Making Graphics Tangible

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Abstract

This is a write-up of material presented at keynote talks at Virtual Reality 2012 and at Shape Modeling International 2012. Producing tangible, physical output becomes a growing role for computer graphics due to the emergence of inexpensive rapid-prototyping machines and on-line fabrication services. While producing small models with a layered manufacturing process has become very easy, creating larger and more durable objects still requires a designer to address an expanded list of issues. In this article some of these issues are discussed based on experiences the author has gained while designing various physical artifacts ranging from mathematical visualization models and geometrical puzzles to large scale sculptures. Tangible artifacts also gain importance at the beginning of a design process and as input to computer graphics tools. An argument is made and an outline is given for new ways and better user interfaces to enter the geometry of inspirational artifacts into our virtual CAD environments.

Keywords: Procedural modeling, tangible modeling tools, rapid prototyping, durable physical output.

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Figure 1: (a) CCD solid state image sensor; (b) Soda hall, U.C. Berkeley.

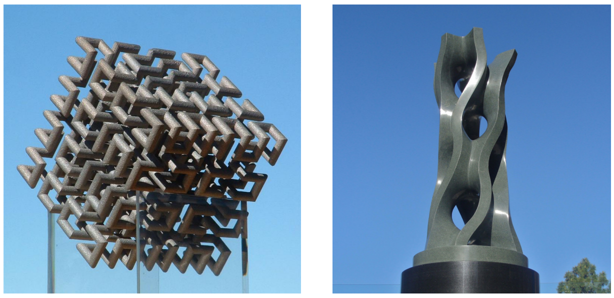

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Figure 2: Large and/or complex physical entities that started out as a computer graphics description: (a) *Hilbert Cube 512* ; (b) *Pillar of Engineering* .

1. Introduction

For most of my life geometry and geometrical constructions have been my profession as well as my hobby. In some instances, the realization of a particular design required an army of people and a major organization for turning the design into a usable physical artifact. My first job after graduation placed me in the group at Bell Labs that had just invented Charge-Coupled Devices (CCD). Over the next few years I designed ever larger solid state image sensors, culminating in 1973 in a CCD imaging chip with enough resolution to be compatible with the American broadcast TV format (Fig.1a). The production of three good chips to be placed around a color separation prism involved the fabrication of tens of silicon wavers, requiring several dozen individual processing steps in a very sophisticated fabrication line for integrated circuits. A decade later I was spearheading the design effort of Soda Hall, the home of Computer Science on the Berkeley campus. There again, the realization of that design took several architects, many contractors and subcontractors, and kept a large number of skilled workers busy for two years to produce the final building (Fig.1b).

Fortunately, not all physical artifacts take that long from initial concept to a first realization. Because of the emergence of rapid-prototyping (RP) machines and services, some smaller objects can now be realized in a matter of hours or days. This has had a tremendous impact on product design in many areas, ranging from car manufacturing to computers and to household items such as toasters, or sports articles such as running shoes. Most of these products now undergo a design cycle that involves a few rapid prototyping models. The final product however, still requires a much more involved and more expensive process, for instance the creation of sophisticated molds for injection molding.

In the world of computer graphics, every year hundreds of stunning renderings and virtual artifacts with which users can interact in video-game settings are being created with interactive modeling tools. Because of the emergence of affordable RP machines, e.g. MakerBots [1] and readily available fabrication services such as Shapeways [2] or Ponoko [3], an ever grow-

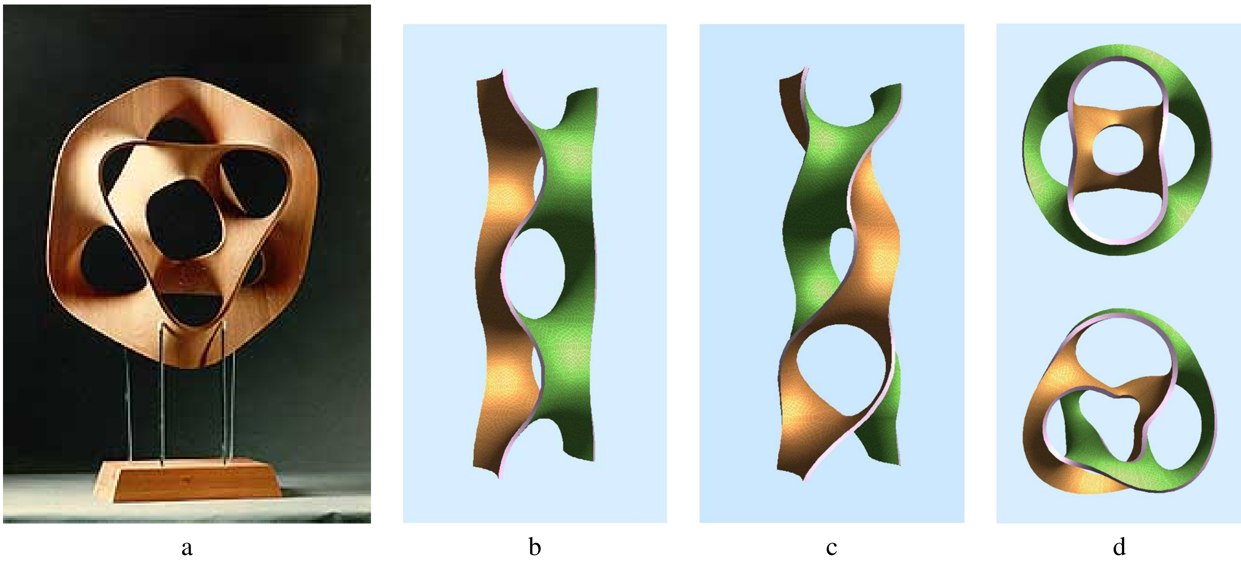

Figure 3: (a) Collins' *Hyperbolic Hexagon* ; (b) Scherk tower; (c) twisted Scherk tower; (d) corresponding Scherk tower loops.

ing fraction of these models may find their way into the world of physical reality. People produce small replicas of their favorite action figures, models of their designs in *Second Life* [4], custom-made coffee mugs or cookie cutters, or pendants and ear rings. But still, to produce an object that serves a functional purpose in your car, in some exercise machine, or in a hand tool like an electric drill, requires a more substantial effort that takes more than a single pass through the rapid-prototyping process.

In this paper I will describe what it takes to turn an appealing virtual design into a physical artifact in the context of mathematical visualization models (Fig.2a), geometrical puzzles, and large and durable public sculptures that can be touched by spectators and can withstand the abuse by weather (Fig.2b). Section 2 gives the background story of how I got started with computer-aided modeling of large sculptures. Sections 3 and 4 describe the realization of two large sculptures implemented with different fabrication technologies. Section 5 discusses additional concerns that come into play when one plans to make multiple copies of an artistic artifact such as an award trophy to be handed to several recipients every year. Section 6 focuses on making models on RP machines with the added constraint of models composed of multiple parts that must fit together smoothly, such as dissection puzzles; it also addresses approaches of reducing the fabrication costs of such models.

Sections 7 and 8 address design and user-interface issues, to the extent that this front-end activity may involve tangible physical artifacts. They conclude with a generalization of the procedural modeling approach that I have used in most of my sculpture designs, called *Inverse 3D Modeling*. They end up with a proposal of how to use tangible pieces of material as design building blocks in an immersive virtual work space.

Rapid prototyping technologies have advanced tremendously in the last decade and have now become quite affordable. Section 9 gives a brief preview of the possibilities that lie ahead in the near future.

2. Scherk Collins Sculptures

My interest in computer-assisted sculpture design started in 1995. It grew out of the need for a virtual prototyping tool that would permit a quick evaluation of a variety of abstract geometrical forms to determine which ones of many conceptual ideas would have the potential to be turned into a 3-dimensional physical sculpture that would look interesting and aesthetically pleasing from most directions. This need arose after I had made contact with sculptor Brent Collins [5], living in Gower, MO, and we started to have weekly phone discussions in which we generated more conceptual ideas than could possibly be examined and evaluated with physical mock-up models.

In 1994 I had come across an article by George Francis [6] in which he analyzed some of Brent Collins' wood sculptures from a topological and knot-theoretical point of view. One of the key elements in Collins' work is a composition of stacked saddles and intertwined tunnels. I was intrigued by the sophisticated way in which the edges of the various saddles merged seamlessly into one another and by the fact that all this complicated geometry resulted in smooth and balanced surfaces. In 1995, after I had come across an image of the graceful *Hyperbolic Heptagon* (Fig.3a), I picked up the phone and contacted Brent Collins. Based on the analysis by George Francis, this sculpture could be understood as a toroidal deformation of a 6-story "Scherk tower." The latter is a finite cylindrical cut-out (Fig.3b) from the center of Scherk's 2nd minimal surface [7].

