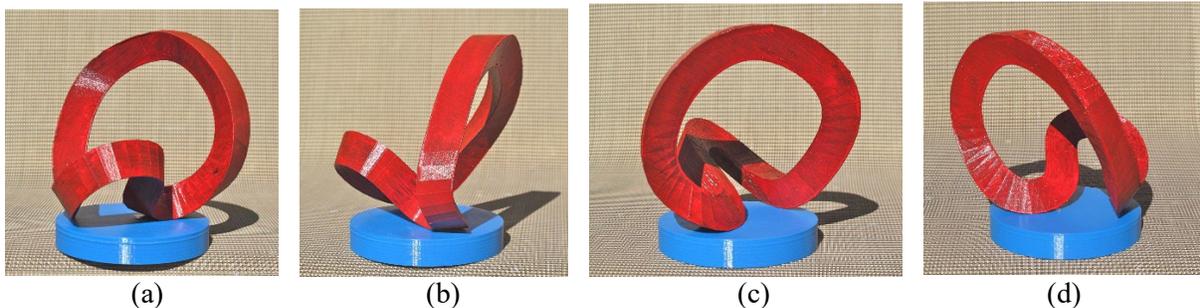
However, extracting the exact shape of the sweep curve underlying this sculpture was still a challenge. I first guessed that the sweep curve might have  $C_2$ -symmetry, and I created a corresponding control polygon (Fig.3a). I started out by designing the tight S-shape in the center of the sculpture, and then I connected its two ends with a big arch above it. This resulted in a sweep that did not seem to be off too far (Fig.3b).

With the sweep curve roughly following the anticipated shape of the sculpture, I tried to fine-tune the azimuth and twist values locally along the space curve to make the prism faces point in the proper directions shown in the five pictures of the sculpture (Figs.1&2). But nothing seemed to work out; I could not get the desired flow from the top face of the arch down to the two ends of the more tightly curved part of the sculpture, forming the central S-shape. Since the S-curve was not lying in the  $x$ - $y$ -plane, but was tilted at an angle of about  $45^\circ$ , the big arch had to connect in two different ways to the two ends of this S-shape. Thus,  $C_2$ -symmetry was out of the question.



**Figure 3:** Modeling "Sea & Sky": (a) First symmetrical control path; (b) resulting triangular sweep. (c) Better modeling of the central S-shape and constructing a first smooth connection to the big arch. (d) Ultimate partitioning of the CAD model into four parts with individual parameters.

To make it easier to assign the proper values of azimuth and twist in the various segments of the sculpture, I decided to model the overall sweep in two separate parts: the central S-shape and the big arch. The three views in Figure 2 show that the two small hairpin turns forming the central S-shape do not have the same radii; the hairpin turn on the left was about 30% smaller than the one on the right. I individually reshaped the two halves as two horseshoes with different radii and connected them back to back. The big arch was modeled with a separate sweep with some control of its radius, its out-of-plane helical distortion, and, of course, its values for azimuth and twist. To obtain a smooth connection with the central S-shape, I overlapped the various cubic B-splines defining these shapes with three control points at either ends. This decomposition resulted in a first CAD model, that seemed to be a reasonable fit to Perry’s sculpture, and I made a corresponding part on a 3D printer (Fig.4). This model has the right amount of twist in all segments of the sculpture, and it properly connects with the supporting base, lying flat with one face of the S-shape, and making a “pointy” contact at the other end of the big arch.

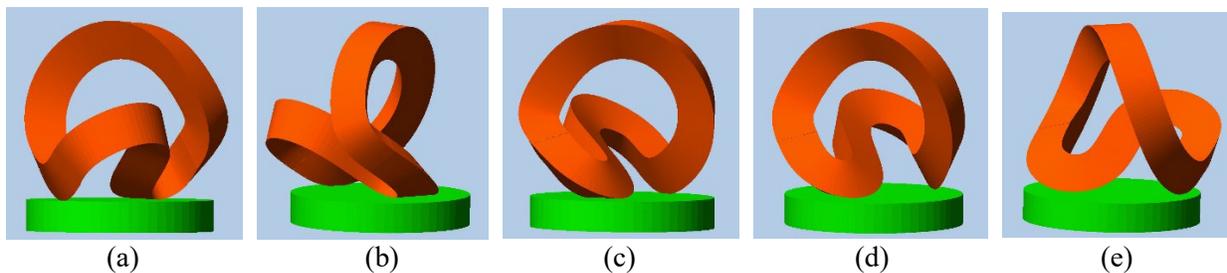


**Figure 4:** *My first model of “Sea & Sky” – four different views.*

But there were still visual differences. First, my red-painted plastic model is too skinny! Studying Figures 1 and 2, the total height of the sculpture appears to be about 4 times the width of the prism faces that encircle the whole sculpture. In my model the sculpture height is between 5 and 6 times the face-width. There are other noticeable differences. Comparing Figure 4b with Figure 1b, the big arch is leaning too much to the right. Comparing Figure 4c with Figure 2a, the arch is bulging too much towards the upper right. Comparing Figure 4d with Figure 2c, the arch is too big in the upper left, and it is leaning left.

I was surprised to learn how difficult it was to determine best values for all the parameter in the sculpture model, – or even to decide how many parameters should be built into the model in the first place!

### Fine-Tuning the Model



**Figure 5:** *Final CAD model of “Sea & Sky”, closely matching Perry’s original sculpture.*

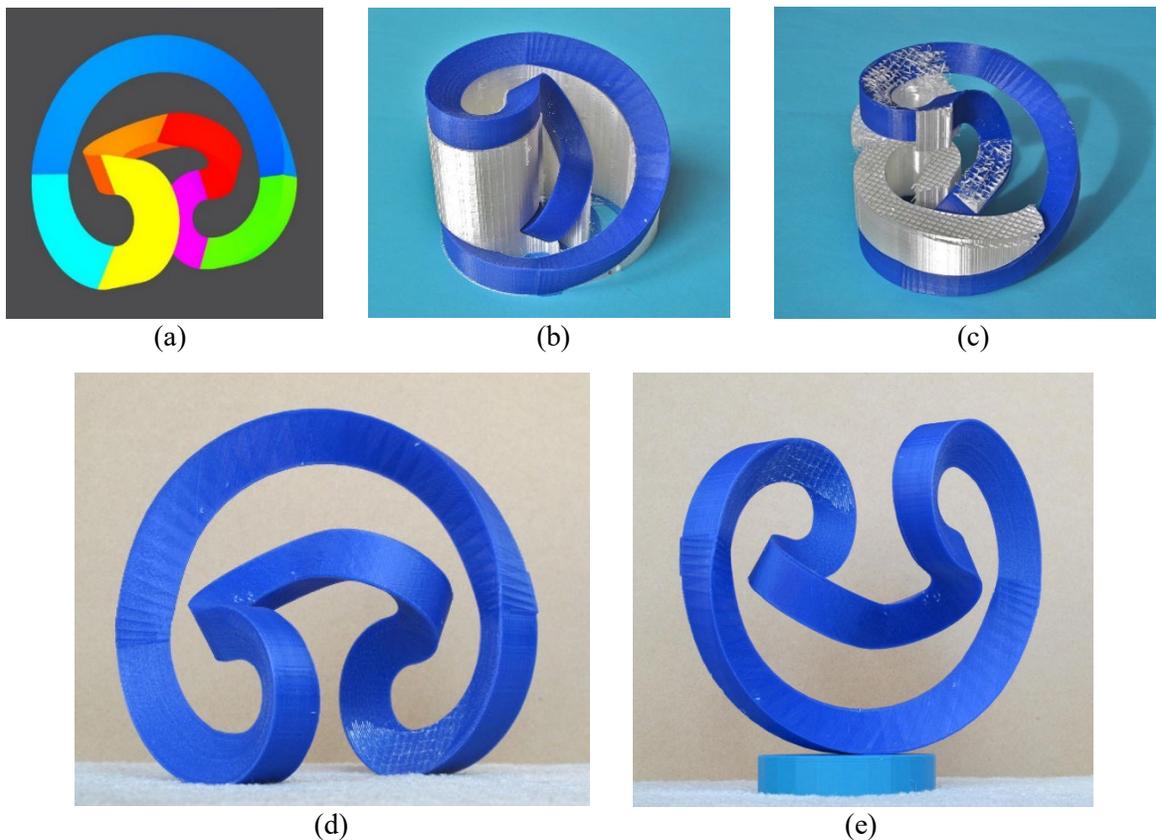
To obtain a model that was a better match to Perry’s sculpture, I partitioned the model into four separate sweep segments (Fig.3d) with their own azimuth and twist values, and I introduced a few parameters that control the geometry of the underlying sweep curve, such as the withs of the two hairpin turns in the central S-shape. By adjusting these parameters and frequently comparing suitable projections with the five images of the actual sculpture, I was able to obtain a much better CAD model. In the final model there are 14

