Today

Approximation Algorithm.

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Facility Location.

Maximum Weight Matching.

Bipartite Graph $G = (V, E)$, $w : E \rightarrow Z$.

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Any “vertex” solution is integer!

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That's what we did.

Facility location

Set of facilities: F, opening cost f_i for facility i

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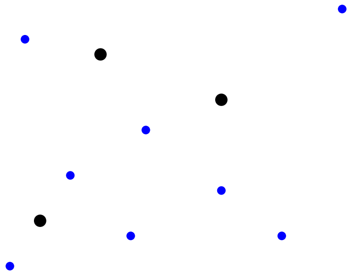

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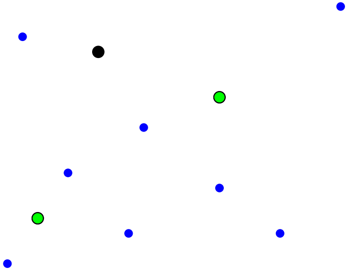

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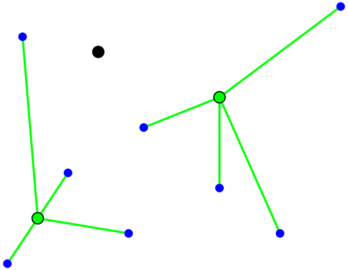

Facility Location

Linear program relaxation:

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“Decision Variables”.

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$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$

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$$\forall i \in F, j \in D \quad x_{ij} \leq y_i,$$

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Facility opening cost.

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Client Connnection cost.
Must connect each client.

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Client Connnection cost.
Must connect each client.
Only connect to open facility.

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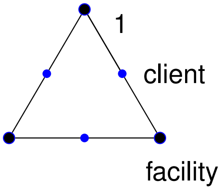

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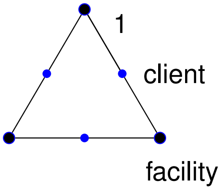

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Facility Cost: $\frac{3}{2}$

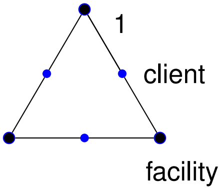
Integer Solution?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$

$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$

$$\forall i \in F, j \in D \quad x_{ij} \leq y_i,$$

$$x_{ij}, y_i \geq 0$$



$x_{ij} = \frac{1}{2}$ edges.
$y_i = \frac{1}{2}$ edges.

Facility Cost: $\frac{3}{2}$ Connection Cost: 3

Integer Solution?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$
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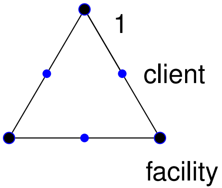

$x_{ij} = \frac{1}{2}$ edges.
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Facility Cost: $\frac{3}{2}$ Connection Cost: 3
Any one Facility:
Facility Cost: 1

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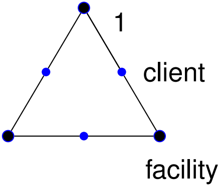

$x_{ij} = \frac{1}{2}$ edges.
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Facility Cost: $\frac{3}{2}$ Connection Cost: 3
Any one Facility:
Facility Cost: 1 Client Cost: 3.7

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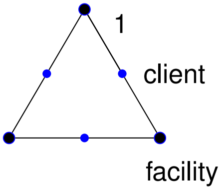

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Facility Cost: 1 Client Cost: 3.7
Make it worse?

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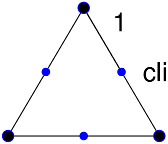

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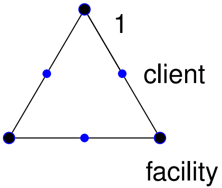

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Facility Cost: $\frac{3}{2}$ Connection Cost: 3
Any one Facility:
Facility Cost: 1 Client Cost: 3.7
Make it worse? Sure. Not as pretty!

Round solution?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$

$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$

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Round independently?

Round solution?

$$
\begin{aligned}
\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij} \\
\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1 \\
\forall i \in F, j \in D \quad x_{ij} \leq y_i, \\
x_{ij}, y_i \geq 0
\end{aligned}
$$

Round independently?

y_i and x_{ij} separately?

Round solution?

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Round independently?

y_i and x_{ij} separately? Assign to closed facility!

Round solution?

$$
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Round x_{ij} and open facilities?
Different clients force different facilities open.

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Any ideas?

Use Dual!

The dual.

$\min cx, Ax \geq b$

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$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$
$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1 \qquad ; \ \alpha_j$$
$$\forall i \in F, j \in D \quad y_i - x_{ij} \geq 0 \qquad ; \ \beta_{ij}$$
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$$\begin{aligned} \max \sum_j \alpha_j \\ \forall i \quad \sum_{j \in D} \beta_{ij} \leq f_i \quad &; \ y_i \\ \forall i \in F, j \in D \quad \alpha_j - \beta_{ij} \leq d_{ij} \quad &; \ x_{ij} \\ \beta_{ij}, \alpha_j \geq 0 \end{aligned}$$

Interpretation of Dual?

