Escher Planetoids

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Figure 1 Double planetoids by Escher, 1949

1. Escher planetoids

The art work "Double planetoids" shown in Figure 1 was drawn by M.C. Escher in 1949. This planet is constructed of two interlocked tetrahedra, and they represent two kinds of planets. One is a natural planet, and the other is a civilized planet. Although these two planets are connected with each other, those cannot share one same world.

2. Goal

In my final project, I designed blocks composing Escher planetoids on SLIDE, and I will make them by the FDM machine. As shown in Figure 2, I make 8 tetrahedra which are 4 natural and 4 civilized tetrahedral planetoids, and 1 octahedral joint part which is

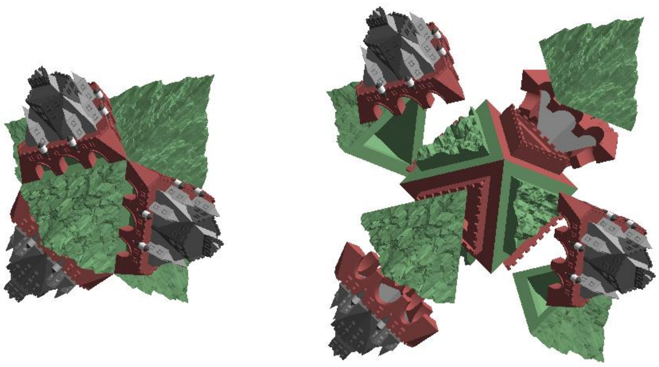

Figure 2 My planetoid has 4 natural tetrahedral planetoids (green), 4 civilized tetrahedral planetoid (red), and one base octahedron with fixing bulge on each face.

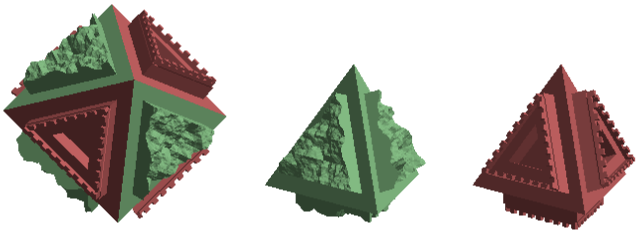

Figure 3 Three kinds of joint parts, which are original octahedral joint with two features, natural tetrahedral joint, and civilized tetrahedral joint.

an intersection part of natural and civilized planet. In addition to them, I designed two more tetrahedral joint parts shown in Figure 3. Each face on all joint parts has a bulge with identical contour designed to joint any tetrahedral parts. You can build a planetoid by putting together those parts and detach them, and also compose alternative kinds of planet by using different combination of tetrahedral planetoids and joint parts.

3. Commodity values, good points

This work has some major good points. One is that this has two contrastive complex shapes on each surface; nature and civilization, or irregular and regular. That contrast and elegant complexity can be very impressive as an art work. Another point is it has a variety of form. By combining 4 natural tetrahedra, 4 civilized tetrahedra, and 3 kinds of joint parts, you can build not only double planetoids but also many kinds of alternative planet. This variety of form is an important element if it is sold as a toy product.

This also has an advantage in the design cost. You can almost automatically design highly realistic natural geographical shape by using the fractal based algorithm, and easily control natures of generated geographic shapes by changing parameters. I will explain about this generating procedure in Section 4.1.

4. Procedure to make each part

4.1. Natural planetoid

To represent realistic natural geographical shapes such as rocks and mountains, I can use fractal based approaches. Many of objects in Nature have self-similar shapes and textures [1]; plants, insects, crystals, and geographical features. To represent such objects, you can use fractal based algorithms. The midpoint displacement algorithm [2] is one of simplest fractal based algorithms to make geographical shapes: rocks, mountains, and coast lines.

The overview of the midpoint is shown in Figure 4. This algorithm repeats two

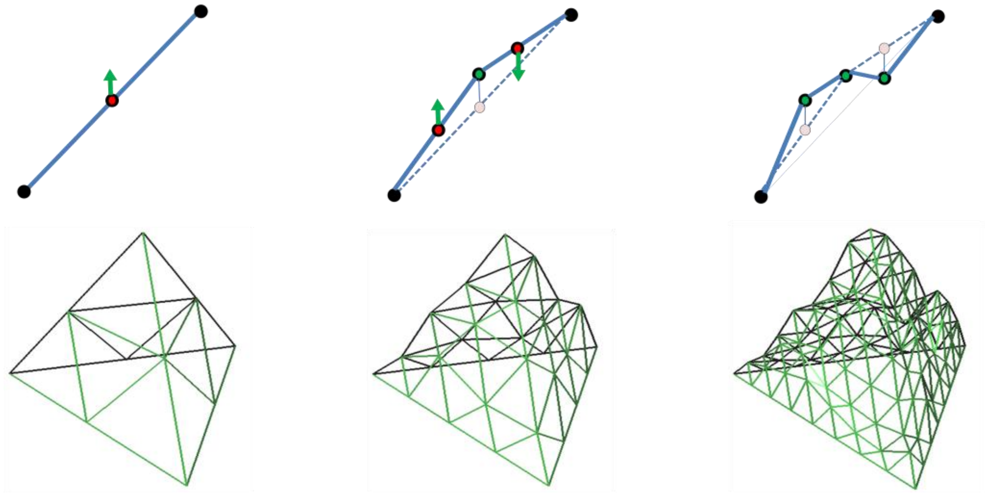

Figure 4 Generation of realistic geographical shapes by using the midpoint displacement algorithm