geometrical parameters. The resulting combined control polygon of a cubic B-spline that defines the complete sweep path has 18 control points. To put all these points into the “right place” would require the adjustment of 54 coordinate values and would likely result in a somewhat “wobbly” sweep path. The geometrical parameters, such as the width of the two hairpin turns in the central S-shape, or the radius of the over-arching lobe allow a more purposeful, high-level fine-tuning of the sculpture. Figure 5 shows five displays of the model that correspond to the sculpture photos in Figures 1 and 2. The match is not perfect; but I don’t know how to make it any better without visiting the actual sculpture and taking direct measurements.

### A First Derivative Sculpture

At this point, I was so fascinated by this triangle-sweep along a curvy space path that I embarked on several additional quests. In particular I wanted to make more intriguing sculptures with more hairpin turns. Without any additional constraints, it is easy to make a plethora of CAD models by sweeping a triangle along an arbitrary closed 3D space curve, adjusting the overall twist to make sure that the twisted prism beam closes smoothly onto itself, and then adjusting the azimuth angle to maximize the aesthetic appeal, or to obtain a convenient connection of the sculpture with an underlying horizontal supporting platform.

Therefore, I gave myself some constraints. I did not want to introduce twisting in the tight S-shape portions of the sculpture. All the twisting that was needed to close the sculpture smoothly – with or without any resulting Möbius twist – had to come through the large outer lobe(s). Moreover, I tried to introduce the overall  $C_2$ -symmetry that I was looking for in my original modeling “Sea & Sky.”



**Figure 6:** *My first derivative sculpture: “Sea & Sky & Space.” (a) CAD model; (b) 3D-print with supporting scaffolding; (c) partly cleaned-up 3D-print. (d,e) Two possible ways of installing this shape as a sculpture.*

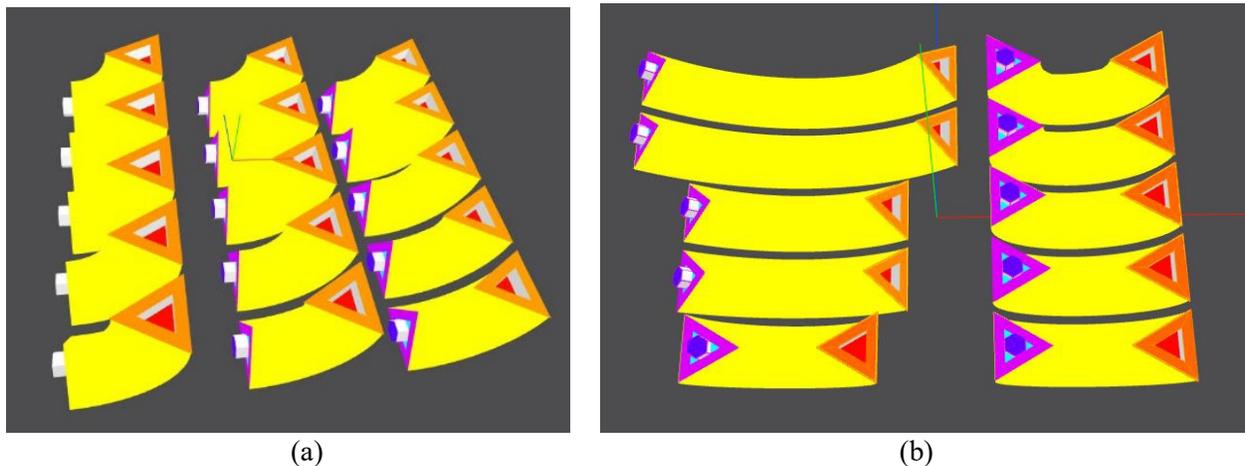
The CAD model of my first derivative composition is shown in Figure 6a. The central double-S-shape is composed of four semi-circular arches. They have no twist, and they all exhibit a perfectly cylindrical outer face while forming a hair-pin turn. Attached to the two ends of the double-S-section are two quarter-toroids with twice the radius of the central S-shapes. This results in a sweep path ending in two coplanar end-faces, which can then readily be joined by a large semi-circular arch that is properly adjusted in radius, azimuth, and twist to provide smooth closure. Overall, the composite shape has  $C_2$  rotational symmetry around a vertical axis. Only the large blue arc is twisted; all other parts are untwisted toroid segments with a cylindrical outer face. If the large arch is given a minimal amount of twist, the overall sculpture still has a prismatic Möbius twist of  $240^\circ$ .

Figures 6b and 6c show how this shape emerges from the 3D-printer and how the white support structure is being removed. This composite shape could now be installed as a sculpture in different ways (Fig.6d,e). I felt that the presence of the additional hairpin turns, forming a more complicated derivative sculpture should also be reflected in a more extensive name of the sculpture; therefore, I call it: “Sea & Sky & Space.”

### Modular Assemblies

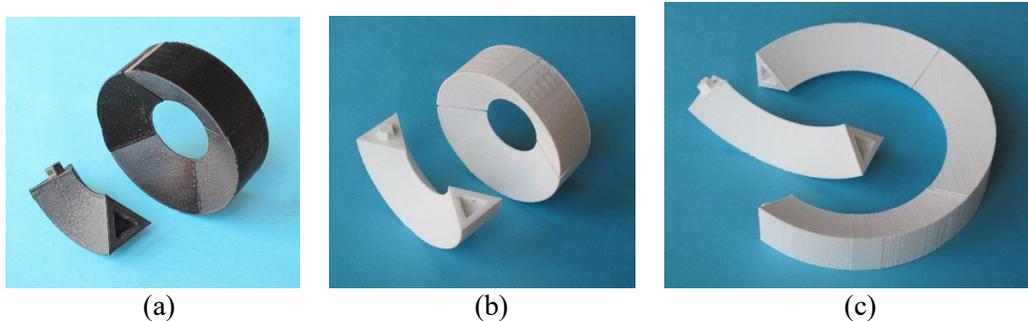
“Sea & Sky & Space” has several identical segments; the four small half-toroids as well as the two quarter toroids with a doubled radius. This raised another question: What interesting and aesthetically pleasing sculptures can be made out of a set of pure toroidal segments that themselves have no twist at all? I also wondered, what sculptures can still be made, if I restrict myself to using only toroidal segments where the outermost prism-face has a symmetrical, cylindrical shape. Could such a modular sculpture still form a closed loop, – perhaps even exhibiting a prismatic Möbius twist?

I found it difficult and tedious to put together a half-dozen or more of such geometrical elements in a typical interactive CAD tool and trying to figure out how they possibly might be closed smoothly into a loop. Thus, I created some physical props to help me in this investigation. Figure 7 shows some of these snap-together, LEGO-like “*MoTo*” (“Modular Torus”) parts. I fabricated parts with three different torus radii of 1.25 inches, 2.5 inches, and 3.75 inches, sweeping through  $90^\circ$  or through  $180^\circ$ .