Our first phone discussion started from the observation that making a good 3D sculpture is not easy, if you want to make it look good from all sides. *Hyperbolic Hexagon* shows some weaknesses in this respect, because from some angles it displays some strange coincidences. This is because it has too much symmetry: All six peripheral holes pass through the dominant plane at ± 45°; and thus from certain viewing angles they all close up simultaneously or show their full, open, circular tunnels.

a b c

Figure 4: Hyperbolic trefoils: (a) Séquin's *Minimal Trefoil* ; (b,c) Collins' *Trefoils* with different azimuth angles.

We contemplated whether this could be fixed by adding some twist into this structure, so that every hole would pass through the dominant plane of the toroidal sweep path at a somewhat different angle. Clearly such twist would have to be inserted in increments of 180°, so that the chain of saddles would still close smoothly onto itself. We also realized that, if there are an odd number of stories in this toroidal loop, an odd multiple of 90° twist is required to close the toroid smoothly; but then the surface becomes single-sided, thus forming a non-orientable Möbius configuration. Moreover, in the original *Hyperbolic Hexagon* all four edges form their own separate, closed loops. Adding twist to the Scherk tower interlinks the edges of this sculpture in interesting ways. For certain amounts of twist, these edges will merge into one another after one pass around the twisted toroid, and it will then take several laps around the toroid to reach the starting point again. Analyzing the multiple intertwined torus knots that can be formed in such twisted structures is rather intriguing.

These insights were mathematically fascinating, but Collins was interested in deriving new ideas for structures that would result in aesthetically pleasing physical sculptures. Typically, Collins first builds small scale models from PVC piping or embroidery hoops, fleshed out with wire meshing and bees wax, before he embarks on carving a large wood sculpture. Making a small scale model takes itself a couple of weeks. Thus it quickly became clear that Brent could not possibly keep up with making models for all the new concepts we discussed in order to judge their visual appeal.

At this point I decided that a computer-based visualization tool was our only hope of keeping up with our stream of ideas. The result was *Sculpture Generator I* [8], a special purpose modeling program written in the C language, the geometry kernel of which comprised only about 3000 lines of code [9]. Within a few months I had a first prototype running that allowed me to specify the number of stories in the loop, their combined total twist, the azimuth orientation of the edges around the smaller radius of the torus, as well as the thickness and extension of the flanges, and various simulated surface properties (Fig.5). Now I could try out many different ideas and parameter combinations in a matter of minutes, and then focus on the most promising configurations and optimize them for their aesthetic appeal. Or I could investigate such questions as: What is the tightest toroid into which a Scherk tower with only three stories could be curled up. The latter question led to my version of the *Minimal Trefoil* (Fig.4a). I gave myself the constraint that the tunnels would start out with a circular profile, trying to avoid any vertical stretching of the 3-story Scherk tower before closing it into a loop. I succeeded in doing this for an azimuthal orientation of 45° of the three saddles in the ring. Collins also hand-designed and built two trefoils with different azimuthal orientations (0° and 45°), but in both instances he stretched the hole-saddle chain substantially, so that he would obtain a central hole that was comparable to the tunnels in the periphery. He wanted to keep all of these tunnels large enough so that he could easily get one of his hand inside for the final carving and polishing (Fig.4b,c).

One key technical issue I had to address in the development of *Sculpture Generator I* was the question how I should best represent the geometry of these shapes to maximize the benefit of the inherent modularity and symmetry in these toroidal structures. Even though these shapes were inspired by the minimal surfaces formed by soap films spanning some boundary loop, they are not typically true minimal surfaces. For aesthetic reasons, most of the twisting ribbons connecting adjacent saddles have a much stronger lateral curvature than their longitudinal curvature. Thus I decided to model one quarter of a standard biped saddle with a stack of hyperbolas with ever more pointy hyperbolic curves that eventually end up as two straight lines crossing at a 90° angle at the saddle point (Fig.6a). Adjacent hyperbolas are connected with triangle strips to form a tessellated surface with an adjustable degree of approximation (Fig.6b). This small piece of surface can then be deformed as needed for a particular sculpture. Properly rotated, mirrored, and translated, it will form a complete 1-story saddle. The desired number of saddles can then be stacked on top of one another, and the whole assembly can be given a suitable amount of twist before it is bent into a loop and closed into the final toroid. If saddles of a higher order are desired, e.g., a 3-way "monkey saddle," the 90° sector forming the fundamental domain of a basic biped saddle can be compressed into a wedge

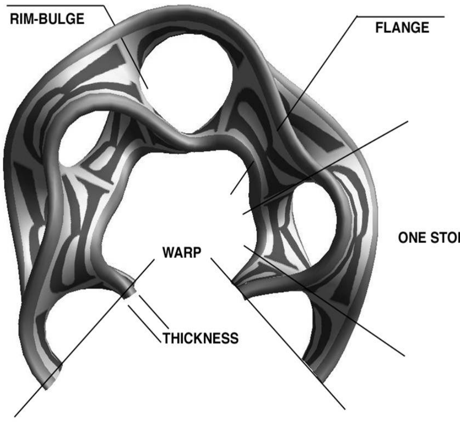

Figure 5: Design parameters in one story of the Scherk-Collins toroid.

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Figure 7: *Hyperbolic Hexagon II* : (a) 1997 wood master carved by Brent Collins; (b) 2009 bronze sculpture cast by Steve Reinmuth, installed in Sutardja Dai Hall at U.C. Berkeley.

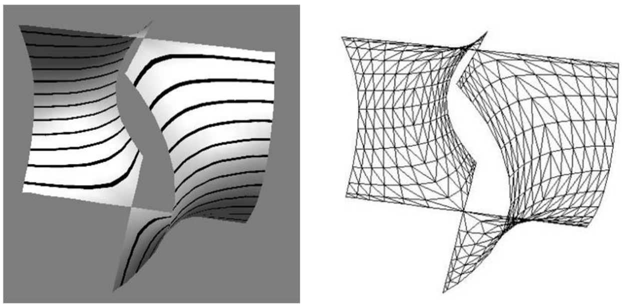

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Figure 6: Internal shape representation: (a) set of stacked, parametrized hyperbolas, (b) approximated with symmetrical pairs of triangle strips.

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Figure 8: *Heptoroid* : (a) blueprint of one of its slices, (b) laminated assembly, (c) final sculpture.

with only a 60° opening, and this wedge is then instantiated six times around the z-axis. Compressing it into $180°/n$ and instantiating it n times in a rotationally symmetrical manner will generate an n-th order saddle.

By the end of 1996, *Sculpture Generator I* [8] had evolved into a fairly robust program with several different output modules. Besides interactive virtual renderings on a computer screen, the program also could output a boundary mesh of these solid shapes in the .STL file format, which is accepted by most layered manufacturing machines. I also added a program module that could output full-size blue prints of cross sections spaced at regular intervals, say 7/8 of an inch apart, corresponding to the thickness of the wooden boards from which Brent Collins was planning to build such sculptures. *Hyperbolic Hexagon II* was the first large sculpture that Collins built from a set of blue prints generated by *Sculpture Generator I*. He cut the individual profiles from 7/8 inch thick walnut boards, assembled them with industrial strength glue, then fine-tuned the shape with hand tools, and honed the surface to perfection. Figure 7a shows our first truly collaborative piece. In 2008 this wood sculpture was used as the master for making a silicon rubber mold, and Reinmuth Bronze Studio [10] produced two bronze replicas. The second one was cast just in time for the opening of the CITRIS headquarter building on the U.C. Berkeley campus (Fig.7b).