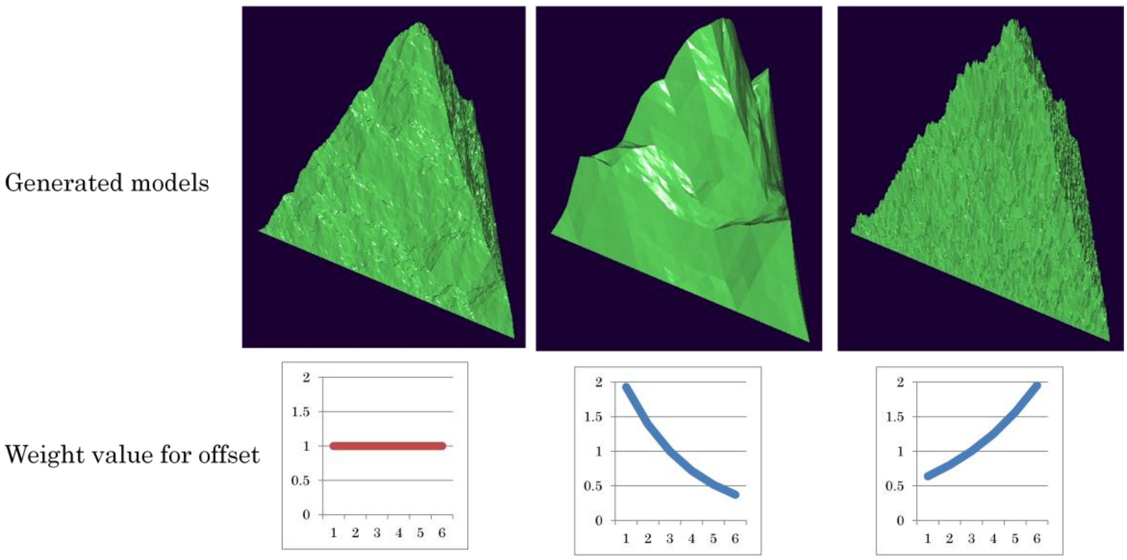

Figure 5 Examples of generated models when using alternative weight values for offsets

procedures. One is equal dividing edges of 3D geometric models, and the other is giving a certain amount of offset to the midpoint generated by division. These procedures continue until the resolution of geometry grows enough. To make a tetrahedral planetoid, you define a tetrahedron at first, and you give vertical vectors as offsets as shown in Figure 4.

The amount of offset can decide the natures of generated geographical shapes. For example, larger offsets in early generations of subdivision and smaller offset in later ones give smooth shapes. On the other hand, smaller offsets in early generations and larger offsets in later ones give precipitous and jaggy shapes. Based on this idea, you can easily control the nature of generated planet by changing the weight value of offset as shown in Figure 5.