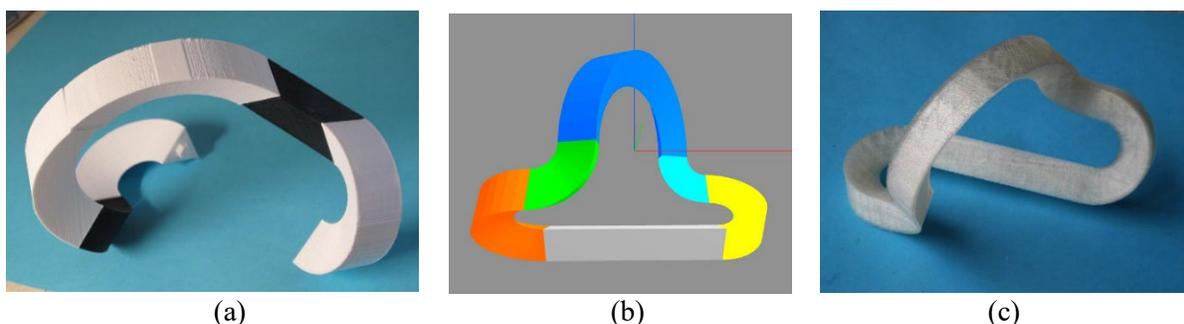
**Figure 7:** CAD models for modular “*MoTo*” (“Modular Torus”) parts: (a) 15 small quarter-toroids; (b) 6 small half-toroids, 2 double-sized quarter-toroids, and 2 triple-sized quarter-toroids.

Figure 8 shows three types of *MoTo* parts fabricated on an inexpensive 3D-printer. They represent simple quarter-toroids (Fig.8a,c) and half-toroids (Fig.8b) and their possible assembly into complete toroids. Playing with these parts, creating some small assemblies, was fun and inspiring, and it gave me a better understanding how these parts might be combined into non-trivial closed loops.



**Figure 8:** Three types of “MoTo” parts for creating modular triangle sweeps.

Figure 9 shows how these *MoTo* parts allowed me to find a first solution to making a prismatic Möbius loop with  $120^\circ$  of twist and  $C_2$ -symmetry. The loop employs six toroidal segments: A large  $180^\circ$ -arc, made of two parts shown in Figure 8c, two small  $180^\circ$ -arcs with half the radius (Fig.8b); two  $90^\circ$ -bends with the same radius (Fig.8a), and a straight prismatic closing rod, which can be understood as an infinitesimal fraction of a toroid with an infinitely large radius. Figure 9c shows a corresponding 3D-print.



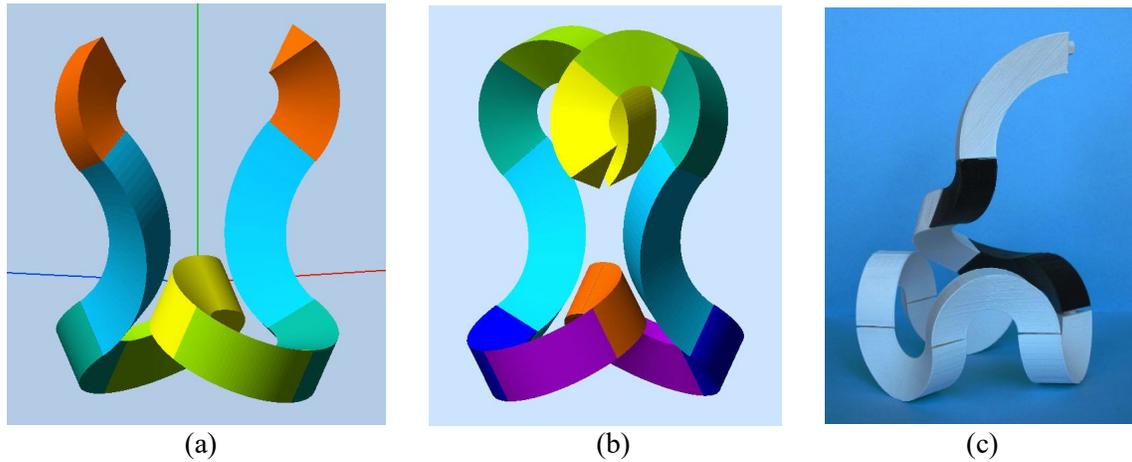
**Figure 9:** A prismatic Möbius loop, using six toroidal arcs: (a) Concept study with “MoTo” parts; (b) CAD model showing the individual components; (c) corresponding FDM print model.

However, for the exploration of more complex composites, the *MoTo* parts were not good enough. The junctions were not snapping together as cleanly and robustly as they do in real LEGO parts. Thus, the resulting angles of rotation were not maintained very accurately; and larger compositions often fell apart when I tried to attach additional *MoTo* elements. Thus, I resorted again to the computer.

The design experiments described here have been carried out in the context of some home-brewed CAD systems. Our design environment has evolved from Berkeley SLIDE (Scene-Language for Interactive Design Environments) [8] to NOME (Non-Orientable Manifold Editor) [7] and to JPCAD (Joint Interactive & Programmable CAD) [1]. They all allow the user to start with some short program fragments to define a sweep curve and its cross-sectional profile. These programs typically have a set of geometrical parameters that can be adjusted interactively through on-screen sliders, while the overall resulting geometry is displayed. SLIDE started out as the design environment used in the Computer Graphics and Modelling course CS 184. NOME and JPCAD incrementally introduced a capability to add and/or modify geometry in an interactive setting on a graphics screen. The main challenge in developing this editing capability was in finding ways to integrate these manually added enhancements into the original program that describes the complete sculpture geometry so that all geometrical parameters remain functional [7].

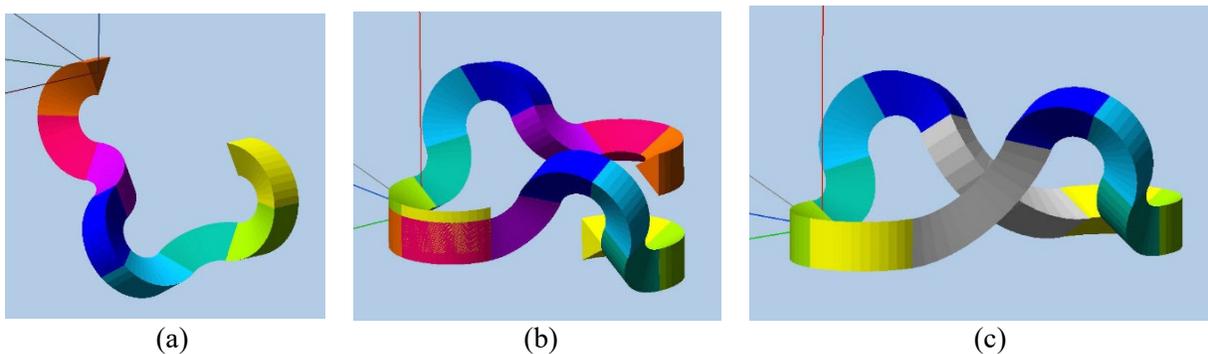
In this environment, I set up a process that allowed me to create sequences of *MoTo* parts with precise alignment of subsequent toroidal components. The set-up mimics a “turtle walk” [9] in 3-dimensional space. Any new toroidal segment to be added to the current chain is always moving the turtle in the direction of the negative  $z$ -axis, with its “starting-face” lying in the  $x$ - $y$ -plane, and with one corner of the triangular cross-section pointing in the negative  $y$ -direction. To make sure that a new piece forms a smooth

continuation of the previously constructed worm, the whole chain must be placed into 3D space in such a way that its “ending-face” lies in that same standard position in the  $x$ - $y$ -plane. The new segment can then be added with three possible azimuth rotations of  $0^\circ$ ,  $120^\circ$ , or  $240^\circ$  degrees around the  $z$ -axis. This new augmented assembly must then be transformed to compensate for the implicit additional transformation introduced by the newly added segment, so that its final end-face lies again in the standard position. In this manner, arbitrary long *MoTo* chains can be composed with full precision. This design environment makes it easy to compose “wild,” open-ended sculptures with the few types of available *MoTo* parts (Figure 10).