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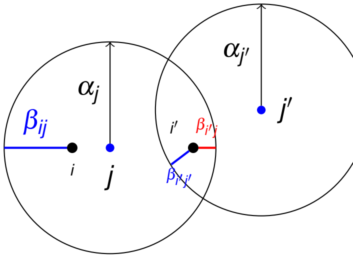

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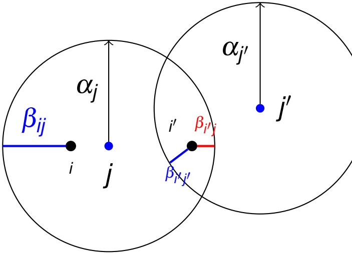

Total payment to facility i at most f_i before opening.

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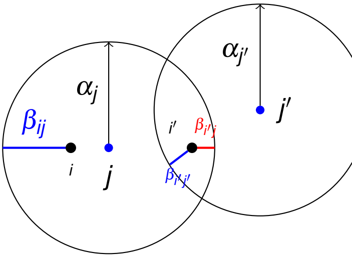
Complementary slackness:



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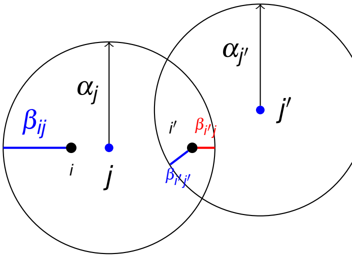

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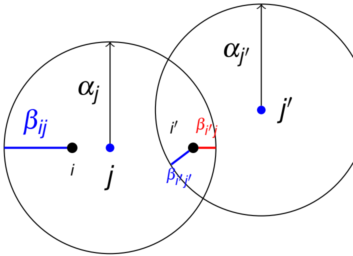
Complementary slackness: $x_{ij} \geq 0$ if and only if $\alpha_j \geq d_{ij}$.
only assign client to "paid to" facilities.



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3. Removed assigned clients, goto 2.

Integral facility cost at most LP facility cost.

Claim: Total facility cost is at most $\sum_i f_i y_i$.

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Integral facility cost at most LP facility cost.

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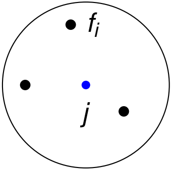

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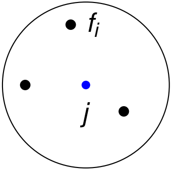
Proof: Step 2 picks client j.
$f_{\min}$ - min cost facility in N_j



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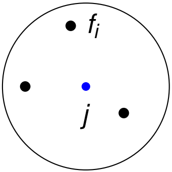

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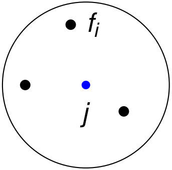

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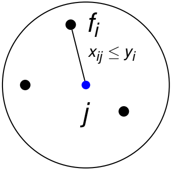

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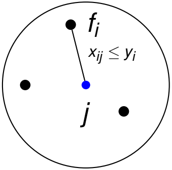

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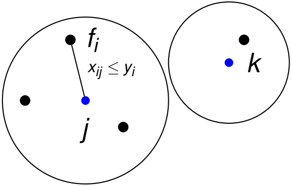

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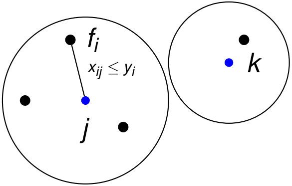

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$N_j \cap N_k = \emptyset$ for j and k in step 2.

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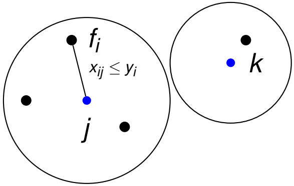

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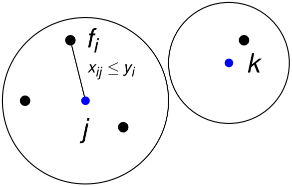

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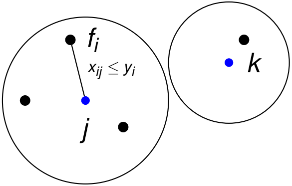

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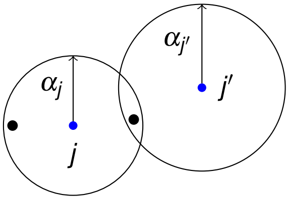

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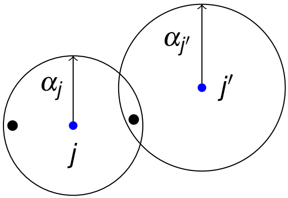

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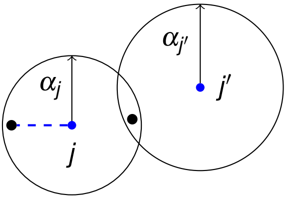

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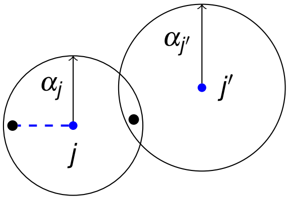