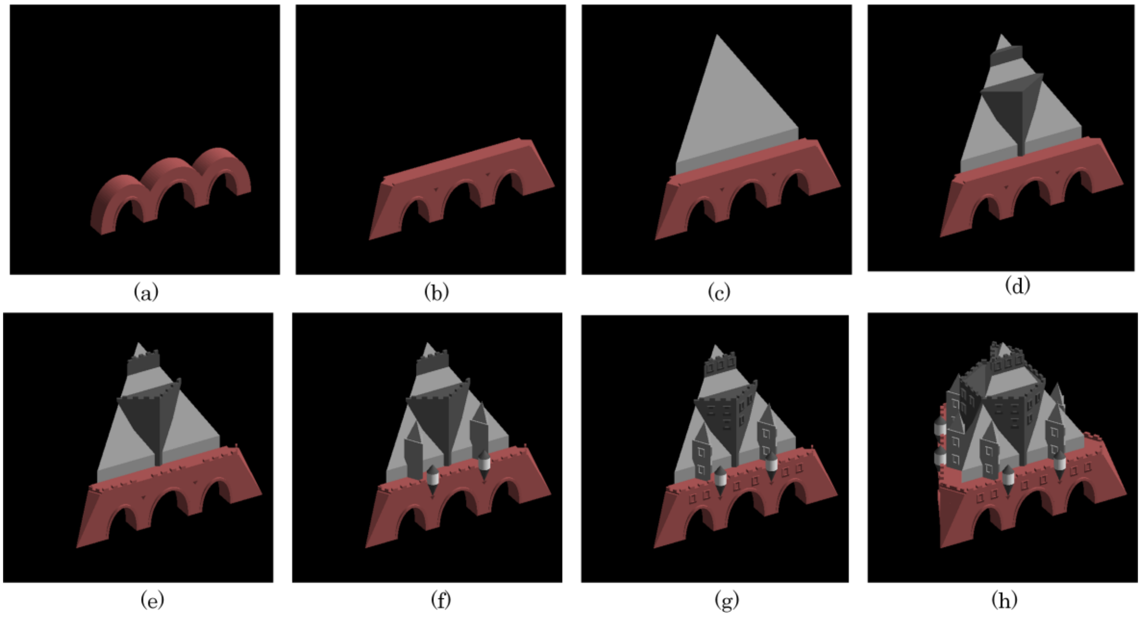

Figure 6 The making procedure of the civilized planetoid. (a) make bridge beam, (b) body of bridge, (c) main roof, (d) decks, (e) fences and blocks along edges of bridge and decks, (f) towers, (g) windows, (h) copy and rotate the constructed face.

4.2. Civilized Planetoid

To make civilized objects, I used some kinds of primitives supported on SLIDE, and composed them. Many of modern objects are based on the combination of simple geometric design, so this simple method is effective enough. I designed it inspired by a castle based on the original Escher's planetoid, which has bridges, towers, and decks. The procedure of making the civilized planetoid is shown in Figure 6.

4.3. Joint Parts

I designed three kinds of joint parts shown in the Figure 3. The octahedral joint part has bulge parts with natural or civilized feature on all faces, and tetrahedral ones also have bulges with either of those features on the faces shown in Figure 7. I designed

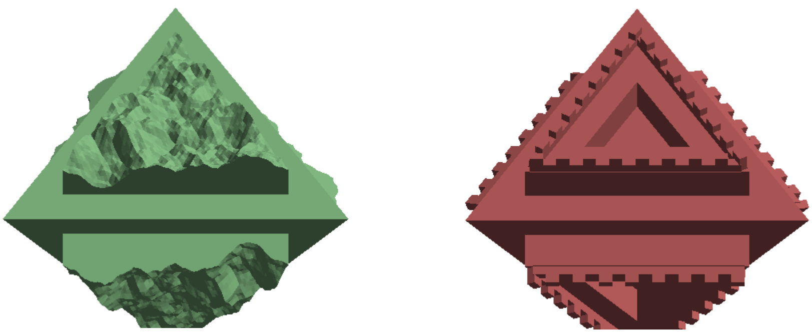

Figure 7 Two different kinds of joint faces, but they have an identical external shape

natural bulges by using same algorithm described in 4.1. In this case, I define a horizontal triangle at first, and apply the algorithm to it. For civilized bulges I designed them inspired by castle's walls. I used simple primitives in the same ways as the case of civilized planetoids, and represent the walls with fences and blocks. These two bulges have different features, but both of them have an identical external shape to joint both types of tetrahedral planetoids as shown in Figure 8. The natural and civilized tetrahedral planetoids also have the concaved hollow with same contours as bulges to joint to them.

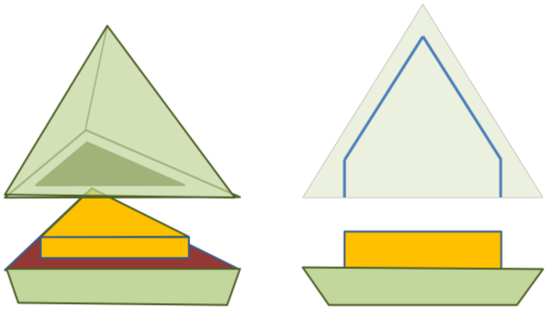

Figure 8　The bulge on the joint part, and concaved hollow of tetrahedral planetoid.

5. Conclusion

In this project, I designed Escher's planetoid on the SLIDE. I proposed to make some kinds of blocks composing Escher's planetoid, and to use them like puzzles. This work has a commodity value that can provide artistic and creative playtime. Additionally, the generation algorithm I used to make geographical shapes enables to design them with minimum costs. The midpoint displacement algorithm can be also applied to make same kinds of objects with natural features.

However, this work still has some problems. The biggest problem is in the civilized planetoid. I used some kinds of objects defined by using B-Spline curves and sweeping operations of the SLIDE, and assembled them to make it. But those parts are not assembled properly; they have violent intersections at some parts in that model. To make the mock-up model of it by the FDM machine, I need to plan the design more exactly, and make it without invalid intersections. The octahedral joint part also has a problem that the bottom part of the cooked model was broken because the bottom part was the pyramid top of the octahedron. Such a narrow shape cannot support the rest of upper parts. I need to modify the direction of the input model to work the FDM machine.

I will correct such problem, and finally I will make mock-ups of all parts by the FDM machine.

References

[1] Fereydoon Family and Tamas Vicsek, “Dynamics of Fractal Surfaces”, pp45, 1991
[2] Fournier,A., Fussel,D., and Carpenter, L., “Random Midpoint Displacement Method”, Computer Rendering of Stochastic Models. Communications of the ACM, 25: pp371-384, 1982