Hyperbolic Hexagon II has enough symmetry that Collins thinks he might have been able himself to generate the basic geometry with ruler and compasses. On the other hand, he admits that he could never have conceived and drawn the geometry of our second collaborative piece, the *Heptoroid* (Fig.8). This is a 7-story structure with 4^{th}-order saddles and a total twist of 135° (3/8 of a full turn). Figure 8a shows one of a dozen actual 3-foot wide blue prints, produced in my *Sculpture Generator I*. The five superposed traces show the geometry of this sculpture at top and bottom of the board, as well as at intermediate levels at 1/4, 1/2, and 3/4 of the board thickness. Again, Collins used a saber saw to cut these shapes out of 7/8-inch thick wood boards and laminated all the 12 profiles together with industrial-strength glue. In this way he obtained a first rough shape that defines all the right symmetries, but shows strong stair-casing on its surface (Fig.8b). He then grinds down the stair-casing and creates a smooth, thinned-down surface, with one continuous rim that travels around the loop eight times before it returns to the starting point. The glue lines provide good cues for the smoothness of the shape – in addition to the haptic feedback gained from running the hand over the surface. This is a manual version of "layered manufacturing." *Heptoroid* was exhibited at Fermi Lab in 1998 (Fig.8c). Many physicists saw something that reminded them of their profession: a stellarator fusion chamber, or some structure relating to string theory.

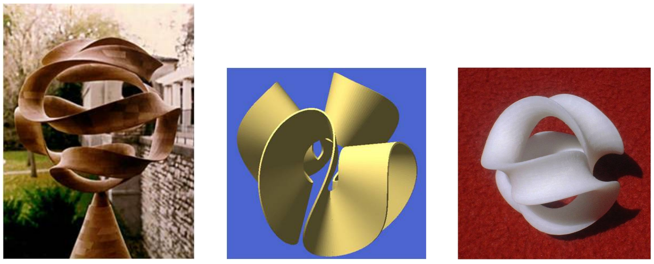

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Figure 9: Sweeps along Gabo curves: (a) Collins' *Pax Mundi* wood master; (b) Gabo-4 ribbon; (c) Gabo-2 (baseball seam) sweep.

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Figure 10: *Viae Globi* sculptures: (a) 5-lobe Gabo path; (b) *Altamont* sculpture; (c) *Chinese Button Knot* .

3. Pax Mundi and Viae Globi

The first commission to build a large metal sculpture concerned a different original model sculpted by Brent Collins. He created *Pax Mundi* completely independently shortly after we had started our collaboration (Fig.9a). It comprised a highly curved ribbon undulating around the surface of a virtual sphere. This new sculpture fascinated me as much as *Hyperbolic Hexagon*, and I immediately set out to also capture it in a procedural description. There was no way that *Sculpture Generator I* could emulate this geometry, because this sculpture was based on quite a different geometrical paradigm. The key module needed was some generalized sweep program that could extrude an arbitrary cross section along a loopy sweep path embedded in the surface of a sphere. The main question was what might be the best way to parametrize the geometry of this sweep path to be able to quickly generate a multitude of different members of this sculpture family. The key association came from some sculptures by Naum Gabo in which he had wound up broad-rimmed annuli so that the outer rim would roughly fall onto a sphere. So I introduced the concept of a "Gabo curve," which is a path that undulates symmetrically up and down around the equator of a globe. The key parameter is how many periods it takes to close the loop; in the case of Figure 9b this number is 4. Correspondingly, the seam on a baseball or on a tennis ball is then a 2-period Gabo curve (Fig.9c). The geometry of the lobes can easily be represented by a B-spline with just 3 parameters that define the amplitude, width, and shape of each lobe. In this light, I could now describe *Pax Mundi* as an "amplitude-modulated 4-period Gabo curve."

Using the SLIDE modeling environment [11], gradually developed by several of my PhD students, I was now able to construct a new and much more modular sculpture generator that could capture this shape and produce many others like it. There are three banks of slider controls. One defines the sweep path on the sphere, another one the shape of the cross section of the ribbon, and the third one the scaling and rotation of the cross section during the sweep. By default the cross section is kept in a fixed orientation with respect to the Frenet frame of the sweep curve. Alternatively it can be propagated in a rotation-minimizing manner along the sweep curve, with additional control through a globally applied azimuthal rotation of the cross section and an evenly spread out gradual twist; the latter makes it possible to obtain an end-to-end alignment of the cross section after a full pass around any arbitrary closed 3D space curve. Adding optional local twist parameters allows the programmer to fine-tune the amount of twisting in the neighborhood of critical loops or turns. This programming environment proved to be particularly fertile and has led to dozens of attractive designs (Fig.10a, 11a). Over the subsequent years I gradually broadened the concept of the *Viae Globi* program and enhanced it to take more complicated undulations (Fig.10b), to allow over/under crossings of the ribbon sweeping along the sphere surface, and eventually leading to completely free-form space curves that could make intriguing knotted structures (Fig.10c).

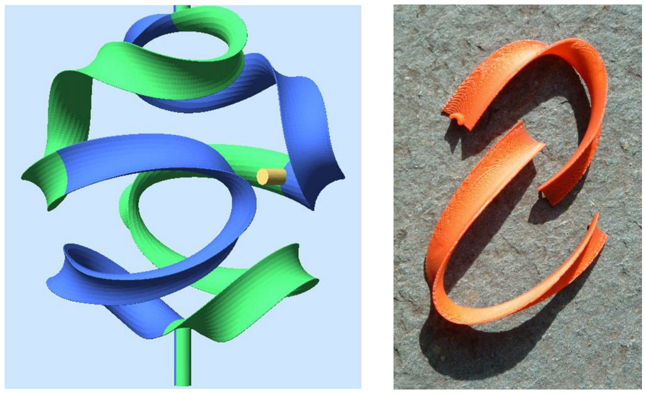

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Figure 11: Segmentation of *Pax Mundi* : (a) Four replicas of the fundamental domain, (b) the domain split into two horseshoes.

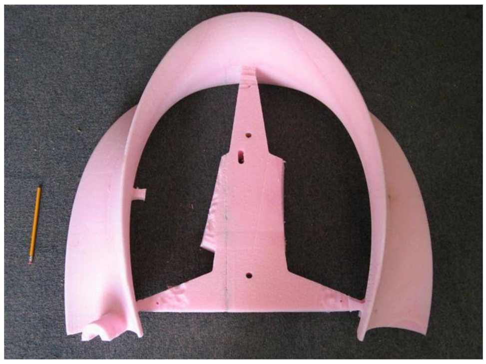

Figure 12: Small horseshoe produced on an NC milling machine.

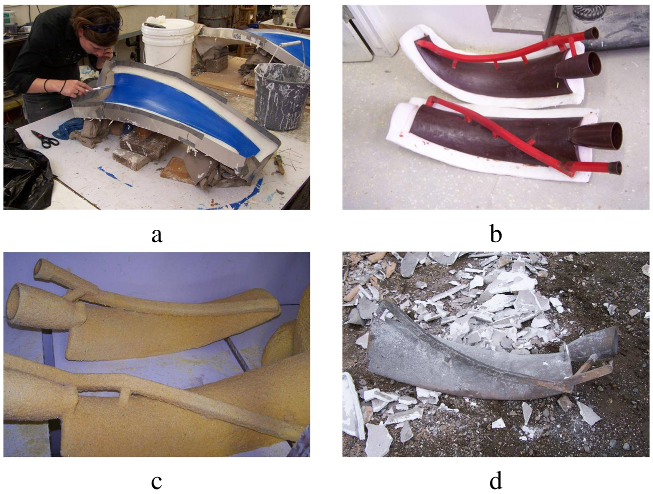

Figure 13: Realization of *Pax Mundi* in bronze: (a) coated part; (b) wax copy; (c) plaster shell; (d) freed up bronze.

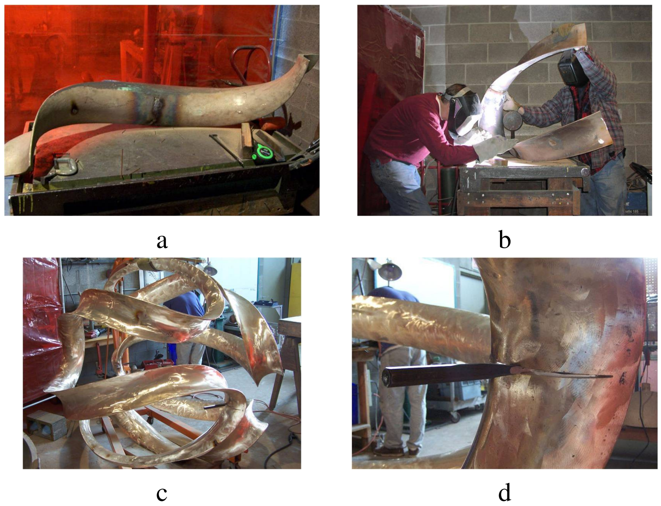

Figure 14: *Pax Mundi* assembly: (a) fitting together two segments, (b) assembling one horse shoe, (c) assembly completed, (d) cuts with wedges to elongate sculpture.

