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Fixing $\|v\|_2$, sparse vectors have small $\|v\|_1$ norm, dense ones have big $\|v\|_1$ norm.

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Linear Program! Exercise.

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Definition: A matrix A is RIP for δ_k if any k-sparse vector x
$$(1-\delta_k)\|x\|_2 \leq \|Ax\|_2 \leq (1+\delta_k)\|x\|_2.$$

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Theorem [Candes-Tao]: For any matrix RIP matrix A with $\delta_{2k} + \delta_{3k} < 1$, for $Ax = b$ with a k-sparse solution, then the solution to $\min \|y\|_1, Ay = b$, has $y = x$.

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Theorem: For a random ± 1, $d \times n$ matrix A, and for any x in $ker(A)$ some $d = \Omega(k \log \frac{n}{k})$ rows, has for any $T \subset [n]$ that

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Note: Parity check matrix of linear code!

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Consider A with property, $x \in ker(A)$, has $\|x\|_2 < \frac{1}{16\sqrt{k}}\|x\|_1$.

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Consider A with property, $x \in ker(A)$, has $\|x\|_2 < \frac{1}{16\sqrt{k}}\|x\|_1$.

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Again: $\sigma_k(x) = \min_{supp(z) \leq k} |x - z|_1$.

Lemma: For $v \in ker(A)$, $T \subset [n]$, $|T| \leq \frac{k}{16}$, $\|v_T\|_1 < \frac{\|v\|_1}{4}$.
Proof of Theorem: T be k largest in magnitude coordinates of x.

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$\|w_{\overline{T}}\|_1 = \|w\|_1 - \|w_T\|_1 \leq \|x\|_1$.

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$$\implies \|x - w\|_1 \leq 4\sigma(x).$$

□

Almost Euclidean Matrices Proof.

Theorem:

For a random ± 1, $d \times n$ matrix, and for any x in with Ax some $d = \Omega(k \log \frac{n}{k})$ rows, has for any $T \subset [n]$ that

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Random dot product is 0 with probability $1/2$. All r rows 0: $(1/2)^r$.

Union bound over $\binom{n}{k}$ vectors.

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"Few" vectors with most of mass in small set of coordinates.

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Credits

Moitra, MIT,6.854. Roughgarden, CS168, Stanford.

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See Jame Lee, TCS Blog, May 2008 for proof of Almost Euclidean Nature of random subspaces.

Possible Topics.

TODO: Long tailed distributions.

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