Today

Experts/Multiplicative Weights.

Today

Experts/Multiplicative Weights.

Experts/Zero-Sum Games Equilibrium.

Today

Experts/Multiplicative Weights.

Experts/Zero-Sum Games Equilibrium.

Boosting and Experts.

Today

Experts/Multiplicative Weights.

Experts/Zero-Sum Games Equilibrium.

Boosting and Experts.

Routing and Experts.

The multiplicative weights framework.

Experts framework.

n experts.

Experts framework.

n experts.

Every day, each offers a prediction.

Experts framework.

n experts.

Every day, each offers a prediction.

“Rain” or “Shine.”

Experts framework.

n experts.

Every day, each offers a prediction.

"Rain" or "Shine."

	Day 1	Day 2	Day 3	$\cdots$	Day T
Expert 1					
Expert 2				$\cdots$	
Expert 3				$\cdots$	
$\vdots$				$\cdots$	

Experts framework.

n experts.

Every day, each offers a prediction.

"Rain" or "Shine."

	Day 1	Day 2	Day 3	$\cdots$	Day T
Expert 1	Shine				
Expert 2	Shine			$\cdots$	
Expert 3	Rain			$\cdots$	
$\vdots$	$\vdots$			$\cdots$	

Rained!

Experts framework.

n experts.

Every day, each offers a prediction.

“Rain” or “Shine.”

	Day 1	Day 2	Day 3	$\cdots$	Day T
Expert 1	Shine	Rain			
Expert 2	Shine	Shine		$\cdots$	
Expert 3	Rain	Rain		$\cdots$	
$\vdots$	$\vdots$	$\vdots$		$\ldots$	

Rained! Shined!

Experts framework.

n experts.

Every day, each offers a prediction.

“Rain” or “Shine.”

	Day 1	Day 2	Day 3	···	Day T
Expert 1	Shine	Rain	Shine		
Expert 2	Shine	Shine	Shine	···	
Expert 3	Rain	Rain	Rain	···	
⋮	⋮	⋮	Shine	···	

Rained! Shined! Shined!

Experts framework.

n experts.

Every day, each offers a prediction.

"Rain" or "Shine."

	Day 1	Day 2	Day 3	$\cdots$	Day T
Expert 1	Shine	Rain	Shine		
Expert 2	Shine	Shine	Shine	$\cdots$	
Expert 3	Rain	Rain	Rain	$\cdots$	
$\vdots$	$\vdots$	$\vdots$	Shine	$\cdots$	

Rained! Shined! Shined! $\cdots$

Experts framework.

n experts.

Every day, each offers a prediction.

"Rain" or "Shine."

	Day 1	Day 2	Day 3	⋯	Day T
Expert 1	Shine	Rain	Shine		
Expert 2	Shine	Shine	Shine	⋯	
Expert 3	Rain	Rain	Rain	⋯	
⋮	⋮	⋮	Shine	⋯	

Rained! Shined! Shined! ⋯

Whose advice do you follow?

Experts framework.

n experts.

Every day, each offers a prediction.

"Rain" or "Shine."

	Day 1	Day 2	Day 3	$\cdots$	Day T
Expert 1	Shine	Rain	Shine		
Expert 2	Shine	Shine	Shine	$\cdots$	
Expert 3	Rain	Rain	Rain	$\cdots$	
$\vdots$	$\vdots$	$\vdots$	Shine	$\cdots$	

Rained! Shined! Shined! $\cdots$

Whose advice do you follow?

"The one who is correct most often."

Experts framework.

n experts.

Every day, each offers a prediction.

"Rain" or "Shine."

	Day 1	Day 2	Day 3	$\cdots$	Day T
Expert 1	Shine	Rain	Shine		
Expert 2	Shine	Shine	Shine	$\cdots$	
Expert 3	Rain	Rain	Rain	$\cdots$	
$\vdots$	$\vdots$	$\vdots$	Shine	$\cdots$	

Rained! Shined! Shined! $\cdots$

Whose advice do you follow?

"The one who is correct most often."

Sort of.

Experts framework.

n experts.

Every day, each offers a prediction.

"Rain" or "Shine."

	Day 1	Day 2	Day 3	$\cdots$	Day T
Expert 1	Shine	Rain	Shine		
Expert 2	Shine	Shine	Shine	$\cdots$	
Expert 3	Rain	Rain	Rain	$\cdots$	
$\vdots$	$\vdots$	$\vdots$	Shine	$\cdots$	

Rained! Shined! Shined! $\cdots$

Whose advice do you follow?

"The one who is correct most often."

Sort of.

How well do you do?

Infallible expert.

One of the experts is infallible!

Infallible expert.

One of the experts is infallible!

Your strategy?

Infallible expert.

One of the experts is infallible!

Your strategy?

Choose any expert that has not made a mistake!

Infallible expert.

One of the experts is infallible!

Your strategy?

Choose any expert that has not made a mistake!

How long to find perfect expert?

Infallible expert.

One of the experts is infallible!

Your strategy?

Choose any expert that has not made a mistake!

How long to find perfect expert?

Maybe..

Infallible expert.

One of the experts is infallible!

Your strategy?

Choose any expert that has not made a mistake!

How long to find perfect expert?

Maybe..never!

Infallible expert.

One of the experts is infallible!

Your strategy?

Choose any expert that has not made a mistake!

How long to find perfect expert?

Maybe..never! Never see a mistake.

Infallible expert.

One of the experts is infallible!

Your strategy?

Choose any expert that has not made a mistake!

How long to find perfect expert?

Maybe..never! Never see a mistake.

Better model?

Infallible expert.

One of the experts is infallible!

Your strategy?

Choose any expert that has not made a mistake!

How long to find perfect expert?