In 2007 Collins obtained the commission to make a large metal version of *Pax Mundi* for the H&R Block headquarters in Kansas City. I used my program to generate the scaled-up geometry at the desired size (6 foot diameter), properly slimmed down to fit within the weight limit (≤ 1500 pounds) and budget constraints (≤ \$ 50'000) while maximizing its aesthetic appeal. Moreover, I had the responsibility to figure out the minimal set of master modules that would have to be produced to create the necessary molds in which segments of this sculpture could be cast. The minimal amount of "master geometry" for this shape turned out to be 1/4 of the whole sculpture. I defined this geometry as a curvy sweep through space (Fig.11a) and also provided this segment with alignment tabs, so that the four pieces could be joined with perfect alignment. Unfortunately the gantry of the NC milling machine available to us had a clearance of only 14 inches and could not handle my complicated, bulky 3D part. So I split the ribbon geometry into two smaller horseshoe pieces as shown in Figure 11b, and provided additional alignment tabs. Both halves are now relatively flat and fit under the gantry of that milling machine.

However both these horseshoes were still too large to fit into the kiln in Steve Reinmuth's bronze foundry [10]. Thus Reinmuth cut the smaller part (Fig.12) in half, and the larger one into three parts. Judiciously he made a suitable jig that later allowed him to re-align the individually cast parts into the required horseshoe shape, since those cut-up segments had no alignment tabs. The five different styro-foam master-geometry modules were coated with (blue) plastic paint (Fig.13a) to yield a smooth surface, suitable for making silicon rubber molds. These molds are then used to cast four identical positive copies each in (brown) wax, to be used in a classical investment-casting process: All wax parts are provided with the (red) sprues and runners and funnels into which the molten bronze will be poured (Fig.13b). This whole assembly then gets dipped repeatedly into plaster slurry to make a ceramic shell (Fig.13c). These shells then get fired in a hot kiln. In this process, the wax runs out, leaving a cavity of the desired shape for the bronze. After the liquid bronze has been cast and has cooled down, the ceramic shell is smashed and removed (Fig.13d) - freeing up the bronze part. On all 20 cast parts, the sprues and runners are cut away, and a lot of cleaning and polishing has to be applied before the pieces are ready for assembly (Fig.14a). First the eight horseshoes are re-assembled (Fig.14b), and then they are combined into the complete undulating ribbon (Fig.14c). All the welds are ground down to a perfectly smooth surface.

But at this stage we had a bad surprise: The whole sculpture sagged under its own weight by about 15% of its overall height – and it was noticeably no longer spherical! Steve Reinmuth found a good solution, but it took several working days to fix the problem: He hung the sculpture from its highest point and let it stretch under its own weight. In addition he cut gaps half-way through the ribbon in four of the mostly horizontal hairpin curves, which enhanced the lengthening even more (Fig.14d). In this elongated state, those gaps were then filled with bronze welds leading to a vertically stretched ellipsoidal overall shape. When the complete sculpture was finally supported from its central point at the bottom, it settled back to an almost perfectly spherical shape. The main lesson learned for future ribbon sculptures of this kind is to do some basic stiffness analysis, because physics is important too – not just geometry!

The assembled and polished shape is then subjected to a select set of chemicals applied with a spray flask and the heat from a hand-held flame torch (Fig.15a). With the skilled hands of an experienced craftsman an even, and possibly smoothly varying patina can be applied, which turns this geometrical form into a true work of art. The installed sculpture is shown in Figure 15b. We think the result is highly successful – and it has indeed led to additional commissions.

Right now a similar bronze ribbon sculpture is in the works as a commission for a new science building for Missouri Western State University. It is based on Brent Collins' *Music of the Spheres*, another one of his original ribbon sculptures carved in wood. The extended version of my *Viae Globi* program

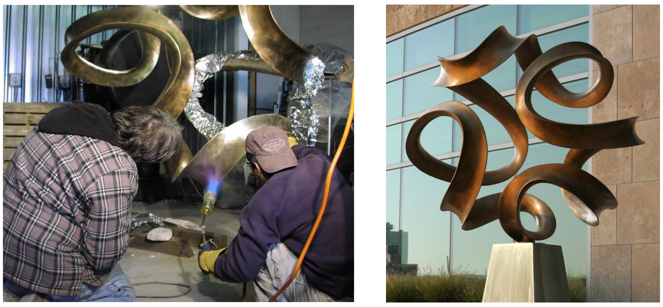

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Figure 15: *Pax Mundi* installation: (a) applying patina; (b) finished sculpture.

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Figure 16: *Music of the Spheres* : (a) FDM maquette. Simulation analysis of gravitational deformation: (b) for *Pax Mundi* and (c) for *Music of the Spheres* .

could easily capture this new shape and produce a maquette (Fig.16a)). Again, to make this into a 6 foot bronze sculpture, the proportions had to be changed. Learning from my negligence with *Pax Mundi*, I subjected the computer model to some displacement analysis, using the *Scan&Solve*^{TM} for Rhino simulator [12]. The colors in Figures 16b,c indicate how much each vertex moves from its design position under the influence of gravity. The red areas showing in the simulation of *Pax Mundi* indicate a 15% displacement from the original position (Fig.16b). The worst-case cyan color in the *Music of the Spheres* simulation predicts a displacement of only about a 3% (Fig.16c). Thanks to a different geometry and a somewhat thicker ribbon we should be much better off with this new sculpture. To be totally on the safe side, Reinmuth cast the upper ribbon segments, which primarily act as pure loads, in the form of hollow extrusions of the crescent profile.

4. Millennium Arch

Shortly after the large-scale realization of *Pax Mundi*, Collins also obtained a commission for a large Scherk-Collins toroid. The design was coming directly out of *Sculpture Generator I* (Fig.17a). It was a 12-story ring with 4^{th}-order saddles and 270° twist. This was the first large sculpture *not* based on an initial model by Collins. This sculpture was destined to be hung indoor under an atrium sky light in a community center in the City of Overland Park, Kansas. Thus it had to be light-weight and somewhat translucent.

In 2007 David Lynn and Nova Blue Studio Arts, L.L.C., in Seymour, MO, [13] helped us realize this 10-foot-diameter

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Figure 17: *Millennium Arch* : (a) computer model; (b) final sculpture.

sculpture (Fig.17b). The master geometry module was carved out of high-density styro-foam on a large NC milling machine (Fig.18a). A mixture of glass-fiber and epoxy was employed to form a reusable multi-piece shell around this module (Fig.18b). Within that shell (Fig.18c) six copies were cast from a mixture of polyester and glass fiber. Two half-circles of three modules each were then formed by fusing together some of the cast modules. These arches were transported to their destination and assembled into a closed ring on the atrium floor (Fig.18d). The completed ring (Fig.17b) was then hoisted into its final display position.

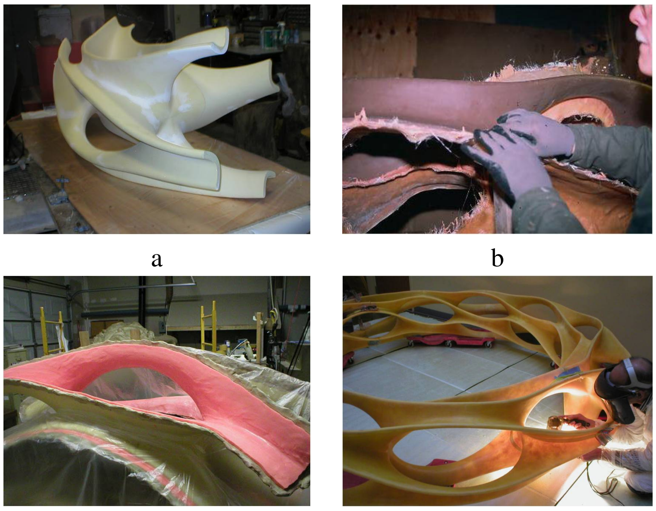

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Figure 18: Realization of *Millennium Arch* : (a) NC-machined master geometry module; (b) fiberglass patches to form the mold; (c) complete mold of master module; (d) assembly of the arch. (Images courtesy of David Lynn, Nova Blue Studio Arts, L.L.C.)

