The Beauty of Knots

by Carlo H. Séquin

A hands-on workshop to study the symmetries and 3D structures of simple mathematical knots and to try to remodel them into aesthetically pleasing sculptural geometries.

Background

A couple of times I have offered a semester-long undergraduate seminar entitled "The Beauty of Knots." In this seminar participants picked one or two simple knot (5 to 9 crossings) from the table of mathematical knots. They first reproduced the 2.5D diagrams in those tables with a piece of string or with pipe-cleaners. Manually deforming that knotted loop, they then tried to find a most symmetrical representation. By further remodeling the geometry of the given knot, they also tried to find aesthetically pleasing configurations that, when scaled up, could make interesting constructivist sculptures.

Given that this was a seminar for CS students, we also investigated the use of procedural deformations that would minimize overall energy of a uniformly charged wire shape by aiming to maximize the distance between adjacent lobes of the knot, and of transformations such as an inside-out reflection on the unit sphere to find new promising geometries for possible sculptures. Sweeping an appropriate profile along the optimized knot curve then resulted in a well-defined solid model of the sculpture geometry. Since we had access to a Fused Deposition Modeling (FDM) machine, the most promising designs were realized as small maquettes standing a few inches tall (Fig.1). Clearly these latter activities would not fit into a single-session workshop.

Figure 1: Knot-sculpture designs realized on a rapid prototyping (FDM) machine (2009-2010): (a) Knot 5-2; (b) Knot 6-1; (c) Knot 7-7; (d) Free-standing trefoil knot, Knot 3-1.

Proposed Workshop

After a short introduction to the mathematical definition and nomenclature of knots and a brief display of some inspirational knot sculptures, each participant will be asked to pick one example from a recommended list of knots. They will then make physical models of the chosen knots using pipe-cleaners, copper wire, or twist-ties, or some kind of tubing (Fig.2). They are encouraged to heavily deform their initial configurations to see the wide range of possible geometrical representations in three-dimensional space of one and the same knot. In this process, they will also look for a skeleton curve for a possible, promising knot sculpture.

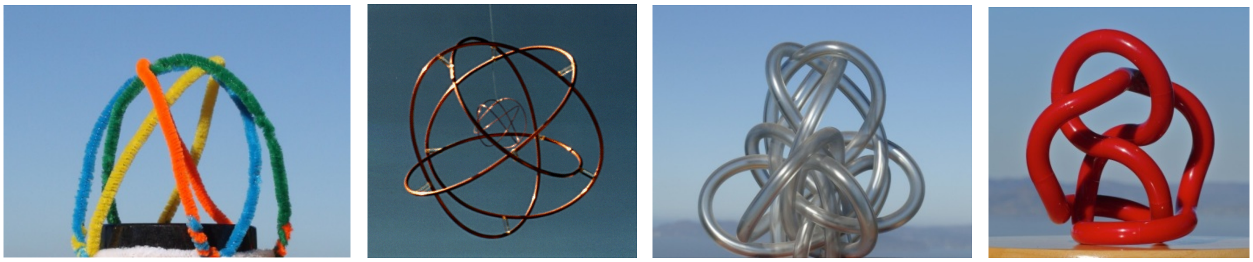

Figure 2: Skeleton curves of possible knot sculptures, using (a) pipe-cleaners, (b) copper wire, (c) plastic tubing, (d) a chain of flexible L-shaped tube segments.

Once a suitable knot curve has been found, a specific medium for implementation as well as the corresponding cross section of the sweep along the knot curve have to be chosen . A simple yet expressive medium is aluminum foil. A large sheet of aluminum foil is repeatedly folded into a flat multi-layered ribbon (about 3/8 of an inch wide). This ribbon is then manually formed to represent the chosen skeleton curve, while different amounts of twist can be applied to the ribbon before it is closed into the knotted loop (Fig.3a). There are other inexpensive materials suitable for making knotted ribbon sculptures: double-wire twist ties, which can be aesthetically enhanced by putting decorative adhesive tape on both sides; sturdy, colorful plastic sheets as are used in file folders; or even flat 3-wire cable. Some flexible snap-together parts such as “Flexeez” http://fixturescloseup.com/2011/01/20/flexeez-p-o-p-whatsit/ (Fig.3b) or “Knüpferli” http://www.dusyma.de/shop/Building-and-Constructing//223218-Knuepferli-Mixed-Packing-New-Colours--223218.html , also allow the formation of broad bendable ribbon structures that then can be closed into a chosen knot configuration.