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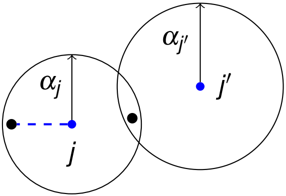

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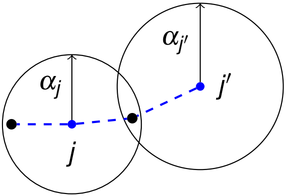

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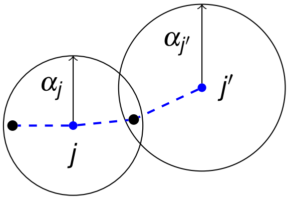

Connection Cost of j: $\leq \alpha_j$.
Connection Cost of j':
$\leq \alpha_{j'} + \alpha_j + \alpha_j$

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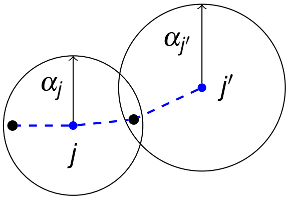

Connection Cost of j: $\leq \alpha_j$.
Connection Cost of j':
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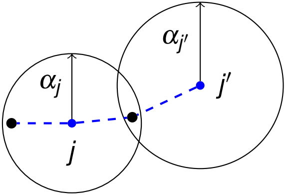

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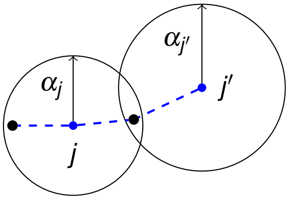

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Total connection cost:
  at most $3\sum_j \alpha_j \leq 3$ times Dual OPT.

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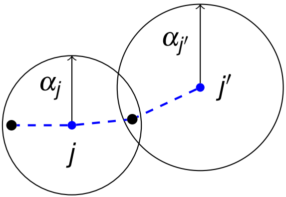

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Previous Slide: Facility cost:

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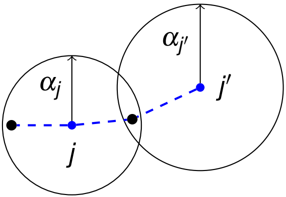

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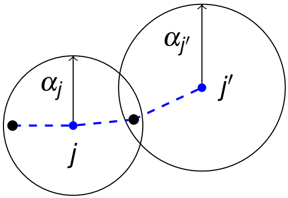

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Total Cost: 4 OPT.

Twist on randomized rounding.

Client j:

Twist on randomized rounding.

Client j: $\sum_i x_{ij} = 1$,

Twist on randomized rounding.

Client j: $\sum_i x_{ij} = 1$, $x_{ij} \geq 0$.

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Probability distribution!

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Expected opening cost:

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$D_j = \sum_i x_{ij} d_{ij}$

Twist on randomized rounding.

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Expected opening cost:
$\sum_{i \in N_j} x_{ij} f_i \leq \sum_{i \in N_j} y_i f_i$.
and separate balls implies total $\leq \sum_i y_i f_i$.

$D_j = \sum_i x_{ij} d_{ij}$ Expected connection cost of primal for j.

Twist on randomized rounding.

Client j: $\sum_i x_{ij} = 1$, $x_{ij} \geq 0$.
Probability distribution! $\rightarrow$ Choose from distribution, x_{ij}, in step 2.

Expected opening cost:
$\sum_{i \in N_j} x_{ij} f_i \leq \sum_{i \in N_j} y_i f_i$.
and separate balls implies total $\leq \sum_i y_i f_i$.

$D_j = \sum_i x_{ij} d_{ij}$ Expected connection cost of primal for j.

Expected connection cost j'

Twist on randomized rounding.

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$D_j = \sum_i x_{ij} d_{ij}$ Expected connection cost of primal for j.

Expected connection cost j' $\alpha_j + \alpha_{j'} + D_j$.

Twist on randomized rounding.

Client j: $\sum_i x_{ij} = 1$, $x_{ij} \geq 0$.
Probability distribution! $\rightarrow$ Choose from distribution, x_{ij}, in step 2.

Expected opening cost:
$\sum_{i \in N_j} x_{ij} f_i \leq \sum_{i \in N_j} y_i f_i$.
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$D_j = \sum_i x_{ij} d_{ij}$ Expected connection cost of primal for j.

Expected connection cost j' $\alpha_j + \alpha_{j'} + D_j$.

In step 2: pick in increasing order of $\alpha_j + D_j$.

Twist on randomized rounding.

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$D_j = \sum_i x_{ij} d_{ij}$ Expected connection cost of primal for j.

Expected connection cost j' $\alpha_j + \alpha_{j'} + D_j$.

In step 2: pick in increasing order of $\alpha_j + D_j$.

$\rightarrow$ Expected cost is $\leq (2\alpha_{j'} + D_{j'})$.
Connection cost: $2\sum_j \alpha_j + \sum_j D_j$.
$2OPT(D)$ plus connection cost of primal.

Twist on randomized rounding.

Client j: $\sum_i x_{ij} = 1$, $x_{ij} \geq 0$.
Probability distribution! $\rightarrow$ Choose from distribution, x_{ij}, in step 2.