Maybe..never! Never see a mistake.

Better model?

How many mistakes could you make?

Infallible expert.

One of the experts is infallible!

Your strategy?

Choose any expert that has not made a mistake!

How long to find perfect expert?

Maybe..never! Never see a mistake.

Better model?

How many mistakes could you make? Mistake Bound.

Infallible expert.

One of the experts is infallible!

Your strategy?

Choose any expert that has not made a mistake!

How long to find perfect expert?

Maybe..never! Never see a mistake.

Better model?

How many mistakes could you make? Mistake Bound.

(A) 1

(B) 2

(C) $\log n$

(D) $n-1$

Adversary designs setup to watch who you choose, and make that expert make a mistake.

Infallible expert.

One of the experts is infallible!

Your strategy?

Choose any expert that has not made a mistake!

How long to find perfect expert?

Maybe..never! Never see a mistake.

Better model?

How many mistakes could you make? Mistake Bound.

(A) 1

(B) 2

(C) $\log n$

(D) $n-1$

Adversary designs setup to watch who you choose, and make that expert make a mistake.

$n-1$!

Concept Alert.

Note.

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Adversary:

Concept Alert.

Note.

Adversary:
makes you want to look bad.

Concept Alert.

Note.

Adversary:
makes you want to look bad.
”You could have done so well...

Concept Alert.

Note.

Adversary:
makes you want to look bad.
”You could have done so well...
but you didn’t!

Concept Alert.

Note.

Adversary:
makes you want to look bad.
”You could have done so well...
but you didn’t! ha..

Concept Alert.

Note.

Adversary:
makes you want to look bad.
”You could have done so well...
but you didn’t! ha..ha...

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Adversary:
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Concept Alert.

Note.

Adversary:
makes you want to look bad.
"You could have done so well...
but you didn't! ha..ha... ha.

Technical Term: Regret.

Concept Alert.

Note.

Adversary:
makes you want to look bad.
”You could have done so well...
but you didn’t! ha..ha... ha.

Technical Term: Regret.

Analysis of Algorithms: do as well as possible!

Concept Alert.

Note.

Adversary:
  makes you want to look bad.
  ”You could have done so well...
    but you didn’t! ha..ha... ha.

Technical Term: Regret.

Analysis of Algorithms: do as well as possible!
  Minimize Regret

Concept Alert.

Note.

Adversary:
makes you want to look bad.
"You could have done so well...
but you didn't! ha..ha... ha.

Technical Term: Regret.

Analysis of Algorithms: do as well as possible!
Minimize Regret $\equiv$ Loss.

Back to mistake bound.

Infallible Experts.

Back to mistake bound.

Infallible Experts.

Alg: Choose one of the perfect experts.

Back to mistake bound.

Infallible Experts.

Alg: Choose one of the perfect experts.

Mistake Bound: $n-1$

Back to mistake bound.

Infallible Experts.

Alg: Choose one of the perfect experts.

Mistake Bound: $n-1$

Lower bound: adversary argument.

Back to mistake bound.

Infallible Experts.

Alg: Choose one of the perfect experts.

Mistake Bound: $n-1$
  Lower bound: adversary argument.
  Upper bound:

Back to mistake bound.

Infallible Experts.

Alg: Choose one of the perfect experts.

Mistake Bound: $n-1$
Lower bound: adversary argument.
Upper bound: every mistake finds fallible expert.

Back to mistake bound.

Infallible Experts.

Alg: Choose one of the perfect experts.

Mistake Bound: $n-1$
  Lower bound: adversary argument.
  Upper bound: every mistake finds fallible expert.

Better Algorithm?

Back to mistake bound.

Infallible Experts.

Alg: Choose one of the perfect experts.

Mistake Bound: $n-1$
Lower bound: adversary argument.
Upper bound: every mistake finds fallible expert.

Better Algorithm?

Making decision, not trying to find expert!

Back to mistake bound.

Infallible Experts.

Alg: Choose one of the perfect experts.

Mistake Bound: $n-1$
  Lower bound: adversary argument.
  Upper bound: every mistake finds fallible expert.

Better Algorithm?

Making decision, not trying to find expert!

Algorithm: Go with the majority of previously correct experts.

Back to mistake bound.

Infallible Experts.

Alg: Choose one of the perfect experts.

Mistake Bound: $n-1$
Lower bound: adversary argument.
Upper bound: every mistake finds fallible expert.

Better Algorithm?

Making decision, not trying to find expert!

Algorithm: Go with the majority of previously correct experts.

What you would do anyway!

Alg 2: find majority of the perfect

How many mistakes could you make?

Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1

(B) 2

(C) $\log n$

(D) $n-1$

Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1

(B) 2

(C) $\log n$

(D) $n-1$

At most $\log n$!

Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1

(B) 2

(C) $\log n$

(D) $n-1$

At most $\log n$!

When alg makes a mistake,
$|$"perfect" experts$|$ drops by a factor of two.

Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1

(B) 2

(C) $\log n$

(D) $n - 1$

At most $\log n$!

When alg makes a mistake,
$|$"perfect" experts$|$ drops by a factor of two.

Initially n perfect experts

Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1

(B) 2

(C) $\log n$

(D) $n-1$

At most $\log n$!

When alg makes a mistake,
$|$"perfect" experts$|$ drops by a factor of two.

Initially n perfect experts
mistake $\rightarrow$ $\leq n/2$ perfect experts

Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1

(B) 2

(C) $\log n$

(D) $n - 1$

At most $\log n$!

When alg makes a mistake,
$|$"perfect" experts$|$ drops by a factor of two.

Initially n perfect experts
mistake $\rightarrow$ $\leq n/2$ perfect experts
mistake $\rightarrow$ $\leq n/4$ perfect experts

Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1

(B) 2

(C) $\log n$

(D) $n-1$

At most $\log n$!

