CS270: Lecture 2.

Admin:

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Check Piazza.

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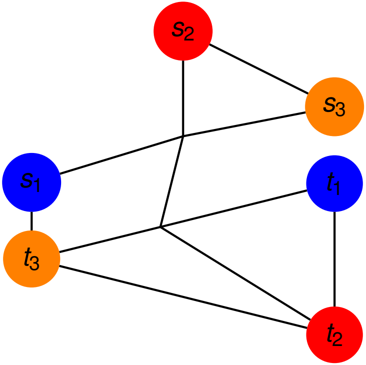
Today:

- Finish Path Routing.
- ????

Path Routing.

Given $G = (V, E)$, $(s_1, t_1), \ldots, (s_k, t_k)$, find a set of k paths connecting s_i and t_i and minimize max load on any edge.



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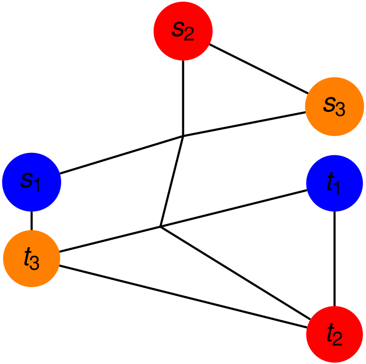

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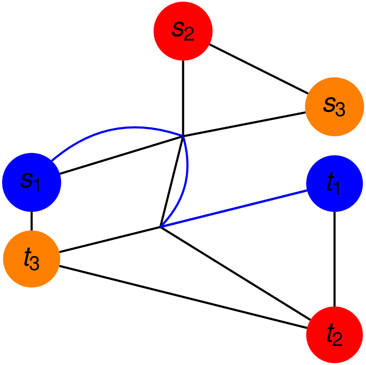

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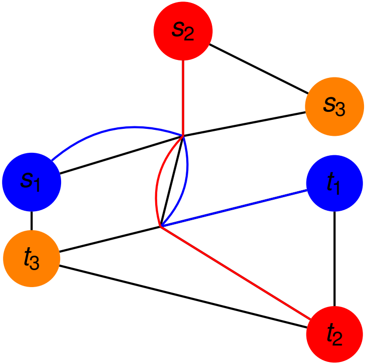

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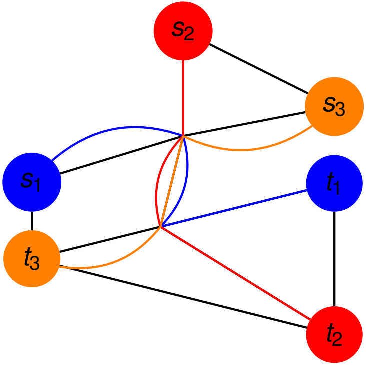

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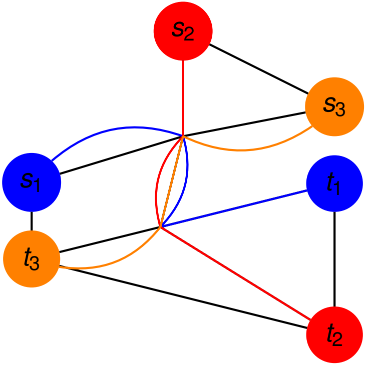

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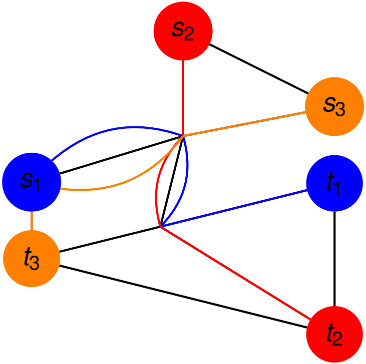

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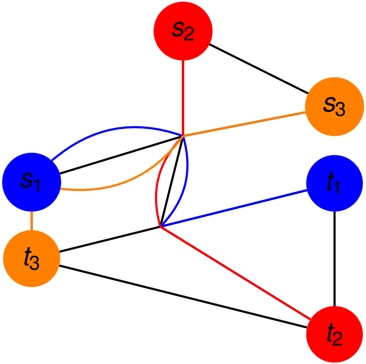