5. Making Multiple Copies

Creating *Pax Mundi* and *Millennium Arch* were both one-time, individual, custom realizations. The set of design concerns expands if one wants to make multiple, perhaps even several dozen copies of a sculpture. And it expands even further if a particular shape is supposed to be mass-produced – perhaps using production techniques such as injection molding. Here

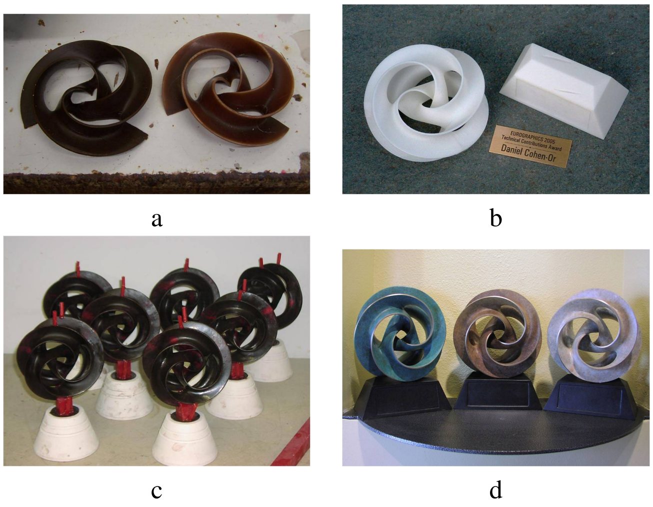


Figure 19: *Eurographics Award* : (a) master; (b) parts; (c) wax assemblies; (d) patinaed bronze awards.

I will just offer a few observations based on my experience with taking one of the Scherk-Collins motives and turning it into an *Award Trophy for EuroGraphics*. Three or four of these awards are being given out every year; thus overall I had to plan for something that would be replicated several dozen times. In this context, the molds should be relatively simple, robust, and able to last for many dozen wax castings. The assembly of the bronze sculptures should also involve as few pieces as possible and require no tricky alignments.

The design that the EuroGraphics committee liked best was based on *Whirled White Web*, a twisted monkey-saddle trefoil from the family of Scherk-Collins toroids, which was carved in snow at the 2003 annual *Snowsculpting Championships* in Breckenridge, CO [14]. Unfortunately this shape is too complicated to be made with a single mold and cast in one piece. It has a depth complexity of 3 or 4 in almost every direction and would require a very complicated multi-piece mold. But it turns out that the shape can be split in its dominant central plane into two identical pieces; and these pieces only have a depth complexity of one or two measured perpendicularly to this plane (Fig.19a). A four-piece silicon rubber mold was able to reproduce such a half-wheel element. Two of those elements could then be fused together and jointly be subjected to a standard investment casting process (Fig.19c). Thus the complete trophy is assembled from only three metal parts: the wheel, the base, and the individual engraved plaques (Fig.19b). Different patinas distinguish the different award categories: The greenish-blue patina is for the *Distinguished Career Award*; the reddish-brown patina is for the *Technical Achievement Award*; the pale tan patina is for the *Young Researcher Award* (Fig.19d).

Fabrication issues are as important as aesthetic concerns, and the former have to be integrated into the design process from a very early stage on. In the end, engineering concerns often cause many more headaches than the artistic design consideration.

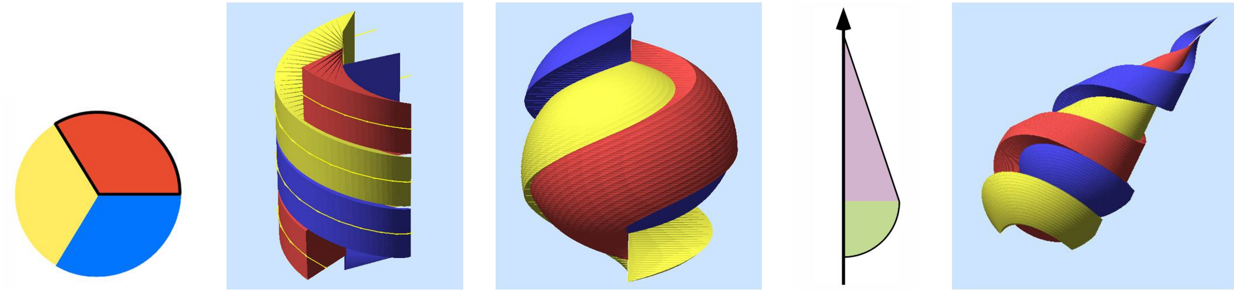
6. Dissection Puzzles and Other RP Models

In the above example the final artifact, its material composition, and the manufacturing process had to be taken carefully into account before the design of the trophy was finalized. Another area, where material properties and tolerances play an important role is whenever several parts are designed that in the end need to fit together snugly - perhaps to make a movable joint or a snap-together junction between parts, such as between a bottom and top half of the casing of a cell phone. A prime tutorial example is the construction of 3D dissection puzzles, and this task domain was explored in a graduate course on “Solid Modeling and Rapid Prototyping” at U.C.Berkeley in the fall of 2011. Such puzzles are an excellent educational tool. They strongly emphasize 3D spatial thinking, and the fabrication gives hands-on feedback about accuracy and tolerances. Moreover, the resulting artifacts are fun to play with and to show off to friends and relatives!

One of the assignments given to the students was: “Design a 2- or 3-piece geometrical puzzle in which a simple shape splits into all congruent parts via a helical screw motion.” All student teams quickly figured out that the parting surfaces would have to be one or more helicoids winding around a common straight line, e.g. the z-axis. It was then their choice to select a suitable overall shape, positioned symmetrically with respect to the chosen system of helicoids, so that the resulting dissection (or trisection) pieces turn out to be congruent. Different teams picked quite different shapes and different modeling approaches. The teams that relied on the Berkeley SLIDE [11] software, which offers very general, versatile sweep constructs, typically defined a cross section (Fig.20a) and then swept that cross section along a helical path (Fig.20b), while applying some uniform scaling as a function of the z-value of the sweep path (Fig.20c). When a teardrop scaling profile is used (Fig.20d), the puzzle shown in Figure 20e results.

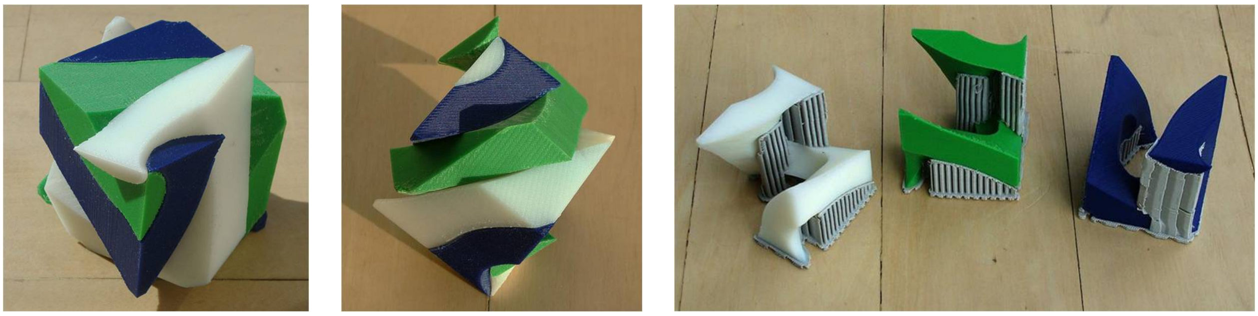
Another team of students, who had access to *SolidWorks* [15], which provides Boolean Constructive Solids Geometry (CSG) operations, decided to partition a cube along a space diagonal into three congruent pieces, which by themselves also exhibited 2-fold rotational symmetry. To accomplish this, they adjusted the pitch of the helicoids so that each cutting surface makes a 780° turn around the z-axis while sweeping from the top to the bottom corner of the cube. Figures 21a,b show two different assembly stages of this puzzle. Figure 21c shows what the individual parts looked like as they came out of the fused deposition modeling (FDM) machine.

In all these puzzles, issues of geometrical design, numerical accuracy, and suitable tolerancing had to be addressed. For some designs like the trisected “teardrop” – or “upside-down ice-cream cone” (Fig.20e) the geometry was so tight and the friction was so high that the three pieces could not be fully screwed together until the helicoidal parting surfaces had been sanded thoroughly. The designers of the trisected cube (Fig.21) struck a good compromise and hollowed-out the central portion of the cube with a diameter equal to about half the cube edge-length. This reduced friction dramatically, and the puzzle slipped together after only minimal sanding. The big question



a b c d e

Figure 20: Helicoidal sweeps: (a) cross section; (b) simple sweep; (c) modulated scaling of cross section; (d) a different scale modulating function; (e) the resulting dissection puzzle geometry.