When alg makes a mistake,
$|$"perfect" experts$|$ drops by a factor of two.

Initially n perfect experts
mistake $\rightarrow$ $\leq n/2$ perfect experts
mistake $\rightarrow$ $\leq n/4$ perfect experts

$\vdots$

Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1

(B) 2

(C) $\log n$

(D) $n - 1$

At most $\log n$!

When alg makes a mistake,
$|$"perfect" experts$|$ drops by a factor of two.

Initially n perfect experts
mistake $\rightarrow$ $\leq n/2$ perfect experts
mistake $\rightarrow$ $\leq n/4$ perfect experts

$\vdots$

mistake $\rightarrow$ ≤ 1 perfect expert

Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1

(B) 2

(C) $\log n$

(D) $n-1$

At most $\log n$!

When alg makes a mistake,
$|$"perfect" experts$|$ drops by a factor of two.

Initially n perfect experts
mistake $\rightarrow$ $\leq n/2$ perfect experts
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Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1

(B) 2

(C) $\log n$

(D) $n-1$

At most $\log n$!

When alg makes a <u>mistake</u>,
$|$"perfect" experts$|$ drops by a factor of two.

Initially n perfect experts
mistake $\rightarrow$ $\leq n/2$ perfect experts
mistake $\rightarrow$ $\leq n/4$ perfect experts

$\vdots$

mistake $\rightarrow$ ≤ 1 perfect expert

≥ 1 perfect expert

Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1

(B) 2

(C) $\log n$

(D) $n-1$

At most $\log n$!

When alg makes a mistake,
$|$"perfect" experts$|$ drops by a factor of two.

Initially n perfect experts
mistake $\rightarrow$ $\leq n/2$ perfect experts
mistake $\rightarrow$ $\leq n/4$ perfect experts

$\vdots$

mistake $\rightarrow$ ≤ 1 perfect expert

≥ 1 perfect expert $\rightarrow$ at most $\log n$ mistakes!

Imperfect Experts

Goal?

Imperfect Experts

Goal?

Do as well as the best expert!

Imperfect Experts

Goal?

Do as well as the best expert!

Algorithm.

Imperfect Experts

Goal?

Do as well as the best expert!

Algorithm. Suggestions?

Imperfect Experts

Goal?

Do as well as the best expert!

Algorithm. Suggestions?

Go with majority?

Imperfect Experts

Goal?

Do as well as the best expert!

Algorithm. Suggestions?

Go with majority?

Penalize inaccurate experts?

Imperfect Experts

Goal?

Do as well as the best expert!

Algorithm. Suggestions?

Go with majority?

Penalize inaccurate experts?

Best expert is penalized the least.

Imperfect Experts

Goal?

Do as well as the best expert!

Algorithm. Suggestions?

Go with majority?

Penalize inaccurate experts?

Best expert is penalized the least.

1. Initially: $w_i = 1$.

Imperfect Experts

Goal?

Do as well as the best expert!

Algorithm. Suggestions?

Go with majority?

Penalize inaccurate experts?

Best expert is penalized the least.

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.

Imperfect Experts

Goal?

Do as well as the best expert!

Algorithm. Suggestions?

Go with majority?

Penalize inaccurate experts?

Best expert is penalized the least.

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \to w_i/2$ if wrong.

Imperfect Experts

Goal?

Do as well as the best expert!

Algorithm. Suggestions?

Go with majority?

Penalize inaccurate experts?

Best expert is penalized the least.

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i/2$ if wrong.

Analysis: weighted majority

Analysis: weighted majority

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \to w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function:

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \to w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$.

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$. Initially n.

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \to w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$. Initially n.

For best expert, b, $w_b \geq \frac{1}{2^m}$.

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$. Initially n.

For best expert, b, $w_b \geq \frac{1}{2^m}$.

Each mistake:

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$. Initially n.

For best expert, b, $w_b \geq \frac{1}{2^m}$.

Each mistake:
  total weight of incorrect experts reduced by

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$. Initially n.

For best expert, b, $w_b \geq \frac{1}{2^m}$.

Each mistake:
total weight of incorrect experts reduced by
-1?

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$. Initially n.

For best expert, b, $w_b \geq \frac{1}{2^m}$.

Each mistake:
  total weight of incorrect experts reduced by
    -1? -2?

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
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Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$. Initially n.

For best expert, b, $w_b \geq \frac{1}{2^m}$.

Each mistake:
total weight of incorrect experts reduced by
-1? -2? factor of $\frac{1}{2}$?

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$. Initially n.

For best expert, b, $w_b \geq \frac{1}{2^m}$.

Each mistake:
total weight of incorrect experts reduced by
-1? -2? factor of $\frac{1}{2}$?
each incorrect expert weight multiplied by $\frac{1}{2}$!

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
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Potential function: $\sum_i w_i$. Initially n.

For best expert, b, $w_b \geq \frac{1}{2^m}$.

Each mistake:
total weight of incorrect experts reduced by
-1? -2? factor of $\frac{1}{2}$?
each incorrect expert weight multiplied by $\frac{1}{2}$!
total weight decreases by

1. Initially: $w_i = 1$.
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Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$. Initially n.

For best expert, b, $w_b \geq \frac{1}{2^m}$.

Each mistake:
  total weight of incorrect experts reduced by
    -1? -2?   factor of $\frac{1}{2}$?
    each incorrect expert weight multiplied by $\frac{1}{2}$!
  total weight decreases by
    factor of $\frac{1}{2}$? factor of $\frac{3}{4}$?

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
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Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$. Initially n.

For best expert, b, $w_b \geq \frac{1}{2^m}$.