Expected opening cost:
$\sum_{i \in N_j} x_{ij} f_i \leq \sum_{i \in N_j} y_i f_i$.
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Total expected cost:

Twist on randomized rounding.

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$\sum_{i \in N_j} x_{ij} f_i \leq \sum_{i \in N_j} y_i f_i$.
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$D_j = \sum_i x_{ij} d_{ij}$ Expected connection cost of primal for j.

Expected connection cost j' $\alpha_j + \alpha_{j'} + D_j$.

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Connection cost: $2\sum_j \alpha_j + \sum_j D_j$.
$2OPT(D)$ plus connection cost of primal.

Total expected cost:
Facility cost is at most facility cost of primal.

Twist on randomized rounding.

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Total expected cost:
Facility cost is at most facility cost of primal.
Connection cost at most $2OPT$ + connection cost of prmal.

Twist on randomized rounding.

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Connection cost: $2\sum_j \alpha_j + \sum_j D_j$.
$2OPT(D)$ plus connection cost of primal.

Total expected cost:
Facility cost is at most facility cost of primal.
Connection cost at most $2OPT$ + connection cost of prmal.
$\rightarrow$ at most $3OPT$.

Primal dual algorithm.

1. Feasible integer solution.

Primal dual algorithm.

1. Feasible integer solution.
2. Feasible dual solution.

Primal dual algorithm.

1. Feasible integer solution.
2. Feasible dual solution.
3. Cost of integer solution $\leq \alpha$ times dual value.

Primal dual algorithm.

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Just did it.

Primal dual algorithm.

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Just did it. Used linear program.

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Typically. (If dual is maximization.)

Primal dual algorithm.

1. Feasible integer solution.
2. Feasible dual solution.
3. Cost of integer solution $\leq \alpha$ times dual value.

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Typically. (If dual is maximization.)
Begin with feasible dual.

Primal dual algorithm.

1. Feasible integer solution.
2. Feasible dual solution.
3. Cost of integer solution $\leq \alpha$ times dual value.

Just did it. Used linear program. Faster?

Typically. (If dual is maximization.)
Begin with feasible dual.
Raise dual variables until tight constraint.

Primal dual algorithm.

1. Feasible integer solution.
2. Feasible dual solution.
3. Cost of integer solution $\leq \alpha$ times dual value.

Just did it. Used linear program. Faster?

Typically. (If dual is maximization.)
Begin with feasible dual.
Raise dual variables until tight constraint.
Set corresponding primal variable to an integer.

Primal dual algorithm.

1. Feasible integer solution.
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3. Cost of integer solution $\leq \alpha$ times dual value.

Just did it. Used linear program. Faster?

Typically. (If dual is maximization.)
  Begin with feasible dual.
  Raise dual variables until tight constraint.
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Recall Dual:

Primal dual algorithm.

1. Feasible integer solution.
2. Feasible dual solution.
3. Cost of integer solution $\leq \alpha$ times dual value.

Just did it. Used linear program. Faster?

Typically. (If dual is maximization.)
  Begin with feasible dual.
  Raise dual variables until tight constraint.
  Set corresponding primal variable to an integer.

Recall Dual:

$$
\begin{aligned}
& \max \sum_j \alpha_j \\
\forall i \in F \quad & \sum_{j \in D} \beta_{ij} \leq f_i \\
\forall i \in F, j \in D \quad & \alpha_j - \beta_{ij} \leq d_{ij} \\
& \alpha_j, \beta_{ij} \leq 0
\end{aligned}
$$

Facility location primal dual.

Phase 1:

Facility location primal dual.

Phase 1: 1. Initially $\alpha_j, \beta_{ij} = 0$.

Facility location primal dual.

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2. Raise α_j for every (unconnected) client.

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When $\alpha_j = d_{ij}$ for some i
raise β_{ij} at same rate

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Intution:Paying β_{ij} to open i.

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Stop when $\sum_i \beta_{ij} = f_i$.

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Intution: facility paid for.

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Temporarily open i.

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Temporarily open i.

Connect all tight *ji* clients *j* to *i*.

Facility location primal dual.

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Intution: facility paid for.

Temporarily open i.

Connect all tight *ji* clients *j* to *i*.

3. Continue until all clients connected.

Facility location primal dual.

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Phase 2:

Facility location primal dual.

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Phase 2:

Make “edge” between two facilities if paid by a common client.

Facility location primal dual.

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Stop when $\sum_i \beta_{ij} = f_i$.

Why? Dual: $\sum_i \beta_{ij} \leq f_i$

Intution: facility paid for.

Temporarily open i.

Connect all tight ji clients j to i.

3. Continue until all clients connected.

Phase 2:

Make "edge" between two facilities if paid by a common client.

Permanently open an independent set of facilities in common client graph.

Facility location primal dual.

Phase 1: 1. Initially $\alpha_j, \beta_{ij} = 0$.

2. Raise α_j for every (unconnected) client.

When $\alpha_j = d_{ij}$ for some i

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Stop when $\sum_i \beta_{ij} = f_i$.