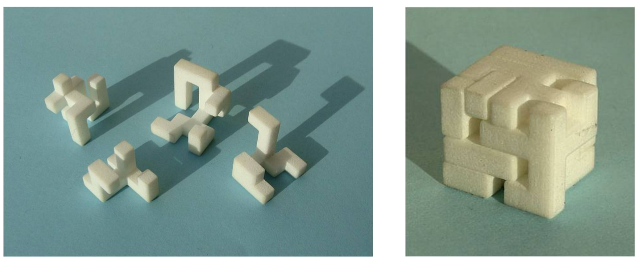
a b c

Figure 21: Helical cube dissection: (a),(b) two different states of assembly; (c) the individual parts coming off the FDM machine.

is: How do you know what the right amount of clearance should be? There were some puzzles that did not hold together well and fell apart under the influence of gravity alone. My experience is that one cannot avoid doing an initial test-run on the machine that one plans to use, submitting two or three design variations that bracket one's best estimate of the proper amount of clearance.

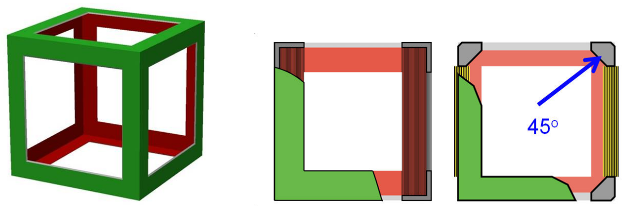
In this course we also addressed the question of how to minimize the costs of making parts on such a layered manufacturing machine. To first order, the cost of such a part is proportional to the material used in making the part. As a dramatic example, I showed the class a neat little burr puzzle that I had purchased from Shapeways [2] for about $10.- (Fig.22), and then I asked the students: "How could I obtain the same puzzle scaled-up by a factor of 10 without having to pay $10'000.- ?" Specifically I asked them to come up with a design that would minimize the material usage on our old FDM machine, a type "1650" from Stratasys [16].

In our discussions we quickly came to the conclusion that it would be advantageous to build just the outline of the puzzle pieces, keeping the interior mostly hollow. However, since the FDM machine builds some kind of scaffolding underneath bridges and overhangs in the part geometry, building a completely hollow cube is not possible, and the internal scaffolding can still use up a lot of build material. A reasonable approach is to decompose each puzzle piece into its constituent cubelets, and then realize each such cubelet as a thickened edge-frame (Fig.23a) - akin to a style used by Leonardo DaVinci to depict some to the regular and semi-regular polyhedra. This approach



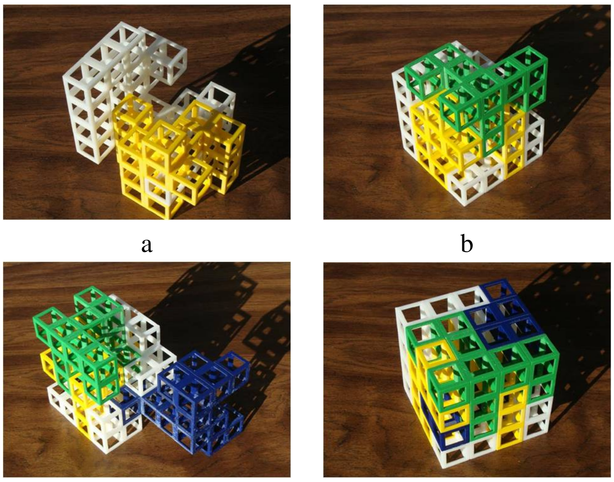
a b

Figure 22: Small burr puzzle: (a) individual parts, (b) assembled puzzle.



a b

Figure 23: Cubelet frame: (a) Leonardo rendering, (b) cross sections through frame, without and with 45-degree bevel.



a b

c d

Figure 24: “Framed” cubic burr puzzle in various stages of assembly.

can save build material as well as support material, if the central portions of the cubelets can be kept free of any type of scaffolding, and if the support material can be restricted to the vertical “windows” in the sidewalls of every cubelet (Fig.23b).

When trying to limit the scaffolding material to the vertical walls, we had to contend with some idiosyncrasies of *QuickSlice*, the software that comes with the Stratasys FDM machine 1650. It slices the geometrical part into 10mil (0.25mm) thick layers and then drives the FDM machine to “paint-in” each such layer with a back-and-forth motion (in the x-y-plane) of the nozzle that dispenses the hot, semi-liquid ABS plastic. In this program, the user has the option to specify what kind of supports the machine is supposed to build. First we tell the machine to simply build straight, vertical support structures. Since we expect all support structures to be small and locally confined to the vertical walls of the cubelets, there is no need to use any tapered lateral growth for stability. Furthermore we specify that overhanging faces of 45° or steeper need no support structures, but can be built by relying on cantilevering outwards the beads in subsequent layers by half their diameter. Moreover, we internally bevel the cube frame at a 45° angle and thereby limit the scaffolding to be constructed to a thin support slab inside each vertical window. Figure 23b shows a schematic cross section through one cubelet with and without the 45° beveling of the frame; the green color represents the outer skin of the front face; pink is the inside of the back face; the cross section of the frame is shown in gray; and the thin, black, vertical lines depict the area of space filled with support material. In this way the material costs can be reduced by about a factor 100 over a simple, 10-fold scaled-up version of the original solid model. Figure 24 shows the complete scaled-up, “framed” burr puzzle in various stages of assembly.

Clearly, in the design of a puzzle viable for mass production, the envisioned manufacturing technique needs to be considered right from the beginning. Such puzzles then become a good exercise in design for manufacturing (CAM). Professor Paul Wright in Mechanical Engineering and I have been running courses at U.C. Berkeley that also focus on the design for manufacturing of more complex products that go beyond pure shape design. Figure 25 shows two models that students have built in this context: a compact travel kit for the maintenance of your contact lenses (Fig.25a) and one of several shapes produced in a study to test the tactile quality for hand-held video game controllers (Fig.25b).

a b

Figure 25: Product prototypes build with layered manufacturing: (a) maintenance kit for contact lenses; (b) hand-held video game controller mock-up.

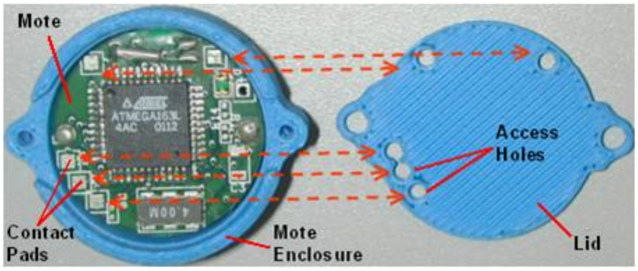


Figure 26: Multi-material prototype: A self-contained sensor node.

New problems that arise when one has to deal with a product design in which several technologies and knowledge domains need to be brought together and interfaced with one another. An example is the self-contained “mote” shown in Figure 26. The contact pads on the printed circuit board need to be brought into alignment with the access holes in the plastic lid of this self-contained sensor / transmitter / networking node.

7. Capturing the Geometry of an Inspirational Model

Now I want to address the issue of how to get started with the design of one of these artifacts discussed above. It turns out that in this context tangible objects also can (and should) play a more important role. Often a design or re-design task starts from some related inspirational artifact. Many of my early abstract geometrical sculptures were inspired by an individual piece of art work sculpted by Brent Collins (Section 2). With *Sculpture Generator I* [9] I custom-built a program to capture the essence of *Hyperbolic Hexagon*. With the more general, modular *Viae Globi* program in SLIDE [11], I extracted the essence of *Pax Mundi* and of other ribbon sculptures (Section 3). In 2009 I started an effort with James Andrews to generalize this approach even more [17]. Our prototypical *Interactive Inverse 3D Modeling* system starts from an existing artifact that may only exist in some low-level, unstructured representation,

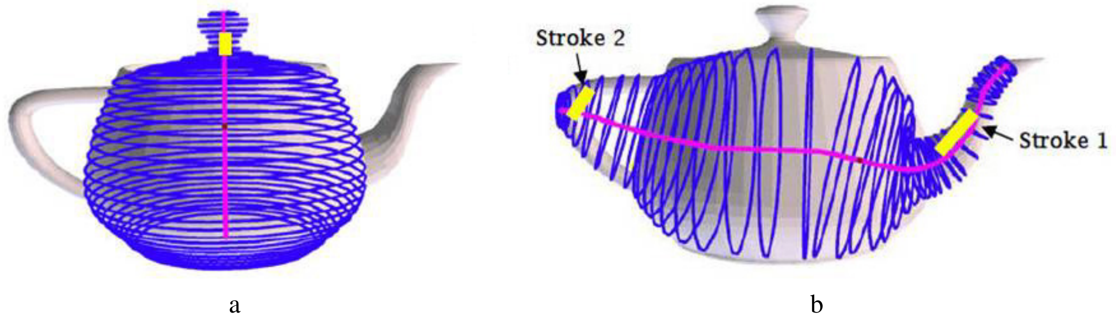


Figure 27: The shape of the Utah teapot interpreted in two different ways: (a) as a rotational sweep, (b) as a progressive sweep from the spout to the handle.