Each mistake:
total weight of incorrect experts reduced by
-1? -2? factor of $\frac{1}{2}$?
each incorrect expert weight multiplied by $\frac{1}{2}$!
total weight decreases by
factor of $\frac{1}{2}$? factor of $\frac{3}{4}$?
mistake $\rightarrow \geq$ half weight with incorrect experts
($\geq \frac{1}{2}$ total.

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i/2$ if wrong.

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Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$. Initially n.

For best expert, b, $w_b \geq \frac{1}{2^m}$.

Each mistake:
  total weight of incorrect experts reduced by
    -1? -2? factor of $\frac{1}{2}$?
    each incorrect expert weight multiplied by $\frac{1}{2}$!
  total weight decreases by
    factor of $\frac{1}{2}$? factor of $\frac{3}{4}$?
mistake $\rightarrow \geq$ half weight with incorrect experts
($\geq \frac{1}{2}$ total.

Mistake $\rightarrow$ potential function decreased by $\frac{3}{4}$.

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i/2$ if wrong.

Analysis: weighted majority

Goal: Best expert makes m mistakes.

Potential function: $\sum_i w_i$. Initially n.

For best expert, b, $w_b \geq \frac{1}{2^m}$.

Each mistake:

total weight of incorrect experts reduced by

-1? -2? factor of $\frac{1}{2}$?

each incorrect expert weight multiplied by $\frac{1}{2}$!

total weight decreases by

factor of $\frac{1}{2}$? factor of $\frac{3}{4}$?

mistake $\rightarrow \geq$ half weight with incorrect experts ($\geq \frac{1}{2}$ total.

Mistake $\rightarrow$ potential function decreased by $\frac{3}{4}$.

We have

$\frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n.$

where M is number of algorithm mistakes.

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i/2$ if wrong.

Analysis: continued.

$$\frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n.$$

Analysis: continued.

$$\frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n.$$

m - best expert mistakes

Analysis: continued.

$$\frac{1}{2^m} \le \sum_i w_i \le \left(\frac{3}{4}\right)^M n.$$

m - best expert mistakes M algorithm mistakes.

Analysis: continued.

$$\frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n.$$

m - best expert mistakes M algorithm mistakes.

$$\frac{1}{2^m} \leq \left(\frac{3}{4}\right)^M n.$$

Analysis: continued.

$$\frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n.$$

m - best expert mistakes M algorithm mistakes.

$$\frac{1}{2^m} \leq \left(\frac{3}{4}\right)^M n.$$

Take log of both sides.

Analysis: continued.

$$\frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n.$$

m - best expert mistakes M algorithm mistakes.

$$\frac{1}{2^m} \leq \left(\frac{3}{4}\right)^M n.$$

Take log of both sides.

$$-m \leq -M \log(4/3) + \log n.$$

Analysis: continued.

$$\frac{1}{2^m} \le \sum_i w_i \le \left(\frac{3}{4}\right)^M n.$$

m - best expert mistakes M algorithm mistakes.

$$\frac{1}{2^m} \le \left(\frac{3}{4}\right)^M n.$$

Take log of both sides.

$$-m \le -M \log(4/3) + \log n.$$

Solve for M.

$M \le (m + \log n)/\log(4/3)$

Analysis: continued.

$$\frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n.$$

m - best expert mistakes M algorithm mistakes.

$$\frac{1}{2^m} \leq \left(\frac{3}{4}\right)^M n.$$

Take log of both sides.

$$-m \leq -M \log(4/3) + \log n.$$

Solve for M.

$$M \leq (m + \log n)/\log(4/3) \leq 2.4(m + \log n)$$

Analysis: continued.

$$\frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n.$$

m - best expert mistakes M algorithm mistakes.

$$\frac{1}{2^m} \leq \left(\frac{3}{4}\right)^M n.$$

Take log of both sides.

$$-m \leq -M \log(4/3) + \log n.$$

Solve for M.

$M \leq (m + \log n)/\log(4/3) \leq 2.4(m + \log n)$

Multiply by $1 - \varepsilon$ for incorrect experts...

Analysis: continued.

$$\frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n.$$

m - best expert mistakes M algorithm mistakes.

$$\frac{1}{2^m} \leq \left(\frac{3}{4}\right)^M n.$$

Take log of both sides.

$$-m \leq -M \log(4/3) + \log n.$$

Solve for M.

$M \leq (m + \log n)/\log(4/3) \leq 2.4(m + \log n)$

Multiply by $1 - \varepsilon$ for incorrect experts...

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Analysis: continued.

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Approaches a factor of two of best expert performance!

Best Analysis?

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Consider two experts: A,B

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Which is worse?

(A) A correct even days, B correct odd days

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Factor of (almost) two worse!

Randomization

Better approach?

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Use?

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Choose each with approximately the same probabilty.

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Proof Idea: $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$

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No factor of 2 loss!

Gains.

Why so negative?

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Summary: multiplicative weights.

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    Choose proportional to weights
        multiply weight by $(1-\varepsilon)^{\text{loss}}$.

Summary: multiplicative weights.

Framework: n experts, each loses different amount every day.

Perfect Expert: $\log n$ mistakes.

Imperfect Expert: best makes m mistakes.

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Multiplicative weights framework!

Applications next!

Two person zero sum games.

$m \times n$ payoff matrix A.

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$$p(x, y) = x^t A y$$

That is,

$$\sum_i x_i \left(\sum_j a_{i,j} y_j \right) = \sum_j \left(\sum_i x_i a_{i,j} \right) y_j.$$

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(No better column strategy, no better row strategy.)

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Example: Roshambo. Value of R?

Row goes first:
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Best Response

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Example: Roshambo. Value of R?

Row goes first:
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Duality.