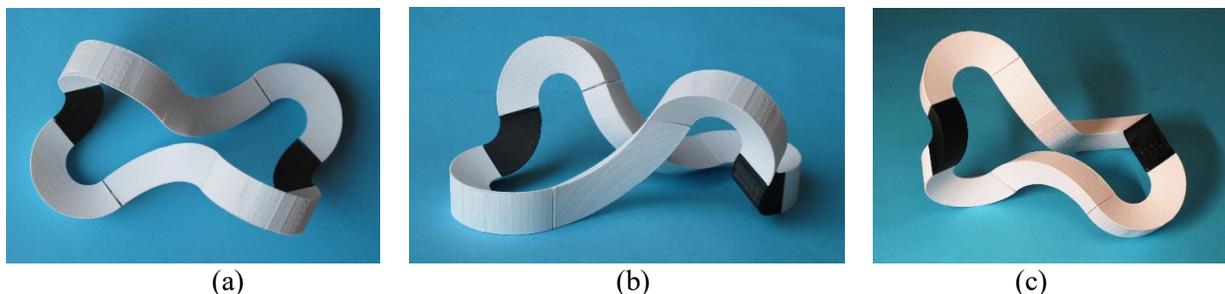
**Figure 10:** *MoTo*-chains forming modular, open-ended, sweep sculptures.

It is a bigger challenge to close such a wild sequence of *MoTo* parts into a smooth loop. One approach that worked for me, was to line up and/or connect two copies of such a wild chain so that they almost form a loop. Then I was looking for possible modifications by inserting an additional part or two, or replacing a small toroidal bend with a larger one. Figure 11 illustrates this process. Starting with the wild chain shown in Figure 11a, I placed two copies of it head-to-toe, with some of the hairpin bends overlapping (Fig.11b). I then removed the overlapping parts and joined the two chains. This brought the other two ends into a vertically aligned configuration. This vertical offset could then be eliminated by replacing the violet quarter-tori with the larger, white components. The result is a  $C_2$ -symmetrical loop with a  $240^\circ$  Möbius twist, composed of 12 quarter-toroids (Fig.11c).



**Figure 11:** *3D Turtle worms:* (a) Some wild trial chain; (b) two copies combined with the goal to form a closed loop; (c) modified chains, stretched by the white segments with twice the basic radius; this indeed forms a closed loop.

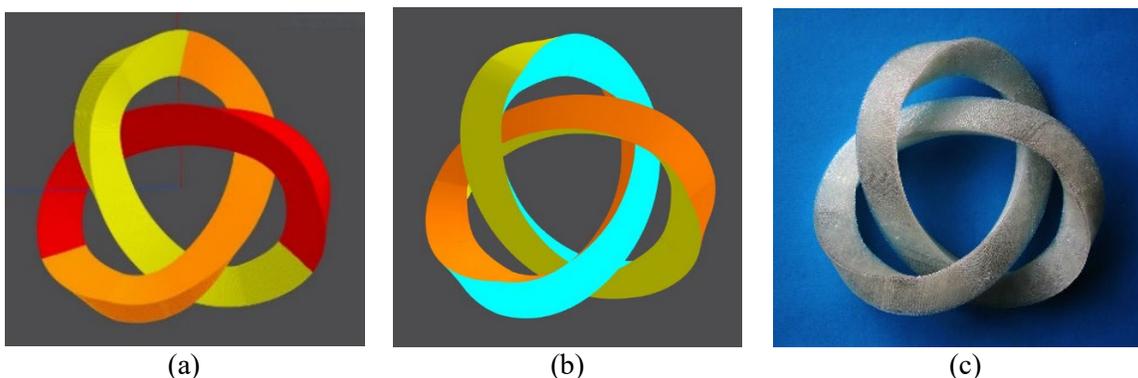
Figure 12 shows three different views of a physical realization of this closed, modular Möbius loop realized with the three types of *MoTo* components introduced in Figure 8: four small  $180^\circ$ -bends (white), two standard  $90^\circ$ -bends (black), and two enlarged  $90^\circ$ -bends (white).



**Figure 12:** A modular Möbius loop realized with eight “MoTo” components; three different views.

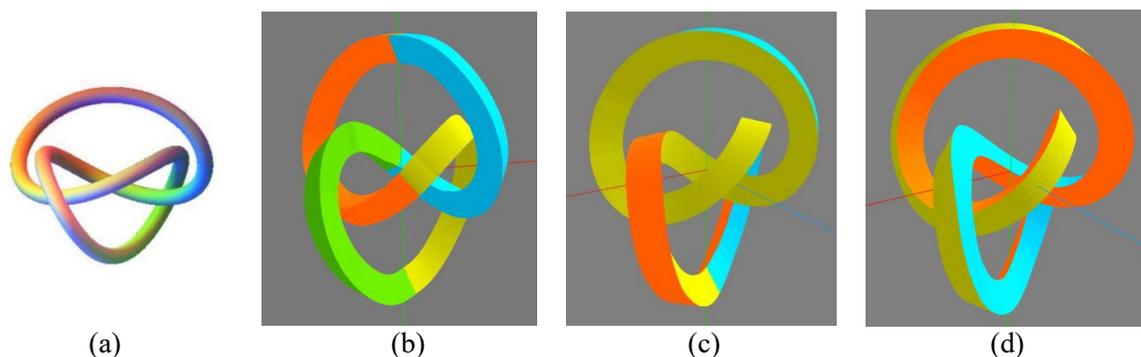
### Knotted and Twisted Space Curves

My next goal was to create prismatic sweeps of this kind through 3D space that form non-trivial mathematical knots, – possibly with some built-in Möbius twist. It is easy to form a smooth, symmetrical, spline-based trefoil with 6-fold  $D_3$ -symmetry with a minimal amount of torsion. Such a triangular prismatic strand does not exhibit any Möbius twist (Fig.13a). If I now want to introduce additional twisting while still maintaining  $D_3$ -symmetry, I must add twist in increments of  $120^\circ$  to each lobe. This then sums up to  $360^\circ$  of twist for the complete knot; thus, the prism beam still does not have a Möbius twist (Fig.13b,c).



**Figure 13:** Trefoil knots: (a) with minimal twist; (b) twist enhanced by  $360^\circ$ ; (c) 3D-print of (b).

To obtain a Möbius-twisted 3-sided prismatic ribbon, I must break the 3-fold symmetry. I can do this in a graceful manner by using a trefoil configuration (Fig. 14a) that has 4-fold  $D_2$  symmetry (Fig. 14b). The least twisted-looking,  $D_2$ -symmetrical shape is achieved for a total twist  $103^\circ$  along the full strand. But this sweep actually has a Möbius twist of  $240^\circ$  (Fig.14c); it takes an extra twisting of  $-240^\circ$  to line up the colored prism faces so that they join with themselves (Fig.14d).



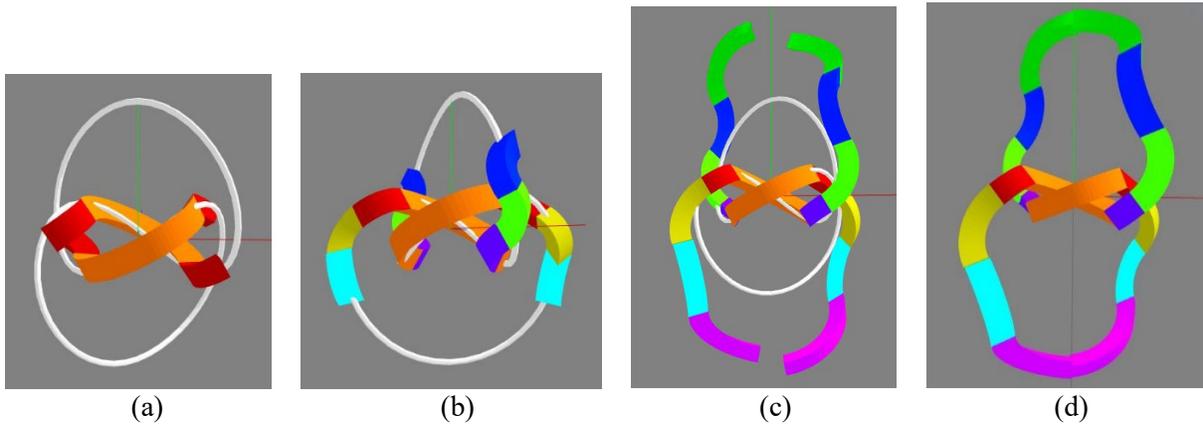
**Figure 14:** Trefoil with  $D_2$  symmetry: (a) Knot diagram; (b) spline-based sweep showing symmetry; (c) displaying the  $240^\circ$  Möbius twist; (d) colored faces lined up by subtracting  $240^\circ$  of twist.