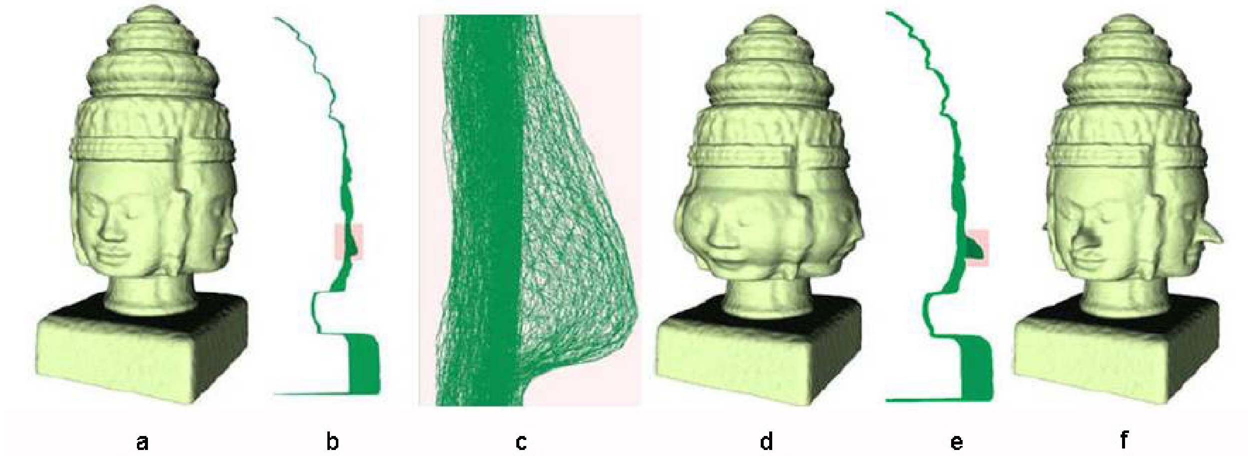


Figure 28: A statue with four-fold symmetry coerced into the framework of a rotational sweep: (a) the original statue, (b) its mesh rotationally collapsed into a thick "profile", (c) the area around the nose selected for editing, (d) the resulting fattening of the whole face region when the selected area is move outwards, (e) an alternative smaller area selecting only the nose tip, (f) the result when this smaller region is pulled outward.

such as a triangle mesh, a point cloud, or a set of images from different directions. With some user guidance the system then extracts a high-level, parametrized, procedural description of this shape in a form that is most useful for the user in any plans for adapting or redesigning it. Bringing physical artifacts into the virtual world of a CAD environment can be quite labor intensive and error prone, but we hope that this system will make this process much more amenable.

The user loads the unstructured part description into the computer and then selects on the surface of the virtual display one feature or one basic shape at a time, while choosing the most desirable CAD representation for that segment. The belly of the famous Utah teapot may be extracted as a rotational sweep (Fig.27a) and its spout and handle as progressive sweeps with slowly varying cross sections. Alternatively the system could also extract the whole teapot as a single progressive sweep from spout to handle (Fig.27b) – but this would probably make much less sense for any reasonable re-design task.

Other objects may be decomposed into simple CSG primitives (half-spaces, cubes, or cylinders) or pieces of quadric surfaces and tori. These structures can then be modified easily by using the parameters of the various extracted segments as handles, perhaps changing the sweep path or the cross section of a progressive sweep or by moving some of the CSG primitives to slightly different locations.

Here is a brief review of two of the parametrized extraction modules that we have implemented so far. First there is the powerful sweep extractor mentioned above: With a first yellow stroke (Fig.27) the user defines the start of a progressive sweep. The program cuts through the local geometry with a plane that is roughly perpendicular to the user-drawn stroke and determines a first tentative cross section. This profile is then swept in the direction of the user stroke, and the profile shape as well as the sweep direction are fine-tuned, so that the generated extruded surface element is as close to the input data as possible. Once a first best-fitting segment has been established, the sweep is continued at both ends in roughly the same direction as the current segment, with possible small corrections to the sweep direction and the orientation and scale of the profile. This process is iteratively repeated at both ends as long as the constructed surface does not deviate by more than some specified error tolerance from the given input geometry. If the process does not cover quite as much of some generalized cylinder geometry as the user would like, an additional stroke can be painted on some additional surface area yet to be covered, and the internal error tolerance will be suitably increased, so that the sweep extraction process can continue as far as the drawn

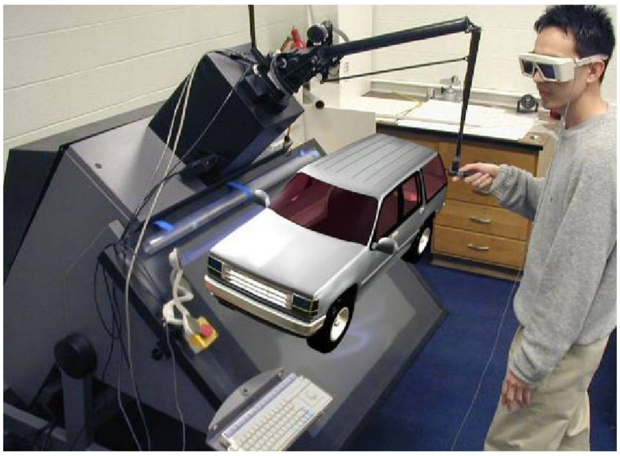
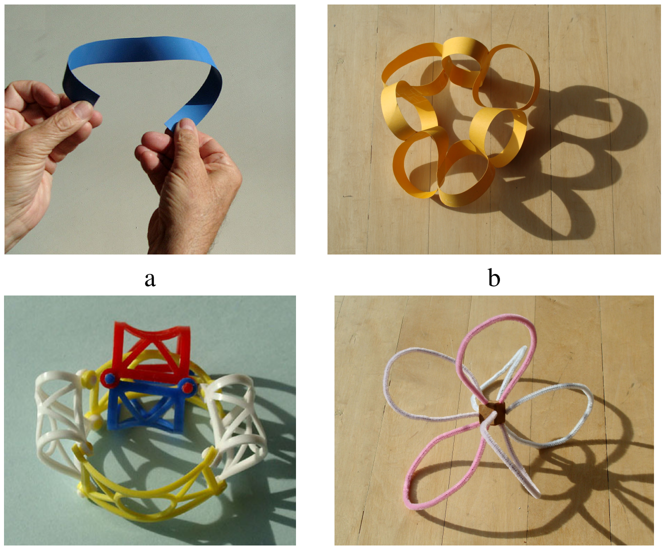


Figure 29: Virtual work space with haptic feedback



a b c d

Figure 30: Shape elements to be inserted: (a) physical spline; (b) paper shapes; (c) plastic parts; (d) pipe cleaners.

stroke indicates (Fig.27b). Conversely, with a different type of stroke, the extent of the sweep extraction can be restricted. Different starting strokes and different error tolerances result in a wide variety of possible extracted sweeps. The selected sweep paths and profiles can then be edited independently at interactive speeds.