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Proof: Better to go second. $\square$

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Strong Duality: There is an equilibrium point! and $R = C$!

Doesn't matter who plays first!

Proof of Equilibrium.

Later.

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Later. Well in just a minute.....

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Aproximate equilibrium ...

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Proof of approximate equilibrium.

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(A) Using geometry.

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(C) Using multiplicative weights.

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How?

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(D) By the skin of my teeth.

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Not hard.

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Not hard. Even easy.

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Not hard. Even easy. Still, head scratching happens.

Games and experts

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Choose column of A that maximizes row's expected loss.

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Let $y^* = \frac{1}{T}\sum_t y_t$ and $x^* = \text{argmin}_{x_t} x_t A y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

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Best expert: L^*- best row against all the columns played.

Approximate Equilibrium!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $y^* = \frac{1}{T}\sum_t y_t$ and $x^* = \text{argmin}_{x_t} x_t A y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* A y$.
Loss on day t, $x_t A y_t \geq C(x^*)$ by the choice of x^* .
Thus, algorithm loss, L, is $\geq T \times C(x^*)$.

Best expert: L^*- best row against all the columns played.

best row against $\sum_t A y_t$

Approximate Equilibrium!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $y^* = \frac{1}{T}\sum_t y_t$ and $x^* = \text{argmin}_{x_t} x_t A y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* A y$.
Loss on day t, $x_t A y_t \geq C(x^*)$ by the choice of x^* .
Thus, algorithm loss, L, is $\geq T \times C(x^*)$.

Best expert: L^*- best row against all the columns played.

best row against $\sum_t A y_t$ and $T \times y^* = \sum_t y_t$

Approximate Equilibrium!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $y^* = \frac{1}{T}\sum_t y_t$ and $x^* = \text{argmin}_{x_t} x_t A y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* A y$.
Loss on day t, $x_t A y_t \geq C(x^*)$ by the choice of x^* .
Thus, algorithm loss, L, is $\geq T \times C(x^*)$.

Best expert: L^*- best row against all the columns played.

best row against $\sum_t Ay_t$ and $T \times y^* = \sum_t y_t$
$\rightarrow$ best row against $T \times Ay^*$.

Approximate Equilibrium!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $y^* = \frac{1}{T}\sum_t y_t$ and $x^* = \text{argmin}_{x_t} x_t A y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* A y$.
Loss on day t, $x_t A y_t \geq C(x^*)$ by the choice of x^* .
Thus, algorithm loss, L, is $\geq T \times C(x^*)$.

Best expert: L^*- best row against all the columns played.

best row against $\sum_t A y_t$ and $T \times y^* = \sum_t y_t$
$\rightarrow$ best row against $T \times A y^*$.
$\rightarrow$ $L^* \leq T \times R(y^*)$.

Approximate Equilibrium!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $y^* = \frac{1}{T}\sum_t y_t$ and $x^* = \text{argmin}_{x_t} x_t A y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* A y$.
Loss on day t, $x_t A y_t \geq C(x^*)$ by the choice of x^* .
Thus, algorithm loss, L, is $\geq T \times C(x^*)$.

Best expert: L^*- best row against all the columns played.

best row against $\sum_t A y_t$ and $T \times y^* = \sum_t y_t$
$\rightarrow$ best row against $T \times A y^*$.
$\rightarrow L^* \leq T \times R(y^*)$.

Approximate Equilibrium!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $y^* = \frac{1}{T}\sum_t y_t$ and $x^* = \text{argmin}_{x_t} x_t A y_t$.

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Loss on day t, $x_t A y_t \geq C(x^*)$ by the choice of x^* .
Thus, algorithm loss, L, is $\geq T \times C(x^*)$.

Best expert: L^*- best row against all the columns played.

best row against $\sum_t A y_t$ and $T \times y^* = \sum_t y_t$
$\rightarrow$ best row against $T \times A y^*$.
$\rightarrow$ $L^* \leq T \times R(y^*)$.

Multiplicative Weights:

Approximate Equilibrium!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $y^* = \frac{1}{T}\sum_t y_t$ and $x^* = \text{argmin}_{x_t} x_t A y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* A y$.
Loss on day t, $x_t A y_t \geq C(x^*)$ by the choice of x^* .
Thus, algorithm loss, L, is $\geq T \times C(x^*)$.

Best expert: L^*- best row against all the columns played.

best row against $\sum_t A y_t$ and $T \times y^* = \sum_t y_t$
$\rightarrow$ best row against $T \times A y^*$.
$\rightarrow$ $L^* \leq T \times R(y^*)$.

Multiplicative Weights: $L \leq (1+\varepsilon)L^* + \frac{\ln n}{\varepsilon}$

Approximate Equilibrium!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $y^* = \frac{1}{T}\sum_t y_t$ and $x^* = \text{argmin}_{x_t} x_t A y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* A y$.
Loss on day t, $x_t A y_t \geq C(x^*)$ by the choice of x^* .
Thus, algorithm loss, L, is $\geq T \times C(x^*)$.

Best expert: L^*- best row against all the columns played.

best row against $\sum_t A y_t$ and $T \times y^* = \sum_t y_t$
$\rightarrow$ best row against $T \times A y^*$.
$\rightarrow$ $L^* \leq T \times R(y^*)$.

Multiplicative Weights: $L \leq (1+\varepsilon)L^* + \frac{\ln n}{\varepsilon}$

$$T \times C(x^*) \leq (1+\varepsilon) T \times R(y^*) + \frac{\ln n}{\varepsilon}$$

Approximate Equilibrium!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $y^* = \frac{1}{T}\sum_t y_t$ and $x^* = \text{argmin}_{x_t} x_t A y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* A y$.
  Loss on day t, $x_t A y_t \geq C(x^*)$ by the choice of x^* .
  Thus, algorithm loss, L, is $\geq T \times C(x^*)$.