## Modular Loops

It is not difficult to form a knotted prismatic Möbius trefoil with a smooth sweep. The real challenge is to construct such a knotted Möbius prism with my very limited set of *MoTo* parts. Also, what might be the minimal number of parts required to do this?

Figure 15 shows an attempt to construct such a sculpture with  $D_2$  symmetry in the spirit of Figure 14b. I was using interactive computer graphics and a 3D turtle-walk to build a suitable chain of *MoTo* parts to represent one of its color components and then obtain the whole sculpture by assembling four suitably rotated copies of that chain. It first seemed natural to start the chain construction on the  $z$ -axis at the tip of one of the big lobes. But it then occurred to me that it would be better to start with the intertwined helical branches to obtain a good geometrical solution in the dense central region of this sculpture. I could then use the wider, open spaces of the big lobes to introduce the necessary sweep path contortions to direct the two branches towards a merger with proper alignment on the  $z$ -axis. Figure 15a shows the placement of the first two *MoTo* parts, including the properly rotated copies to maintain  $D_2$ -symmetry.

In Figure 15b I have added a couple more *MoTo* parts. They coarsely follow the desired knot curve. But the face-normals on their end-faces point in rather weird directions, and it is not clear how I might achieve closure between them with my small set of *MoTo* parts. Perhaps it would be better to keep track of the critical end-face normal directions and accept much stronger displacements from the desired knot curve. If I find an arrangement that lets the normals at the ends of the chains point in the proper directions, then I can try to use *MoTo* parts with different radii to bring the ends of the two chains into close proximity (Fig.15c). At this stage, I can then rotate the two chains slightly around the  $y$ -axis (going through the center of the two orange *MoTo* pairs) and also translate them along this axis to bring the chain ends into contact (Fig.15d). If I am lucky, I may get the desired smooth connection between the two prismatic sweeps.



**Figure 15:** *Composing a modular trefoil: (a) Start with the central, intertwined branches; (b) add MoTo parts along the knot curve; (c) add more parts to have a chance at closure; (d) rotate and translate the two separate chains to “join” their ends.*

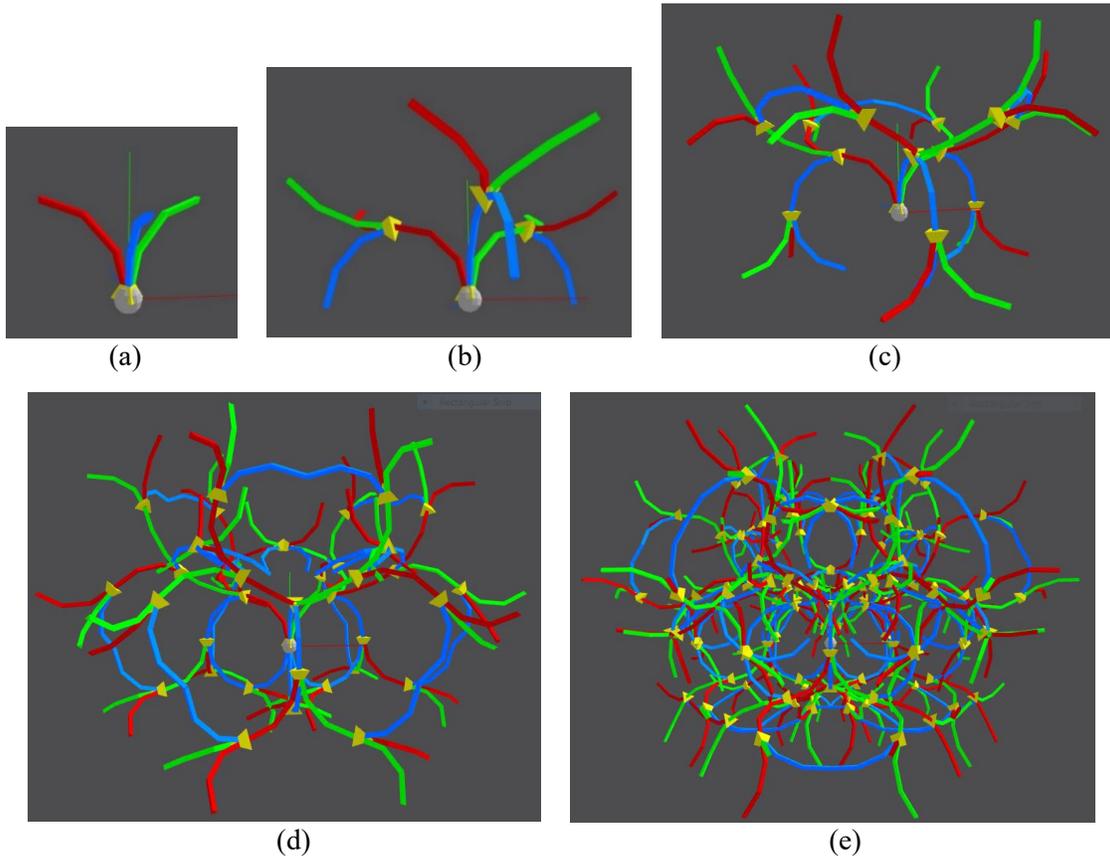
Close inspection of the tips of the two lobes (Fig.15d) reveals that there is a slight misalignment in the azimuth angles of the two prism ends that need to be joined! Since I am working with a small set of standard, predefined parts, I have no “twist parameter” that could take care of this problem. It may seem rather hopeless to work through all possible combinations of *MoTo* parts to find one that yields perfect closure. Perhaps finding the Möbius loop with perfect closure shown in Figures 11 and 12 was just a fluke?

At this point it occurred to me that perhaps all possible end-face normals in a perfectly formed *MoTo* chain might occupy a relatively sparse set of all directions in 3D space, since each quarter-toroid bends through exactly  $90^\circ$ , and the connection to the subsequent part can only introduce azimuth rotations in increments of  $120^\circ$ . Thus, the Möbius loop (Fig.11) was not a fluke; it just followed, forwards and backwards, through a small set of such discrete normal-directions. Since I started my Trefoil design with

two independent *MoTo* chains, I would have to make sure that their end-face normals were lying in the same direction sets. With two chains covering the same set of end-face normals and leaving all rotations unchanged, I can then look for appropriate translations obtained by changing the radii of some of the *MoTo* parts in the two chains.