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Phase 2:

Make "edge" between two facilities if paid by a common client.
Permanently open an independent set of facilities in common client graph.

For client j, connected facility i is opened.

Facility location primal dual.

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Intution:Paying β_{ij} to open i.

Stop when $\sum_i \beta_{ij} = f_i$.

Why? Dual: $\sum_i \beta_{ij} \leq f_i$

Intution: facility paid for.

Temporarily open i.

Connect all tight ji clients j to i.

3. Continue until all clients connected.

Phase 2:

Make "edge" between two facilities if paid by a common client.
Permanently open an independent set of facilities in common client graph.

For client j, connected facility i is opened. Good.

Facility location primal dual.

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When $\alpha_j = d_{ij}$ for some i
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Stop when $\sum_i \beta_{ij} = f_i$.
Why? Dual: $\sum_i \beta_{ij} \leq f_i$
Intution: facility paid for.
Temporarily open i.
Connect all tight ji clients j to i.

3. Continue until all clients connected.

Phase 2:
Make "edge" between two facilities if paid by a common client.
Permanently open an independent set of facilities in common client graph.

For client j, connected facility i is opened. Good.
Connected facility not open

Facility location primal dual.

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When $\alpha_j = d_{ij}$ for some i
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Intution:Paying β_{ij} to open i.
Stop when $\sum_i \beta_{ij} = f_i$.
Why? Dual: $\sum_i \beta_{ij} \leq f_i$
Intution: facility paid for.
Temporarily open i.
Connect all tight ji clients j to i.

3. Continue until all clients connected.

Phase 2:
Make "edge" between two facilities if paid by a common client.
Permanently open an independent set of facilities in common client graph.

For client j, connected facility i is opened. Good.
Connected facility not open
$\rightarrow$ exists client j' paid i and connected to open facility.

Facility location primal dual.

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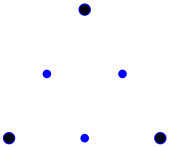
Intution:Paying β_{ij} to open i.
Stop when $\sum_i \beta_{ij} = f_i$.
Why? Dual: $\sum_i \beta_{ij} \leq f_i$
Intution: facility paid for.
Temporarily open i.
Connect all tight ji clients j to i.

3. Continue until all clients connected.

Phase 2:
Make "edge" between two facilities if paid by a common client.
Permanently open an independent set of facilities in common client graph.

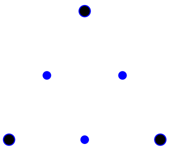
For client j, connected facility i is opened. Good.
Connected facility not open
$\rightarrow$ exists client j' paid i and connected to open facility.
Connect j to j''s open facility.

Constraints for dual.



Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$



Constraints for dual.

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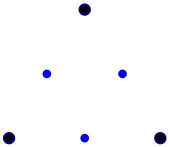
$\alpha_i - \beta_{ij} \leq d_{ij}$.



Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

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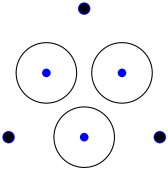
Grow α_j.



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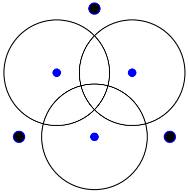


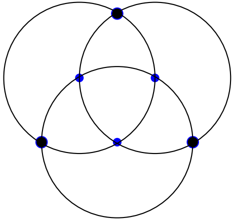
Constraints for dual.

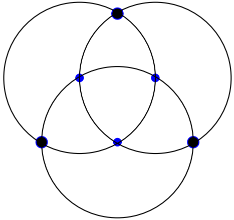
$\sum_j \beta_{ij} \leq f_i$

$\alpha_i - \beta_{ij} \leq d_{ij}$.

Grow α_j.

$\alpha_j = d_{ij}$!





Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

$\alpha_i - \beta_{ij} \leq d_{ij}$.

Grow α_j.

$\alpha_j = d_{ij}$!

Tight constraint:

Constraints for dual.

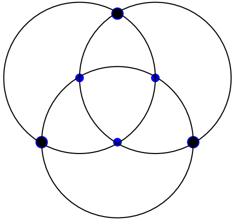
$\sum_j \beta_{ij} \leq f_i$

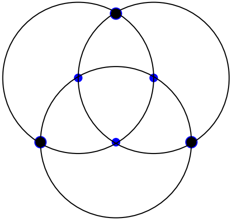
$\alpha_i - \beta_{ij} \leq d_{ij}$.

Grow α_j.

$\alpha_j = d_{ij}$!

Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.





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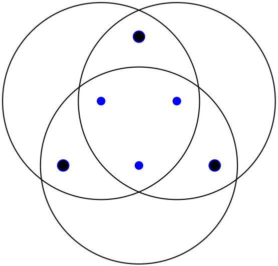
$\alpha_j = d_{ij}$!

Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

Grow β_{ij} (and α_j).



Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

$\alpha_i - \beta_{ij} \leq d_{ij}$.

