Today

Experts/Zero-Sum Games Equilibrium.

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Boosting and Experts.

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Routing and Experts.

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Linear Programming Introduction (Gentle)

Games and experts

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n Experts, T days, L^* -total loss of best expert.

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$L \leq (1+\varepsilon)L^* + \frac{\log n}{\varepsilon}$

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Multiplicative Weights: $L \leq (1+\varepsilon) L^* + \frac{\ln n}{\varepsilon}$

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$TC(x^*) \leq (1+\varepsilon)TR(y^*) + \frac{\ln n}{\varepsilon}$

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$$TC(x^*) \leq (1+\varepsilon)TR(y^*) + \frac{\ln n}{\varepsilon} \rightarrow C(x^*) \leq (1+\varepsilon)R(y^*) + \frac{\ln n}{\varepsilon T}$$

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$T = \frac{\ln n}{\varepsilon^2}$, $R(y^*) \leq 1 \rightarrow C(x^*) - R(y^*) \leq 2\varepsilon$.

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$T = \frac{\ln n}{\varepsilon^2} \to O(nm\frac{\log n}{\varepsilon^2})$.

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Versus Linear Programming: $O(n^3 m)$ Basically quadratic.
(Faster linear programming: $O(\sqrt{n+m})$ linear solution solves.)

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$T = \frac{\ln n}{\varepsilon^2} \to O(nm\frac{\log n}{\varepsilon^2})$. Basically linear!

Versus Linear Programming: $O(n^3 m)$ Basically quadratic.
(Faster linear programming: $O(\sqrt{n+m})$ linear solution solves.)
Still much slower

Comments

For any ε, there exists an ε-Approximate Equilibrium.

Does an equilibrium exist? Yes.

Something about math here? Fixed point theorem.

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"In practice."

Boosting...

Learning

Learning just a bit.

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Example: set of labelled points, find hyperplane that separates.

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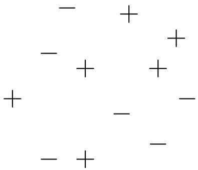

Looks hard.

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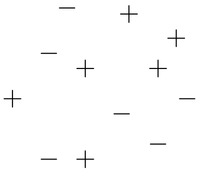

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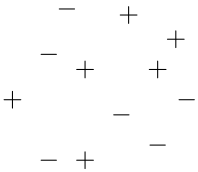

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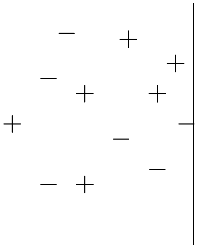

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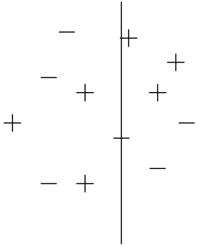

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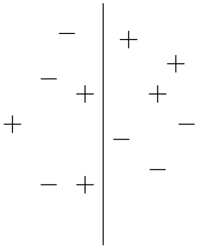

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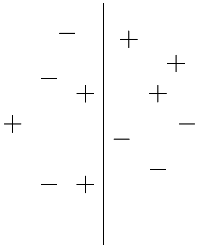

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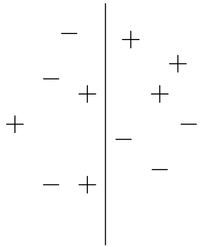

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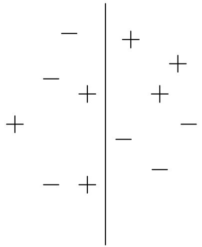

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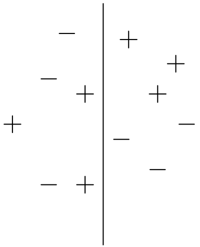

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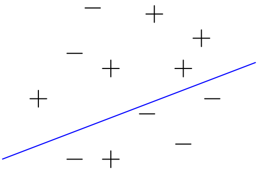

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That's a really strong learner!

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Can one use weak learning to produce strong learner?

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Boosting: use a weak learner to produce strong learner.

Poll.

Given a weak learning method (produce ok hypotheses.)

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Can we do this?

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Can we do this?

(A) Yes

(B) No

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(A) Yes
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If yes.

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Multiplicative Weights!

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Multiplicative Weights!

The endpoint to a line of research.

Boosting/MW Framework

Experts are points.

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Experts are points. “Adversary” weak learner.

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Points (experts) suffer loss when classified correctly.

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Really?

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majority of $h_t(x)$ are wrong for $x \in S_{bad}$.

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$|S_{bad}|(1-\varepsilon)^{T/2} \leq W(T) \leq ne^{-\varepsilon(\frac{1}{2}+\gamma)}T$

Calculation..

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Set $\varepsilon = \gamma$, take logs.

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Set $\varepsilon = \gamma$, take logs.

$$\ln\left(\frac{|S_{bad}|}{n}\right) + \frac{T}{2}\ln(1-\gamma) \leq -\gamma T(\frac{1}{2}+\gamma)$$

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$G \leq T \times C^*$ – each day, gain is avg. congestion $\leq$ opt congestion.

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$G \geq (1-\varepsilon)G^* - \frac{\rho \log n}{\varepsilon}$.

Let $T = \frac{k \log n}{\varepsilon^2}$

1. Row player runs multiplicative weights on edges:
   $w_i = w_i(1+\varepsilon)^{g_i/k}$.
2. Column routes all paths along shortest paths.
3. Output the average of all routings: $\frac{1}{T}\sum_t f(t)$.

Claim: The congestion, c_{max} is at most $C^* + 2k\varepsilon$.

Proof:

$G \geq G^*(1-\varepsilon) - \frac{k \log n}{\varepsilon T} \rightarrow G^* - G \leq \varepsilon G^* + \frac{k \log n}{\varepsilon}$

$G^* = T * c_{\max}$ – Best row payoff against average routing (times T).

$G \leq T \times C^*$ – each day, gain is avg. congestion $\leq$ opt congestion.

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$c_{max} - C^* \leq \varepsilon C^* + \varepsilon$ $\square$

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Thus expected log of the ratio of the algorithm to the best stock is within $O(\sqrt{\frac{\log n}{T}})$ of the best. ($\log r \leq 1$).

See you on Tuesday.