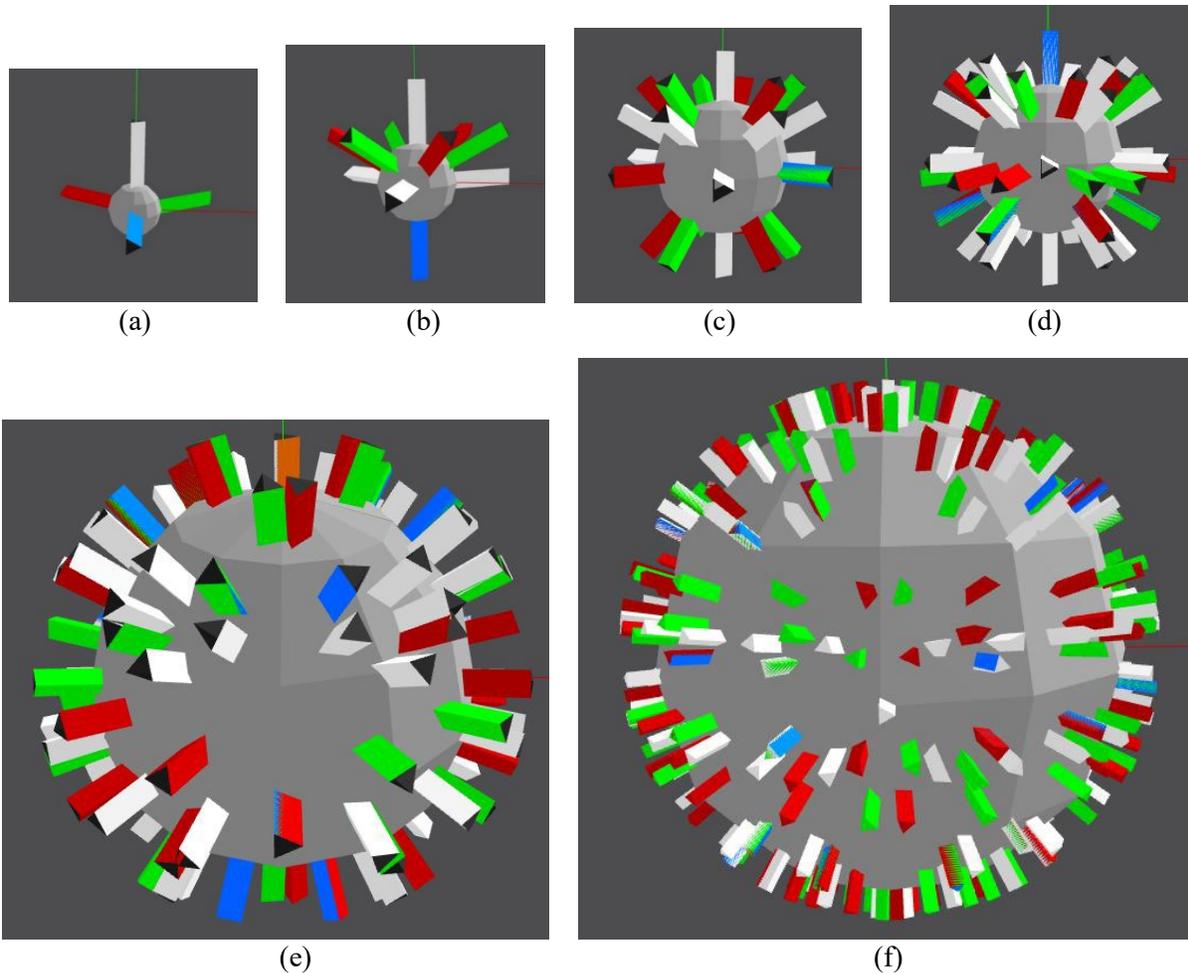
### The *MoTo* Direction Space

To explore the direction-space of quarter-toroids with a triangular cross-section, I tried to construct a complete lattice that a turtle crawling up the  $z$ -axis could visit. As its next move, the turtle must perform a  $90^\circ$ -turn in three possible planes that contain the  $z$ -axis. Showing all three possibilities in (R, B, G), forms a little “bush” with three branches (Fig.16a). Next, I place three such bushes at the ends of the original three branches, thus forming a first “tree” structure (Fig.16b). Now, placing three such trees at the ends of the initial bush, forms a first “super-tree” structure (Fig.16c). Continuing this construction recursively, I obtain the next two tree structures, shown in Figures 16d and 16e.



**Figure 16:** Connection possibilities yielding: (a) a level-1 “bush” at first junction, (b) a level-2 “tree,” (c) a level-3 “super-tree,” (d) a level-4 “network,” (e) a level-5 “network.”

These networks do not look like they are converging into some well-defined lattice structure. So, rather than rendering the actual quarter tori, Figure 17 just displays the directions of the end-face-normals (from the center of the grey sphere), so that the ends of those direction-vectors stick out slightly beyond the surface of the sphere. Progressing through just six recursion-steps makes it clear that the achievable directions of the end-face normals do not form a small discrete set, but comprise a dense distribution. This is good news and bad news. It implies that any two arbitrary directions can probably be connected with a legal turtle-walk path composed of quarter-toroids with triangular cross sections. On the other hand, this may take an arbitrary large number of turtle steps!

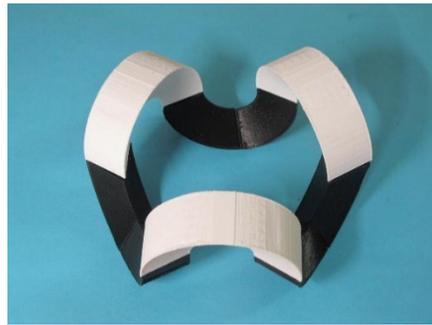


**Figure 17:** Direction vectors for: (a) level-1 “bush,;” (b) level-2 “tree,” (c) level-3 “super-tree,” (d) level-4 “network,” (e) level-5 “network” (side view), (f) level-6 “network” (both directions).

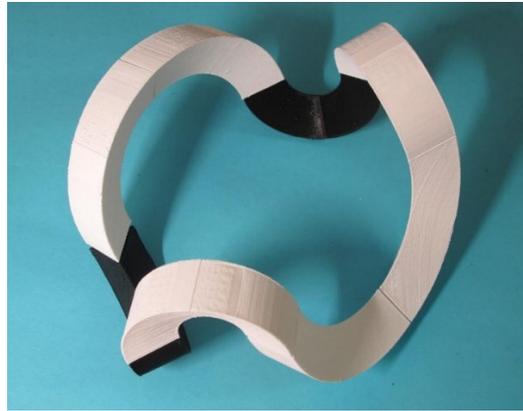
In view of this, when trying to form a closed loop, it might be good to voluntarily restrict ourselves to a limited set of end-face directions and use some kind of compensation scheme, where every bend in the turtle path is matched with an equivalent bend somewhere else in the loop. Figure 18 shows a basic example. Six half-toroids bending through  $180^\circ$  can readily be combined into an undulating, “Hex-ring” (Fig.18a). In this closed loop, we can now replace the white half-toroid in the upper-left and the black half-toroid in the lower-right with two half-toroids with a doubled major radius. This elongates the *hex-ring* along a line going from lower-left to upper-right (Fig.18b). No angles have been changed, all the joints are still perfectly aligned, and only four end-face directions have been used, – one along the z-axis and three more in the x-y-plane.

Moreover, we can even replace these larger half-tori with asymmetrical arches composed of one large white  $90^\circ$ -bend and one small black  $90^\circ$ -bend (Fig.18c, at left and right extremes of the loop) and still maintain alignment at all joints. The *hex-ring* can be further distorted by also replacing the small black half-torus in the lower-left and the small white half-torus in the upper-right with such asymmetrical arches.

It is also useful to look at substitutions that break the closed loop. Figure 18d demonstrates that we can replace opposite pairs of the original small *MoTo* parts with larger  $90^\circ$ -bends, so that all expansions in the x-y-plane balance, but the offsets in the z-direction accumulate and result in a net vertical offset between the pair of end-faces in the broken ring. This whole twelve-part *MoTo* chain could now replace a simple, straight, vertical “rod” with a triangular cross-section, shown in grey in Figure 18d.



