The Symmetry in Music

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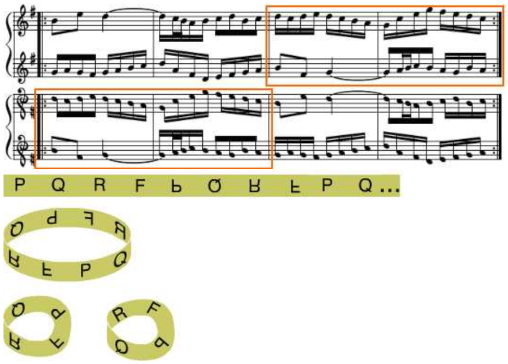
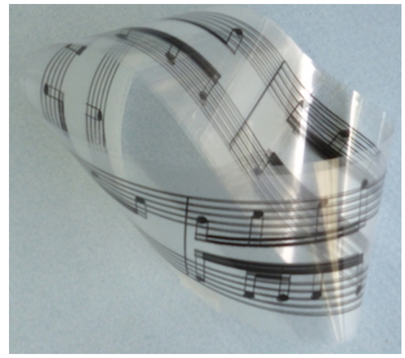
Music and math may seem fundamentally different, but really music has roots in mathematical principles, a lot of which can be showcased through symmetry. There is an obvious visual symmetry in instruments such as the piano and the guitar. The piano has very clear translational symmetry. An octave has 12 keys and the piano has 88 keys. This means there are approximately seven octaves on a piano, so if we picture the keys of a piano wrapped around a cylinder, there would be C_7 symmetry. Many guitars, particularly acoustic guitars, have a very distinct vertical mirror axis.

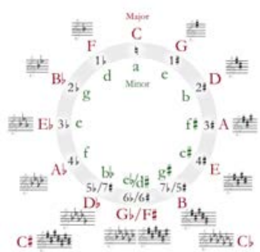
Besides instruments, however, there are several elements of symmetry that exist in musical score as well. For instance, if we think about lines of musical score wrapped around a cylinder, we can see frieze symmetries. Excellent examples can be found in Claude Debussy's work. Debussy lived from 1862 to 1918 and is considered a famous name in impressionist music, although he himself hated the term. When people think of Debussy, they often recall his famous piece Clair de Lune. Several mathematical patterns can be seen in his works including the golden ratio and the Fibonacci sequence. In fact, author Roy Howat wrote a book on the mathematical influences in Debussy's work. One of his compositions, the Three Nocturnes, happens to actually contain all seven of the frieze symmetries. The images displayed are shown with Conway's footstep notation. He uses hops, steps, jumps, etc. to show the different kinds of symmetry. On top of basic translational symmetry, they show horizontal and vertical mirror symmetries, glide symmetries and rotational axes. For some of these, to see the frieze symmetry, they have to be looked at without the stems of the notes such as the S_n symmetry and the D_n symmetry.

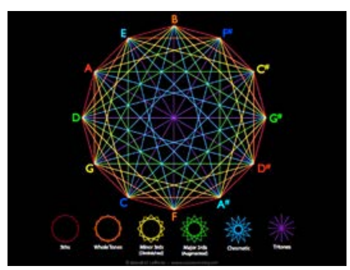
Because of certain frieze symmetries, you can find some interesting topology in certain pieces. Johann Sebastian Bach was a Baroque composer who happened to be particularly fond of glide symmetries. His specialty was canons. A canon is where a certain melody starts, and then the same melody starts again a little after the first with a layering effect. One of the most famous examples of a canon is Frere Jacques. Bach composed 14 of these canons. His Canon 5 contains a very unique quality. Due to the existing glide symmetry, the line can be turned into a perfectly overlaid mobius band. If we take a line of Bach's Canon 5 and flip it upside down, creating a horizontal mirror line, there are very present glide symmetries ignoring all of the stems. This allows for a mobius strip that goes around twice with all of the notes lining up perfectly, as seen in the image.





Another interesting example of symmetry comes from the circle of fifths. The circle of fifths is one of the first principles taught in music theory. It is used mostly for the sake of visualizing intervals in the music scale. The circle of fifths comprises of all the notes in a scale arranged in fifth intervals. This creates several radial symmetries in the form of a 12-point star. The chromatic scale, or every note on the scale in order, forms a full 12-point star. Whole tones, or every other note going up the chromatic scale, form a 12-point star made up of two overlaid hexagons. Minor 3rds, or third intervals that typically sound sad, form a 12-point star made up of three overlaid squares. Major 3rds, or third intervals that typically sound happy, form a 12-point star made up of four overlaid triangles. Lastly, the notes directly across from each other are what are known as tritones. Tritones are an interval created by three adjacent whole tones put together. These, when played at the same time, tend to sound very unpleasant to the ear. In fact, there was a time where diabolus in musica, or "the devil in music," was applied to the tone. There are stories where people would get excommunicated from the church early on for the use of this interval. Its use began during the Baroque and Classical era.





With all of this there lies the question of whether or not symmetry is present in music intentionally. Did these composers intentionally use symmetry in the making of their pieces? For the older composers like Debussy and Bach it's hard to know for sure, but it's unlikely. The more likely explanation is that symmetry is what sounds naturally sounds pleasant to our ears. The inherent symmetries present in the physics behind sound likely make some melodies and harmonies sound more enjoyable than others. However, there are artists that have used mathematical principles to their advantage. For instance, famous jazz musician John Coltrane was known to have researched Einstein and applied some Einsteinian principles to jazz composition.

The connection between math and music is very present. Symmetry appears in several forms through all elements of music, whether is be instruments, musical score, or music theory. It is field you wouldn't automatically associate with math, but the existing mathematical applications are incredibly unique.