Best expert: L^*- best row against all the columns played.

  best row against $\sum_t A y_t$ and $T \times y^* = \sum_t y_t$
  $\rightarrow$ best row against $T \times A y^*$.
  $\rightarrow L^* \leq T \times R(y^*)$.

Multiplicative Weights: $L \leq (1+\varepsilon)L^* + \frac{\ln n}{\varepsilon}$

$$T \times C(x^*) \leq (1+\varepsilon) T \times R(y^*) + \frac{\ln n}{\varepsilon} \rightarrow C(x^*) \leq (1+\varepsilon) R(y^*) + \frac{\ln n}{\varepsilon T}$$

Approximate Equilibrium!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $y^* = \frac{1}{T}\sum_t y_t$ and $x^* = \text{argmin}_{x_t} x_t A y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* A y$.
Loss on day t, $x_t A y_t \geq C(x^*)$ by the choice of x^* .
Thus, algorithm loss, L, is $\geq T \times C(x^*)$.

Best expert: L^*- best row against all the columns played.

best row against $\sum_t A y_t$ and $T \times y^* = \sum_t y_t$
$\rightarrow$ best row against $T \times A y^*$.
$\rightarrow L^* \leq T \times R(y^*)$.

Multiplicative Weights: $L \leq (1+\varepsilon)L^* + \frac{\ln n}{\varepsilon}$

$T \times C(x^*) \leq (1+\varepsilon) T \times R(y^*) + \frac{\ln n}{\varepsilon} \rightarrow C(x^*) \leq (1+\varepsilon) R(y^*) + \frac{\ln n}{\varepsilon T}$
$\rightarrow C(x^*) - R(y^*) \leq \varepsilon R(y^*) + \frac{\ln n}{\varepsilon T}$.

Approximate Equilibrium!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $y^* = \frac{1}{T}\sum_t y_t$ and $x^* = \text{argmin}_{x_t} x_t A y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* A y$.
Loss on day t, $x_t A y_t \geq C(x^*)$ by the choice of x^* .
Thus, algorithm loss, L, is $\geq T \times C(x^*)$.

Best expert: L^*- best row against all the columns played.

best row against $\sum_t A y_t$ and $T \times y^* = \sum_t y_t$
$\rightarrow$ best row against $T \times A y^*$.
$\rightarrow L^* \leq T \times R(y^*)$.

Multiplicative Weights: $L \leq (1+\varepsilon)L^* + \frac{\ln n}{\varepsilon}$

$T \times C(x^*) \leq (1+\varepsilon)T \times R(y^*) + \frac{\ln n}{\varepsilon} \rightarrow C(x^*) \leq (1+\varepsilon)R(y^*) + \frac{\ln n}{\varepsilon T}$
$\rightarrow C(x^*) - R(y^*) \leq \varepsilon R(y^*) + \frac{\ln n}{\varepsilon T}$.

$T = \frac{\ln n}{\varepsilon^2}$, $R(y^*) \leq 1$

Approximate Equilibrium!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $y^* = \frac{1}{T}\sum_t y_t$ and $x^* = \text{argmin}_{x_t} x_t A y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* A y$.
Loss on day t, $x_t A y_t \geq C(x^*)$ by the choice of x^* .
Thus, algorithm loss, L, is $\geq T \times C(x^*)$.

Best expert: L^*- best row against all the columns played.

best row against $\sum_t A y_t$ and $T \times y^* = \sum_t y_t$
$\rightarrow$ best row against $T \times A y^*$.
$\rightarrow L^* \leq T \times R(y^*)$.

Multiplicative Weights: $L \leq (1+\varepsilon)L^* + \frac{\ln n}{\varepsilon}$

$T \times C(x^*) \leq (1+\varepsilon)T \times R(y^*) + \frac{\ln n}{\varepsilon} \rightarrow C(x^*) \leq (1+\varepsilon)R(y^*) + \frac{\ln n}{\varepsilon T}$
$\rightarrow C(x^*) - R(y^*) \leq \varepsilon R(y^*) + \frac{\ln n}{\varepsilon T}$.

$T = \frac{\ln n}{\varepsilon^2}$, $R(y^*) \leq 1$
$\rightarrow C(x^*) - R(y^*) \leq 2\varepsilon$.

Approximate Equilibrium: slightly different!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Approximate Equilibrium: slightly different!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $x^* = \frac{1}{T}\sum_t x_t$

Approximate Equilibrium: slightly different!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $x^* = \frac{1}{T}\sum_t x_t$ and $y^* = \frac{1}{T}\sum_t y_t$.

Approximate Equilibrium: slightly different!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $x^* = \frac{1}{T}\sum_t x_t$ and $y^* = \frac{1}{T}\sum_t y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Approximate Equilibrium: slightly different!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $x^* = \frac{1}{T}\sum_t x_t$ and $y^* = \frac{1}{T}\sum_t y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* Ay$.

Approximate Equilibrium: slightly different!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $x^* = \frac{1}{T}\sum_t x_t$ and $y^* = \frac{1}{T}\sum_t y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* Ay$.
Let y_r be best response to $C(x^*)$.

Approximate Equilibrium: slightly different!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $x^* = \frac{1}{T}\sum_t x_t$ and $y^* = \frac{1}{T}\sum_t y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* Ay$.
Let y_r be best response to $C(x^*)$.
Day t, $x_t A y_t \geq x_t A y_r$ – y_t is best response to x_t.

Approximate Equilibrium: slightly different!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $x^* = \frac{1}{T}\sum_t x_t$ and $y^* = \frac{1}{T}\sum_t y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* Ay$.

Let y_r be best response to $C(x^*)$.