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Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

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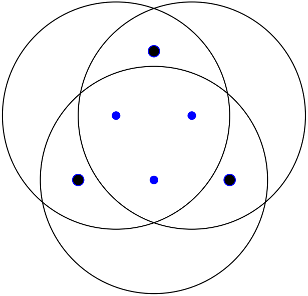

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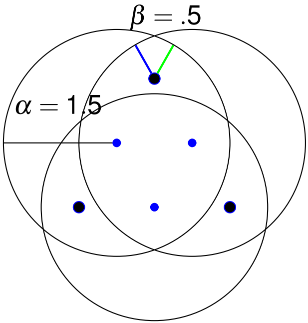

Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

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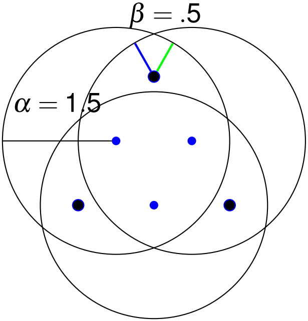
$\alpha_j = d_{ij}$!

Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

Grow β_{ij} (and α_j).

$\sum_j \beta_{ij} = f_i$ for all facilities.



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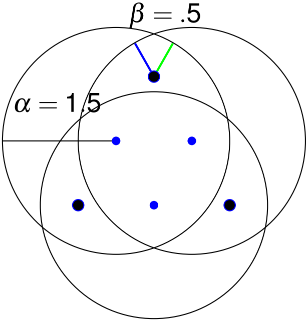

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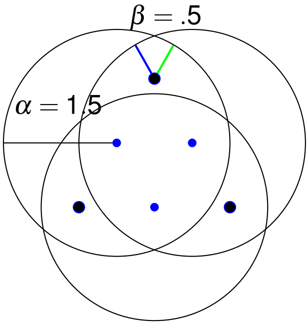

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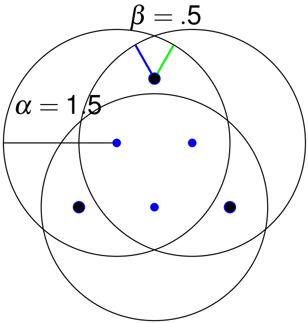
Grow β_{ij} (and α_j).

$\sum_j \beta_{ij} = f_i$ for all facilities.

Tight: $\sum_j \beta_{ij} \leq f_i$

LP Cost: $\sum_j \alpha_j$



Constraints for dual.

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Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

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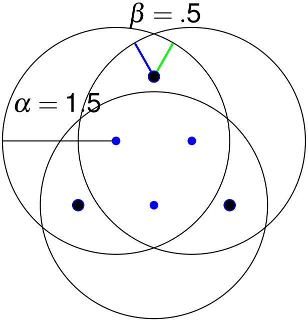
$\sum_j \beta_{ij} = f_i$ for all facilities.

Tight: $\sum_j \beta_{ij} \leq f_i$

LP Cost: $\sum_j \alpha_j = 4.5$



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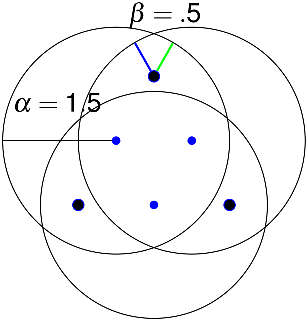
LP Cost: $\sum_j \alpha_j = 4.5$

Temporarily open all facilities.



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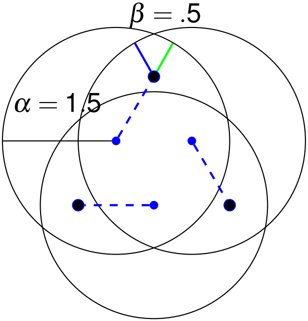

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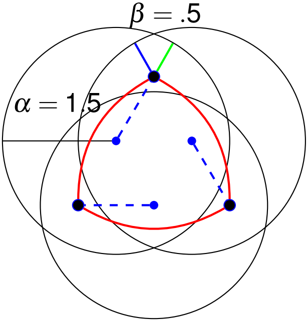
LP Cost: $\sum_j \alpha_j = 4.5$

Temporarily open all facilities.

Assign Clients to “paid to” open facility.



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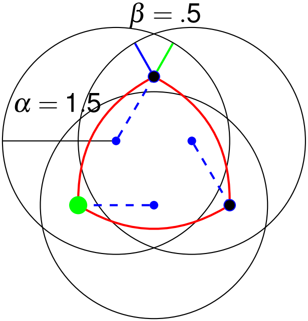
LP Cost: $\sum_j \alpha_j = 4.5$

Temporarily open all facilities.

Assign Clients to “paid to” open facility.

Connect facilities with common client.



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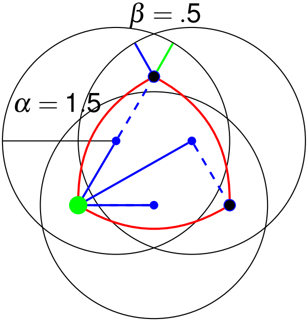
LP Cost: $\sum_j \alpha_j = 4.5$

Temporarily open all facilities.

Assign Clients to “paid to” open facility.

Connect facilities with common client.

Open independent set.