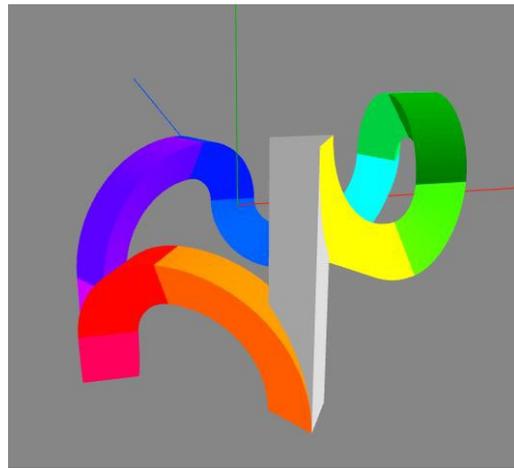
(a)



(b)

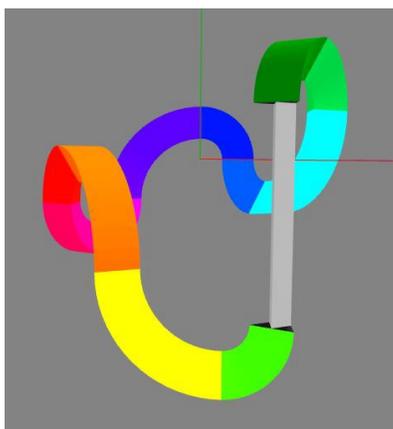


(c)

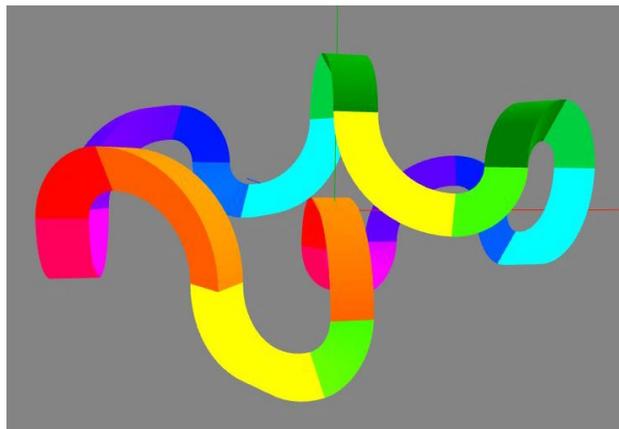


(d)

**Figure 18:** (a) A symmetrical hexagonal ring. (b) Linearly extended version of this ring using two larger half-tori. (c) Larger half-tori replaced by combinations of black / white quarter toroids. (d) Unbalanced substitution leading to a pair of vertically offset end-faces, mimicking a “rod.”



(a)



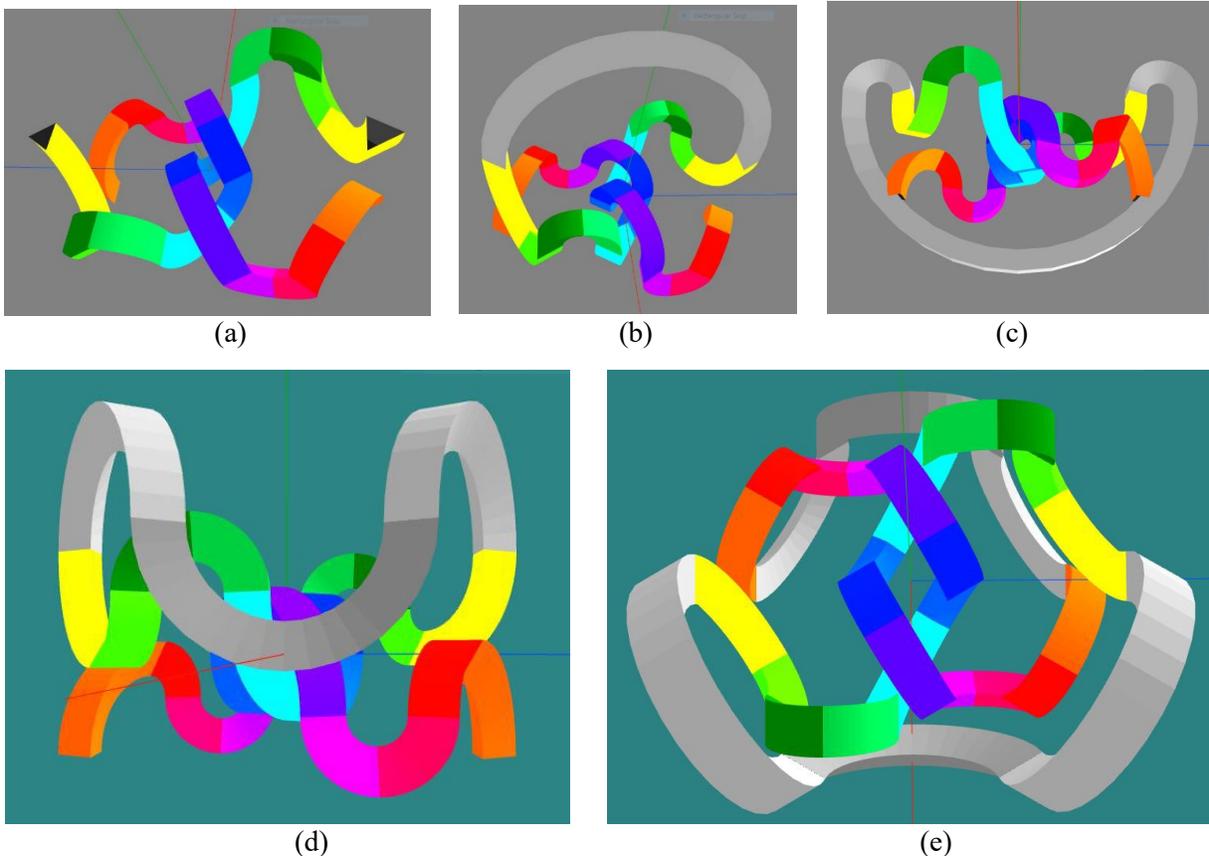
(b)

**Figure 19:** (a) Opposite imbalance of the vertical offsets, leading to a “clamp” configuration. (b) Combining the “clamp” and the “rod” versions of the imbalanced undulating hex-ring.

By driving the imbalance in the opposite direction, we can also produce a “negative” vertical offset between the pair of end-faces (Fig.19a). This *MoTo*-chain now acts more like a “clamp” than like a “rod.” We can also combine a “clamp”-like *hex-ring* and a “rod”-like chain with identical vertical offsets. This results in an intricate looking figure-8 shape. All the directions of the end-face normals are still exactly the same as they were in the initial undulating *hex-ring*. Thus, all the joints between the *MoTo* parts are geometrically precise. Such an approach can then lead to many more modular derivative sculptures of “Sea & Sky.”

Since the *hex-ring* proved to be such a versatile structure, I started to use it in my attempts to make a knotted path composed of my set of *MoTo* parts. I started by interlinking two of the “rod”-like *hex-rings* (Fig.20a). Connecting pairwise the two yellow and the two orange stubs with some closing arches, would produce a Trefoil knot. Fortunately, the two pairs of end-faces lie in the same planes and are nicely lined up, so that a simple semicircular arch can connect them (Fig.20b). However, this arch is not one of my “standard” *MoTo* parts; it features a ridge on the outside, rather than the desired cylindrical surface.

I tried to fix this problem by bending the arch “the other way around” and looping it around the bottom of the sculpture. To do this, I need to introduce two small half-toroids that stick out like “ears” at the top of the assembly (Fig.20c). This works well for one arch. But the other arch, connecting the two orange stubs, lies in the same central plane, and the two arches would interfere. I can move this arch into a different vertical plane by rotating the two “ears” out of the central plane. By aiming them towards each other and making them larger, I can also reduce the size of the closing arch (Fig.20d). By carefully choosing the offset between the two hex-rings, I can achieve that the four “ears” and the two closing arches all can be made with standard *MoTo* parts of radius 2. This produces the modular trefoil knot shown in Figure 20e.

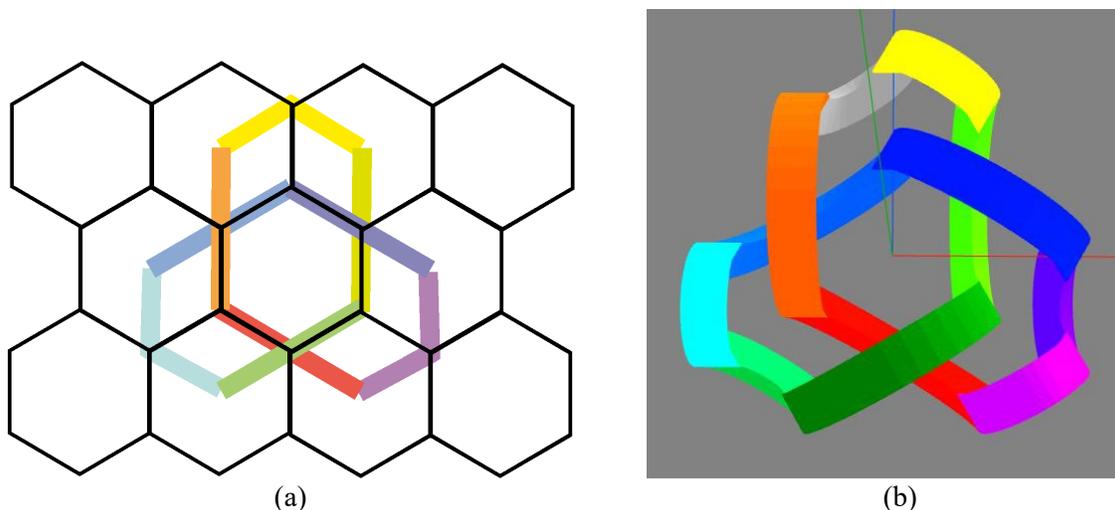


**Figure 20:** (a) Interlinked hex-rings of the “rod” type. (b) A non-standard arch above the top. (c) An “everted” arch around the bottom. (d) The “everted” arch moved to a new plane. (e) The completed modular trefoil knot with two “everted” arches.