A different extraction module tries to fit a generalized rotational sweep to the surface areas selected by the user. This module cannot only match surfaces with strict rotational symmetry but can also extract helical and spiral sweeps [18] [19]. In all our extraction modules, the reconstructed, parametrized surface can either be used directly as the starting point for a new design, or the surface details of the input model can be preserved and reused. To allow this, the difference between the actual input geometry and the extracted surface is stored as a displacement map, which can be re-applied to any edited version of the reconstructed surface. This is demonstrated in Figure 28 with a roughly cylindrical statue. The whole shape is first approximated by a rotational sweep, and all the vertices as well as their interconnecting edges are then rotationally collapsed into the green profile bundle shown in Figure 28b. Portions of this bundle can now be selected and modified interactively with respect to their distance from (and position along) the z-axis. If we select the whole region around the nose (Fig.28c) and move it to the right, all the nose and cheek areas will be puffed up (Fig.28d). If, on the other hand, we only select the tip of the nose (Fig.28e), only those vertices will change their radial component, and we simply obtain four stretched noses (Fig.28f). The variety of the extraction modules, and the versatility with which they can be applied and the extracted geometry can be used and transformed, make this a powerful way to get a new design started, – if an artifact close enough to the envisioned design can be found.

8. Capturing an Initial Inspiration Existing in Your Mind

But sometimes a model close enough to the intended design may not exist, and the design is just a vision in one's mind. How do we get this into the computer? It would be nice if we could wave our hands and describe shapes in this way. Several such sculpting systems have been presented over the last two decades, e.g. [20]. A few years ago together with Professor Sara McMains in Mechanical Engineering, we built such an interactive shape editor in an immersive 3D environment [21]. Through a head-tracked pair of shutter glasses, the user was seeing a 3D rendering of the design object in front of a rear-projected display wall. With a haptic stylus attached to a position-sensing articulating arm the user could interact with that object. Vertices could be moved around, or the model could be annotated or painted (Fig.29). However, the project was not a smashing success! The tactile feeling was not solid; it was more like drawing on a balloon with a ball point pen. Also, the whole set-up was tedious and tiring. I found it impossible to create conceptually new sculpture models in this environment.

Looking back over the last two decades of my own involvement with sculpting and model making, I find that most novel ideas start with something tangible in my hands. The things that I am forming and combining are: wire, paper, scotch-tape, paper clips, styro foam, clay, etc. Touch and proprioception (knowing where your hands are even when blind-folded) play an important role. Another important aspect of tangible objects is that the materials properties contribute an important part of the shape formation process. Bending a plastic strip or a thin steel band creates nicely behaved spline curves. A knot made from thick plastic tubing can be pulled tight mechanically to produce some tightest configuration, – a process that would be rather complicated to program in virtual space. Even famous architects like Frank Gehry [22] occasionally use heavy cloth or velvet to outline the roof of a new concert hall or museum.

Thus, what I really would like to do is to use whatever material is most appropriate to form components of my design and then integrate them with what is already there. One way in which this could be accomplished is to work in an immersive virtual environment (but comfortably sitting down) in which I can hold up various elements of the types shown in Figure 30, positioning and aligning them visually with the emerging composite, and then, by tapping my foot (or clicking my tongue) send a command to capture this proffered shape and add it into the design description. This capture could be accomplished with a fast 3D scanner, a structured light system [23], or a future version of something like the Kinect [24]. The captured shape would be presented to the designer through the *Interactive Inverse 3D Modeling* system described above (Section 7), so that it can be represented with the most beneficial parametrization for further editing, refinement, and optimization.

An environment like this could offer the best of both the physical world and the virtual domain of computer graphics. It should allow us to readily grab a wide variety of tangible things, or unstructured geometry descriptions, – whatever serves as a possible inspiration or take-off point for our designs. However, once such elements have entered the CAD design environment, these pieces of geometry can take on a new identity in the form of materials that mimic the best of: clay, wire, paper, scotch-tape, styro foam, etc, but without the adversity of: messy glue, gravity, or strength limitations; and they can represent ideal pseudo-physical materials that bend as nicely as steel wire, or stretch like a nylon hose, but are as strong as titanium, and as transparent as quartz.

9. Possibilities of Emerging RP Technologies

Rapid prototyping technologies are currently in a phase of fast development. Every year several new models of "MakerBots" and other layered fabrication machines are introduced [25]. Many companies, as well as individuals, are experimenting with new materials, different binders and support structures, and are exploring new application domains for these emerging technologies.

The large annual graphics conferences recently have devoted whole sessions to the topic of rapid prototyping and to novel ways of producing physical, tangible output from computer graphics designs. Some of these applications are quite different from the traditional task of producing a physical 3D object from a virtual model developed with one of the commercial graphics or CAD programs. For instance, the development of a bas relief of some arbitrary 3D scene [26] requires sophisticated depth compression schemes, which are similar in their approach to techniques used to compress high dynamic range images into renderings that can be displayed optimally on standard graphics screens with limited dynamic range [27]. Other work explores ways to make bas reliefs structured as height fields at the pixel level to generate physical panels that will produce one or possibly several images when illuminated at oblique angles from suitable directions [28] [29]. Such systems typically require multi-pass optimization, and the outputs are often plagued with low contrast and ghosting or cross talk between the different images.

Instead of structuring the geometry of the output artifact to produce images through shadows, other researchers have used optimization algorithms to tailor the shape of a reflector so as to create a desired illumination pattern [30]. In yet another approach, transparent materials are micro-machined, and refraction is used to form the desired image as a superposition of properly tailored caustics [31].

The emergence of multi-material RP machines [32] makes it possible to create physical output with novel optical properties or varying stress-strain relationships. In the first sector researchers have created flat panels or 3D models from materials with custom-designed subsurface-scattering properties [33] [34]. This approach, which is also based on physical simulation and multi-pass optimization, makes it possible to emulate the look of marble, jade, or even a slice of salmon. By varying the deposited materials from layer to layer, as well as adjusting the porosity with which these layers are deposited, the strength and pliability of the resulting composite also can be controlled and finely tuned. Researchers have been able to closely model and reproduce the bending and compression behavior of the soles of flip-flops or slippers [35].

An even tougher challenge is to create complex 3D models that can change shape. Among the many different possible approaches, there are some inspiring trend-setters. Some of the rapid prototyping methods allow in-situ construction of interlocking assemblies of movable parts. This allows the production of demonstration models of wrenches [36] or of complete gear boxes [37]. Another approach is to create articulated figures from snap-together parts, so that phantasy creatures designed in *Spore* [38] can be brought to life as toys that can assume many different poses [39]. Another highly intriguing problem is the design and fabrication of balloon hulls that, when inflated to the proper size, will assume a desired shape. Since such membranes do not inflate in a linear manner at all, the initial limp hulls have to be given some quite different shapes. Again, a multi-pass optimization step comprising detailed physical simulation can lead to success [40]. There seems to be no limit to the modalities by which computer graphics design and physical output can be related.

10. Summary and Conclusion

Interactive computer graphics has come a long way since Ivan Sutherland first demonstrated his *Sketchpad* [41]. Remarkable progress has been made in photo-realistic rendering, simulation, immersive 3D worlds, and haptic feedback. Further developments will couple the virtual domain of computer graphics and the 3D world of physical artifacts in ever more diverse and intriguing ways. Most of the techniques mentioned in the previous section are at a stage of demonstrating feasibility. It will take some time before they turn into mainstream output technologies that can readily be applied at the push of a button.

To produce substantial and durable physical artifacts is still a venture that takes special attention for most new models. I have learned that every time when I work on a new sculpture family, or switch materials from bronze to stone, to sintered steel, or to translucent polyester resins, a new set of problems emerge; those need careful attention and often require some preliminary testing of the modeling as well as of the fabrication technologies. The 3D Hilbert Cube (Fig.2a), made with the

ExOne metal-sintering process [42], required three attempts, comprising an ever-increasing number of internal support rods. These auxiliary struts were necessary to hold this shape together in its green state as it is inserted into the sintering oven, because at this stage the model has the weight of iron, but only the strength of the selectively deposited binder that holds the stainless steel particles together.

For many of the techniques mentioned in Section 9, mature processes and services will develop over time, and will eventually allow a one-click order of the desired physical output. One such example is the stone-carving service offered by Dingli in China [43]. All I had to do to obtain the four foot tall granite sculpture shown in Figure 2b was to send them a six inch tall RP model of the top part; they did not even use the CAD files for the actual production.

As discussed in Sections 7 and 8, the coupling between computer graphics and the physical world is not a one-way street. Pre-existing artifacts, as well as RP models with tailor-made deformation properties, could serve as design elements for inputting shapes into a CAD environment at the early stages of a design process. However, both the input end, as well as the output end of a computer-based design effort, require significant advances in tool development and user-interface design to make these transition points reliable and easy to use.

11. Acknowledgements

This work is supported in part by the National Science Foundation: NSF award #CMMI-1029662 (EDI).

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