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$\sum_j \beta_{ij} = f_i$ for all facilities.

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LP Cost: $\sum_j \alpha_j = 4.5$

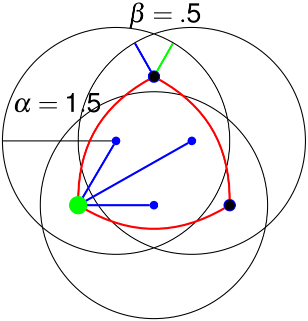
Temporarily open all facilities.

Assign Clients to “paid to” open facility.

Connect facilities with common client.

Open independent set.

Connect to “killer” client’s facility.



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Temporarily open all facilities.

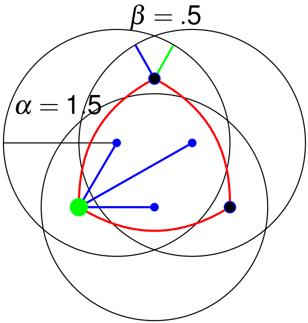
Assign Clients to “paid to” open facility.

Connect facilities with common client.

Open independent set.

Connect to “killer” client’s facility.

Cost: 1 + 3.7



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LP Cost: $\sum_j \alpha_j = 4.5$

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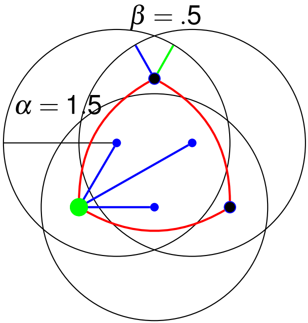
Assign Clients to “paid to” open facility.

Connect facilities with common client.

Open independent set.

Connect to “killer” client’s facility.

Cost: 1 + 3.7 = 4.7.



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Grow β_{ij} (and α_j).

$\sum_j \beta_{ij} = f_i$ for all facilities.

Tight: $\sum_j \beta_{ij} \leq f_i$

LP Cost: $\sum_j \alpha_j = 4.5$

Temporarily open all facilities.

Assign Clients to “paid to” open facility.

Connect facilities with common client.

Open independent set.

Connect to “killer” client’s facility.

Cost: 1 + 3.7 = 4.7.

A bit more than the LP cost.

Analysis

Claim: Client only pays one facility.

Analysis

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Independent set of facilities.

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Claim: S_i - directly connected clients to open facility i.

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Claim: S_i - directly connected clients to open facility i.
$f_i + \sum_{j \in S_i} d_{ij} \leq \sum_j \alpha_j$.

Analysis

Claim: Client only pays one facility.

Independent set of facilities.

Claim: S_i - directly connected clients to open facility i.
$f_i + \sum_{j \in S_i} d_{ij} \leq \sum_j \alpha_j$.

Proof:

Analysis

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Claim: S_i - directly connected clients to open facility i.
$f_i + \sum_{j \in S_i} d_{ij} \leq \sum_j \alpha_j$.

Proof:
$f_i = \sum_{j \in S_i} \beta_{ij}$

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$f_i + \sum_{j \in S_i} d_{ij} \leq \sum_j \alpha_j$.

Proof:
$f_i = \sum_{j \in S_i} \beta_{ij} = \sum_{j \in S_i} \alpha_j - d_{ij}$.

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Claim: Client only pays one facility.

Independent set of facilities.

Claim: S_i - directly connected clients to open facility i.
$f_i + \sum_{j \in S_i} d_{ij} \leq \sum_j \alpha_j$.

Proof:
$f_i = \sum_{j \in S_i} \beta_{ij} = \sum_{j \in S_i} \alpha_j - d_{ij}$.
Since directly connected: $\beta_{ij} = \alpha_j - d_{ij}$. $\square$

Analysis.

Claim: Client j is indirectly connected to i

Analysis.

Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.

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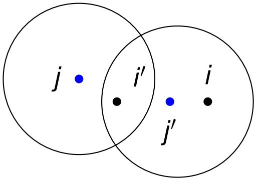
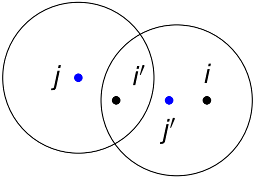
Directly connected to (temp open) i'



Analysis.

Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.



Directly connected to (temp open) i' conflicts with i.

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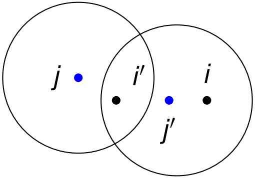
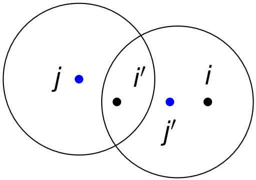


Directly connected to (temp open) i' conflicts with i.

exists j' with $\alpha_{j'} \geq d_{ij'}$ and $\alpha_j \geq d_{i'j'}$.

Analysis.

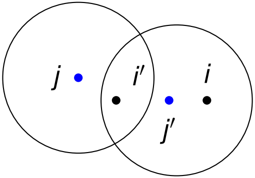
Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.