The resulting trefoil knot is quite a “baroque” construction! But it is a proof of concept that a knotted structure can be built from all untwisted, cylindrical quarter-toroids. It uses 12 small and 24 larger quarter-toroids. It employs only four end-face directions. And it gave me hope that there are also simpler modular solutions with fewer parts. Starting with one or more closed loops and then making incremental substitutions seems like a useful approach. In particular, the universe of the undulating *hex-ring*, using only four end-face-normal directions, looks quite promising.

### An Elegant Modular Trefoil

Then one night, I suddenly saw a good, and in hind-sight rather obvious approach: Start with a hexagonal grid and draw the desired knot onto it. As a first example, the  $D_3$ -symmetric Trefoil is a prime candidate. Figure 21a shows how the Trefoil naturally fits onto the grid. Every straight line segment corresponds to a half-toroid above or below the  $x$ - $y$ -plane. Figure 21b shows the corresponding modular assembly of six small and six larger half-toroid *MoTo* parts. I am almost certain that no simpler solution exists.



**Figure 21:** (a) *Fitting a Trefoil knot onto a hexagonal grid.* (b) *The corresponding modular assembly of half-toroid MoTo parts.*

### Summary and Conclusions

This report has two main parts. The first one has an analytic nature. When attempting to create a CAD model of an existing, abstract, geometrical sculpture, that is available only through a few images, a crucial step is to find a good partitioning of the sculpture into some coherent geometrical *elements* that can be modeled conveniently using a few adjustable geometrical parameters. To model “Sea & Sky” by Charles O. Perry (see also [2], pages 56-57), two natural *elements* emerged in the form of the hair-pin turns in the central S-shape and the over-arching lobe that connects the two ends of the S-shape, thus forming a closed loop. Because of a lack of obvious symmetry in Perry’s sculpture, I further partitioned these elements into two slightly different halves. In other sculptures, such *elements* may occur more than once; they can then be used to capture some inherent symmetry or modularity of the sculpture. They also play an important role in the construction of new derivative sculptures.

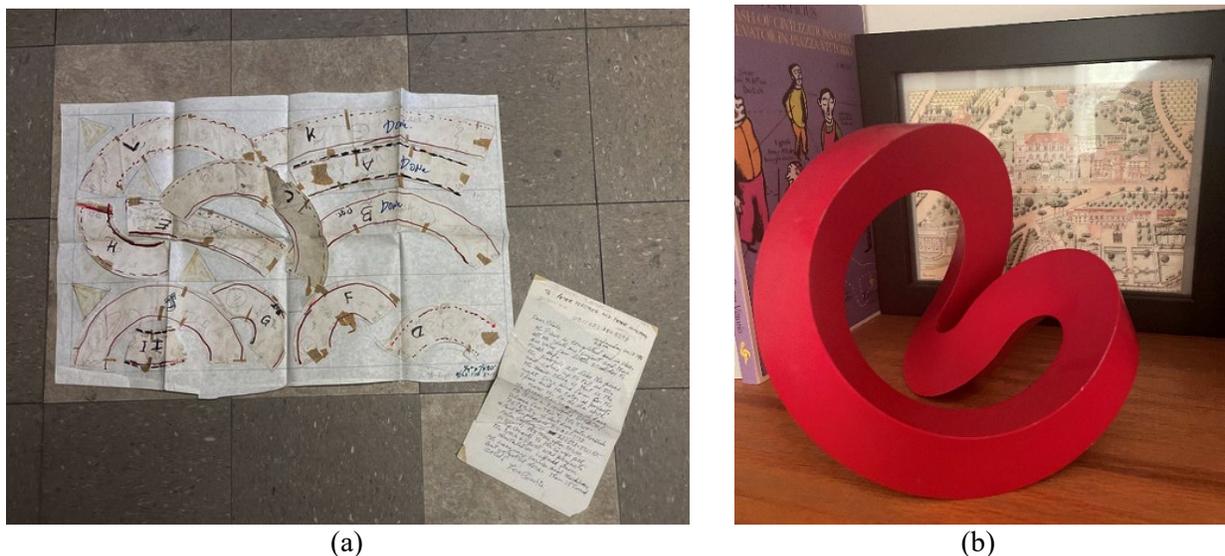
The second part of this report is more constructive. The geometrical *elements* found in “Sea & Sky” are reused in different combinations to form new sculptures that can be understood as belonging to the same family of shapes. A crucial issue is how to connect individually generated prismatic sweep *elements* into an overall smooth and elegant sweep structure. One approach is to form an overall smooth B-spline curve by suitably overlapping the ends of the control polygons of the individual *elements*.

A more challenging problem is to define a small set of modular components, such as quarter-toroids of different radii, that can then form a large variety of interesting sculptures through simple assembly. I found it difficult to explore this new domain in a completely virtual, interactive graphics approach. Therefore, I fabricated a set of physical snap-together parts to allow quick and open-ended exploration. However, the low quality of these 3D-printed parts limited the size of stable models that I could build to about ten parts. The approach that worked best for me was to construct small physical combinations of about 4 to 8 toroidal parts, that looked promising when combined into larger assemblies. Short chains of *MoTo* parts could then be combined into more complex 3D assemblies using our JIPCAD [1] modeling environment. This combination of virtual and physical exploration turned out to be vital.

Thus, many of the explorative experiments and resulting designs employed all components of the “SCULPT” concept. I sketched layouts (“L”) of virtual *MoTo* parts in an interactive graphics tool; I also programmed (“P”) turtle-walk sequences of *MoTo* parts that guaranteed perfect alignment between subsequent components; and I used 3D-print technology (“T”) to make tangible components for conceptual studies of possible *MoTo* chains. A proper interplay of all three design approaches (“P”, “L”, and “T”) will also be needed to construct additional models of elegant modular knots.

### Acknowledgments

I am infinitely grateful to Charles O. Perry for creating intriguing sculptures that are not only beautiful to look at, but also stimulate my own creativity. I am also grateful to his sons, Paul and Carlo, who helped me gain a deeper understanding of their father’s creations. Recently they provided me with crucial insights how “Sea & Sky” had been designed. Charles needed no computer. He was able to sketch individual surface pieces (Fig. 22a) that would then form cardboard models of various types of half-tori, which he used for design exploration. Different assemblies of such half-tori would then form perfect scale models for several different sculptures corresponding to sweeps of a triangular cross-section. These maquettes were so well done that, with a coat of paint on them (Fig.22b), they were often difficult to distinguish from the actual sculptures ([2], pages 62-65,74).



**Figure 22:** (a) Perry’s sketch of various surface pieces that form various half-tori.  
(b) Resulting assembled maquette made from plastic pieces and painted red.

Siebren Versteeg, one of the fabricators who helped building the large-scale sculpture had this to say (slightly paraphrased): "... It's interesting how some artists were imagining and executing geometries that seem so impossible to realize without digital computation. Charles Perry prepared complete scale models

from several surface pieces that were ready for enlargement to be cut from sheet metal and assembled. Charlie was truly a visionary!”

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