Day t, $x_t Ay_t \geq x_t Ay_r$ – y_t is best response to x_t.

Algorithm loss: $\sum_t x_t Ay_t \geq \sum_t x_t Ay_r$

Approximate Equilibrium: slightly different!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $x^* = \frac{1}{T}\sum_t x_t$ and $y^* = \frac{1}{T}\sum_t y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* A y$.
Let y_r be best response to $C(x^*)$.
Day t, $x_t A y_t \geq x_t A y_r$ – y_t is best response to x_t.
Algorithm loss: $\sum_t x_t A y_t \geq \sum_t x_t A y_r$
$L \geq T \times C(x^*)$.

Approximate Equilibrium: slightly different!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $x^* = \frac{1}{T}\sum_t x_t$ and $y^* = \frac{1}{T}\sum_t y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* Ay$.
Let y_r be best response to $C(x^*)$.
Day t, $x_t A y_t \geq x_t A y_r$ – y_t is best response to x_t.
Algorithm loss: $\sum_t x_t A y_t \geq \sum_t x_t A y_r$
$L \geq T \times C(x^*)$.

Best expert: L^*- best row against all the columns played.

Approximate Equilibrium: slightly different!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $x^* = \frac{1}{T}\sum_t x_t$ and $y^* = \frac{1}{T}\sum_t y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* Ay$.
  Let y_r be best response to $C(x^*)$.
  Day t, $x_t Ay_t \geq x_t Ay_r$ – y_t is best response to x_t.
  Algorithm loss: $\sum_t x_t Ay_t \geq \sum_t x_t Ay_r$
  $L \geq T \times C(x^*)$.

Best expert: L^*- best row against all the columns played.

  best row against $\sum_t Ay_t$

Approximate Equilibrium: slightly different!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $x^* = \frac{1}{T}\sum_t x_t$ and $y^* = \frac{1}{T}\sum_t y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* Ay$.
Let y_r be best response to $C(x^*)$.
Day t, $x_t Ay_t \geq x_t Ay_r$ – y_t is best response to x_t.
Algorithm loss: $\sum_t x_t Ay_t \geq \sum_t x_t Ay_r$
$L \geq T \times C(x^*)$.

Best expert: L^*- best row against all the columns played.

best row against $\sum_t Ay_t$ and $Ty^* = \sum_t y_t$

Approximate Equilibrium: slightly different!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $x^* = \frac{1}{T}\sum_t x_t$ and $y^* = \frac{1}{T}\sum_t y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* A y$.
Let y_r be best response to $C(x^*)$.
Day t, $x_t A y_t \geq x_t A y_r$ – y_t is best response to x_t.
Algorithm loss: $\sum_t x_t A y_t \geq \sum_t x_t A y_r$
$L \geq T \times C(x^*)$.

Best expert: L^*- best row against all the columns played.

best row against $\sum_t A y_t$ and $T y^* = \sum_t y_t$
$\rightarrow$ best row against $T A y^*$.

Approximate Equilibrium: slightly different!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $x^* = \frac{1}{T}\sum_t x_t$ and $y^* = \frac{1}{T}\sum_t y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* Ay$.
Let y_r be best response to $C(x^*)$.
Day t, $x_t Ay_t \geq x_t Ay_r$ – y_t is best response to x_t.
Algorithm loss: $\sum_t x_t Ay_t \geq \sum_t x_t Ay_r$
$L \geq T \times C(x^*)$.

Best expert: L^*- best row against all the columns played.

best row against $\sum_t Ay_t$ and $Ty^* = \sum_t y_t$
$\rightarrow$ best row against TAy^*.
$\rightarrow L^* \leq T \times R(y^*)$.

Approximate Equilibrium: slightly different!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $x^* = \frac{1}{T}\sum_t x_t$ and $y^* = \frac{1}{T}\sum_t y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* Ay$.
Let y_r be best response to $C(x^*)$.
Day t, $x_t A y_t \geq x_t A y_r$ – y_t is best response to x_t.
Algorithm loss: $\sum_t x_t A y_t \geq \sum_t x_t A y_r$
$L \geq T \times C(x^*)$.

Best expert: L^*- best row against all the columns played.

best row against $\sum_t A y_t$ and $T y^* = \sum_t y_t$
$\rightarrow$ best row against TAy^*.
$\rightarrow$ $L^* \leq T \times R(y^*)$.

Approximate Equilibrium: slightly different!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $x^* = \frac{1}{T}\sum_t x_t$ and $y^* = \frac{1}{T}\sum_t y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* Ay$.
Let y_r be best response to $C(x^*)$.
Day t, $x_t Ay_t \geq x_t Ay_r$ – y_t is best response to x_t.
Algorithm loss: $\sum_t x_t Ay_t \geq \sum_t x_t Ay_r$
$L \geq T \times C(x^*)$.

Best expert: L^*- best row against all the columns played.

best row against $\sum_t Ay_t$ and $Ty^* = \sum_t y_t$
$\rightarrow$ best row against TAy^*.
$\rightarrow$ $L^* \leq T \times R(y^*)$.

Multiplicative Weights:

Approximate Equilibrium: slightly different!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $x^* = \frac{1}{T}\sum_t x_t$ and $y^* = \frac{1}{T}\sum_t y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* A y$.
  Let y_r be best response to $C(x^*)$.
  Day t, $x_t A y_t \geq x_t A y_r$ – y_t is best response to x_t.
  Algorithm loss: $\sum_t x_t A y_t \geq \sum_t x_t A y_r$
  $L \geq T \times C(x^*)$.

Best expert: L^*- best row against all the columns played.

  best row against $\sum_t A y_t$ and $T y^* = \sum_t y_t$
  $\rightarrow$ best row against TAy^*.
  $\rightarrow$ $L^* \leq T \times R(y^*)$.