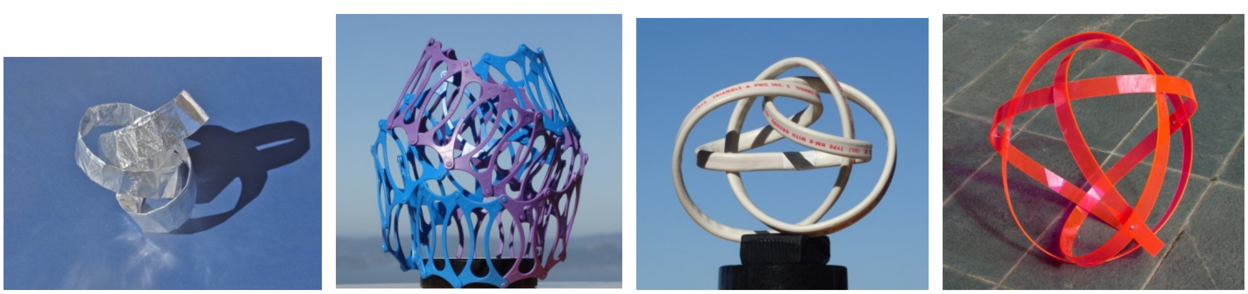

Figure 3: Ribbon sweeps with a flat profile along various knot curves, realized with: (a) aluminium foil, (b) *Flexeez* snap-together parts, (c) 3-wire electrical cable, (d) plastic strips.

Larger and sturdier knot sculptures can be obtained from vinyl or aluminum drier duct (Fig.4a,b), from flexible plastic tubing (Fig.4c), or from PVC pipe segments and elbows (Fig.4d). The latter approach would probably require making available at the workshop a simple PVC pipe cutter, to cut pipe segments to any desired length.

Figure 4: Knot sculptures made from: (a) 4” vinyl drier duct, (b) 6” aluminum drier duct, (c) flexible plastic piping, (d) PVC pipe segments and elbows.

The workshop will conclude with a pictorial presentation of some larger sculptures derived from mathematical knots and a brief outline of the rather lengthy and elaborate process required to realize the large bronze sculptures described in the next section.

Larger, Permanent Sculpture

While some of the geometries created in such a workshop may look already quite attractive and could certainly serve as temporary displays in the National Museum of Mathematics, they may look even better and more inspiring when done at a larger scale and in a more durable material. Carlo Séquin, collaborating with Steve Reinmuth from the Bronze Studio in Eugene, OR, and artist Brent Collins from Gower, MO, have created some such sculptures in bronze during the last few years. Two examples are shown in Figure 5. The defining geometrical elements were modeled in a parameterized computer program, which then produced a detailed boundary representation that could be read by a numerically controlled milling machine. All unique geometrical elements were then milled from high-density styro-foam and used as the master geometry to make negative molds in silicone rubber. Multiple copies were then cast in wax in these molds and used in a classical investment casting process to produce corresponding bronze elements. These parts were welded together, ground and polished to provide a smooth surface, onto which an attractive patina could be applied.

Overall, the aim of this workshop is to give participants a better understanding of mathematical knots and of the many different ways of embedding them in 3D space – but also to inspire and encourage some of the participants to undertake an effort to pursue the creation of art work in the form of larger, more durable realizations.

Figure 5: Large Bronze Sculptures:
(a) "Torus Knot-5-3 (version 2)" (August 2010) - Bronze, 16" tall. -- This sculpture won 2nd prize at the AMS annual exhibition of Mathematical Art in January 2011.
(b) "Music of the Spheres" (January 2013)", (With Steve Reinmuth and Brent Collins) -- a 10-foot tall bronze sculpture, installed at Missouri Western State University, St Joseph, MO.