Directly connected to (temp open) i' conflicts with i.
exists j' with $\alpha_{j'} \geq d_{ij'}$ and $\alpha_j \geq d_{i'j'}$.
When i' opens, stops both α_j and α_j'.

Analysis.

Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.



Directly connected to (temp open) i' conflicts with i.

exists j' with $\alpha_{j'} \geq d_{ij'}$ and $\alpha_j \geq d_{i'j'}$.

When i' opens, stops both α_j and α'_j.

$\alpha_{j'}$ stopped no later

Analysis.

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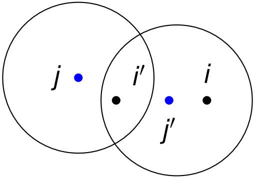
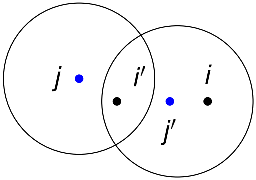


Directly connected to (temp open) i' conflicts with i.
exists j' with $\alpha_{j'} \geq d_{ij'}$ and $\alpha_j \geq d_{i'j'}$.
When i' opens, stops both α_j and α'_j.
$\alpha_{j'}$ stopped no later (..maybe earlier..)

Analysis.

Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.



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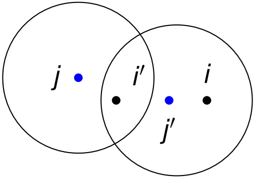
$\alpha_{j'}$ stopped no later (..maybe earlier..)
$\alpha_{j'} \leq \alpha_j$.

Analysis.

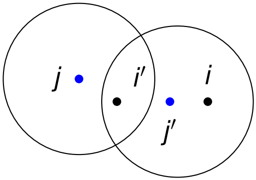
Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.



Directly connected to (temp open) i' conflicts with i.
exists j' with $\alpha_{j'} \geq d_{ij'}$ and $\alpha_j \geq d_{i'j'}$.
When i' opens, stops both α_j and α'_j.
$\alpha_{j'}$ stopped no later (..maybe earlier..)
$\alpha_{j'} \leq \alpha_j$.
Total distance from j to j'.

Analysis.

Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.



Directly connected to (temp open) i' conflicts with i.
exists j' with $\alpha_{j'} \geq d_{ij'}$ and $\alpha_j \geq d_{i'j'}$.
When i' opens, stops both α_j and α_j'.
$\alpha_{j'}$ stopped no later (..maybe earlier..)
$\alpha_{j'} \leq \alpha_j$.
Total distance from j to j'.
$d_{ji'} +$

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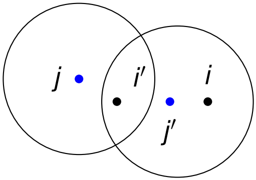

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$\alpha_{j'}$ stopped no later (..maybe earlier..)

$\alpha_{j'} \leq \alpha_j$.

Total distance from j to j'.

$d_{ji'} + d_{i'j'} +$

Analysis.

Claim: Client j is indirectly connected to i
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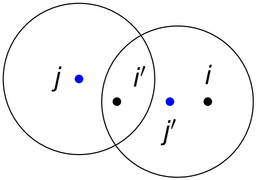

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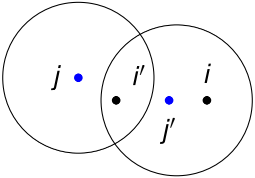
When i' opens, stops both α_j and α_j'.
$\alpha_{j'}$ stopped no later (..maybe earlier..)
$\alpha_{j'} \leq \alpha_j$.
Total distance from j to j'.
$d_{ji'} + d_{i'j'} + d_{j'i}$

Analysis.

Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.



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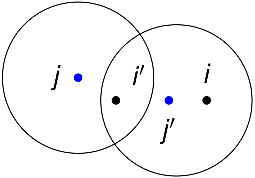
$\alpha_{j'} \leq \alpha_j$.
Total distance from j to j'.
$d_{ji'} + d_{i'j'} + d_{j'i} \leq 3\alpha_j$

Analysis.

Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.



Directly connected to (temp open) i' conflicts with i.

exists j' with $\alpha_{j'} \geq d_{ij'}$ and $\alpha_j \geq d_{i'j'}$.

When i' opens, stops both α_j and α_j'.

$\alpha_{j'}$ stopped no later (..maybe earlier..)

$\alpha_{j'} \leq \alpha_j$.

Total distance from j to j'.

$d_{ji'} + d_{i'j'} + d_{j'i} \leq 3\alpha_j$ □

Putting it together!

Claim: Client only pays one facility.

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Total Cost:

Putting it together!

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Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.

Total Cost:
direct clients dual (α_j) pays for facility and own connections.

Putting it together!

Claim: Client only pays one facility.
Claim: S_i - directly connected clients to open facility i.
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3 OPT.

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3 OPT.

Fast!

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3 OPT.

Fast! Cheap!

Putting it together!

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3 OPT.

Fast! Cheap! Safe!

Check: if time.

Check: if time.

Won’t see you on Tuesday.

Check: if time.

Won’t see you on Tuesday.

Guest Speaker: Tselil Schramm.

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Semidefinite Programming and Approximation.