Multiplicative Weights: $L \leq (1+\varepsilon)L^* + \frac{\ln n}{\varepsilon}$

Approximate Equilibrium: slightly different!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $x^* = \frac{1}{T}\sum_t x_t$ and $y^* = \frac{1}{T}\sum_t y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* A y$.
Let y_r be best response to $C(x^*)$.
Day t, $x_t A y_t \geq x_t A y_r$ – y_t is best response to x_t.
Algorithm loss: $\sum_t x_t A y_t \geq \sum_t x_t A y_r$
$L \geq T \times C(x^*)$.

Best expert: L^*- best row against all the columns played.

best row against $\sum_t A y_t$ and $T y^* = \sum_t y_t$
$\rightarrow$ best row against $T A y^*$.
$\rightarrow$ $L^* \leq T \times R(y^*)$.

Multiplicative Weights: $L \leq (1+\varepsilon)L^* + \frac{\ln n}{\varepsilon}$

$TC(x^*) \leq (1+\varepsilon)TR(y^*) + \frac{\ln n}{\varepsilon}$

Approximate Equilibrium: slightly different!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $x^* = \frac{1}{T}\sum_t x_t$ and $y^* = \frac{1}{T}\sum_t y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* Ay$.
Let y_r be best response to $C(x^*)$.
Day t, $x_t A y_t \geq x_t A y_r$ – y_t is best response to x_t.
Algorithm loss: $\sum_t x_t A y_t \geq \sum_t x_t A y_r$
$L \geq T \times C(x^*)$.

Best expert: L^*- best row against all the columns played.

best row against $\sum_t Ay_t$ and $Ty^* = \sum_t y_t$
$\rightarrow$ best row against TAy^*.
$\rightarrow$ $L^* \leq T \times R(y^*)$.

Multiplicative Weights: $L \leq (1+\varepsilon)L^* + \frac{\ln n}{\varepsilon}$

$TC(x^*) \leq (1+\varepsilon)TR(y^*) + \frac{\ln n}{\varepsilon} \rightarrow C(x^*) \leq (1+\varepsilon)R(y^*) + \frac{\ln n}{\varepsilon T}$

Approximate Equilibrium: slightly different!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $x^* = \frac{1}{T}\sum_t x_t$ and $y^* = \frac{1}{T}\sum_t y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* Ay$.

Let y_r be best response to $C(x^*)$.

Day t, $x_t A y_t \geq x_t A y_r$ – y_t is best response to x_t.

Algorithm loss: $\sum_t x_t A y_t \geq \sum_t x_t A y_r$

$L \geq T \times C(x^*)$.

Best expert: L^*- best row against all the columns played.

best row against $\sum_t Ay_t$ and $Ty^* = \sum_t y_t$

$\rightarrow$ best row against TAy^*.

$\rightarrow L^* \leq T \times R(y^*)$.

Multiplicative Weights: $L \leq (1+\varepsilon)L^* + \frac{\ln n}{\varepsilon}$

$TC(x^*) \leq (1+\varepsilon)TR(y^*) + \frac{\ln n}{\varepsilon} \rightarrow C(x^*) \leq (1+\varepsilon)R(y^*) + \frac{\ln n}{\varepsilon T}$

$\rightarrow C(x^*) - R(y^*) \leq \varepsilon R(y^*) + \frac{\ln n}{\varepsilon T}$.

Approximate Equilibrium: slightly different!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $x^* = \frac{1}{T}\sum_t x_t$ and $y^* = \frac{1}{T}\sum_t y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* Ay$.
Let y_r be best response to $C(x^*)$.
Day t, $x_t A y_t \geq x_t A y_r$ – y_t is best response to x_t.
Algorithm loss: $\sum_t x_t A y_t \geq \sum_t x_t A y_r$
$L \geq T \times C(x^*)$.

Best expert: L^*- best row against all the columns played.

best row against $\sum_t Ay_t$ and $Ty^* = \sum_t y_t$
$\rightarrow$ best row against TAy^*.
$\rightarrow L^* \leq T \times R(y^*)$.

Multiplicative Weights: $L \leq (1+\varepsilon)L^* + \frac{\ln n}{\varepsilon}$

$TC(x^*) \leq (1+\varepsilon)TR(y^*) + \frac{\ln n}{\varepsilon} \rightarrow C(x^*) \leq (1+\varepsilon)R(y^*) + \frac{\ln n}{\varepsilon T}$
$\rightarrow C(x^*) - R(y^*) \leq \varepsilon R(y^*) + \frac{\ln n}{\varepsilon T}$.

$T = \frac{\ln n}{\varepsilon^2}$, $R(y^*) \leq 1$

Approximate Equilibrium: slightly different!

Experts: x_t is strategy on day t, y_t is best column against x_t.

Let $x^* = \frac{1}{T}\sum_t x_t$ and $y^* = \frac{1}{T}\sum_t y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A.

Column payoff: $C(x^*) = \max_y x^* A y$.
Let y_r be best response to $C(x^*)$.
Day t, $x_t A y_t \geq x_t A y_r$ – y_t is best response to x_t.
Algorithm loss: $\sum_t x_t A y_t \geq \sum_t x_t A y_r$
$L \geq T \times C(x^*)$.

Best expert: L^*- best row against all the columns played.

best row against $\sum_t A y_t$ and $T y^* = \sum_t y_t$
$\rightarrow$ best row against TAy^*.
$\rightarrow L^* \leq T \times R(y^*)$.

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$T = \frac{\ln n}{\varepsilon^2}$, $R(y^*) \leq 1 \rightarrow C(x^*) - R(y^*) \leq 2\varepsilon$.

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See you on Thursday.