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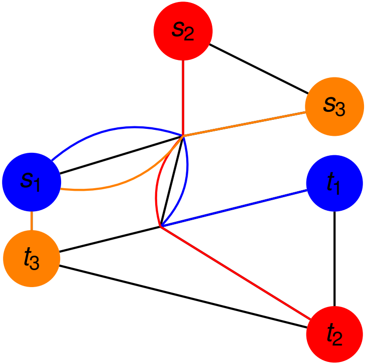

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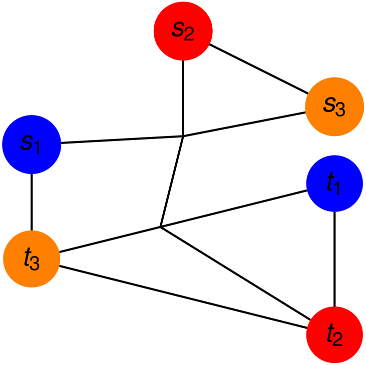

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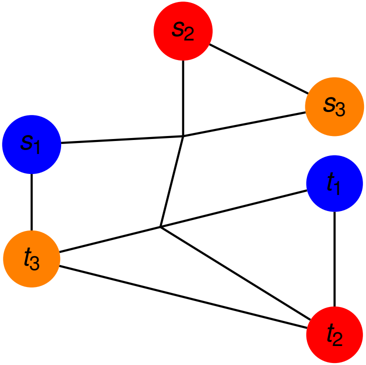

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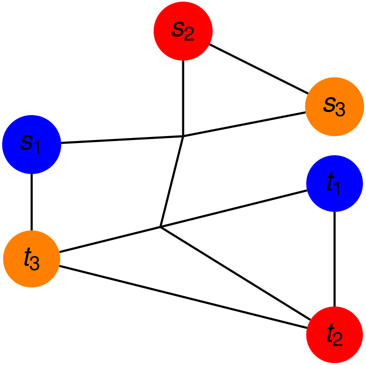

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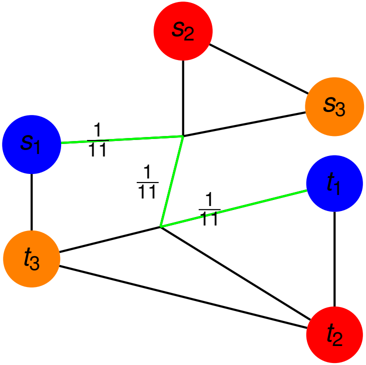

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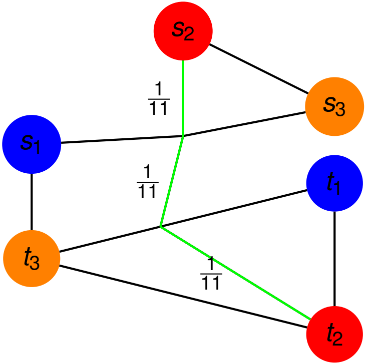

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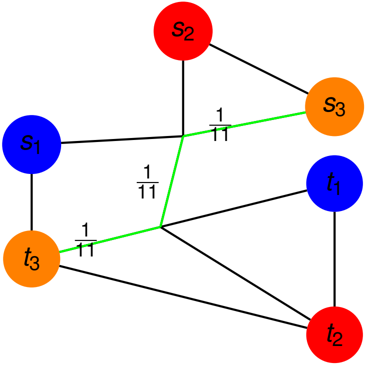

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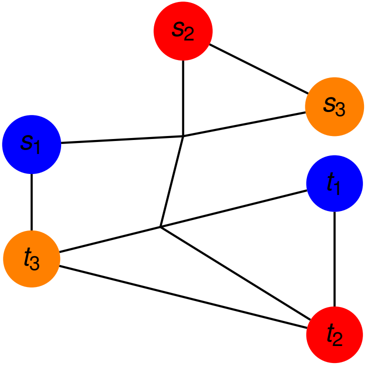

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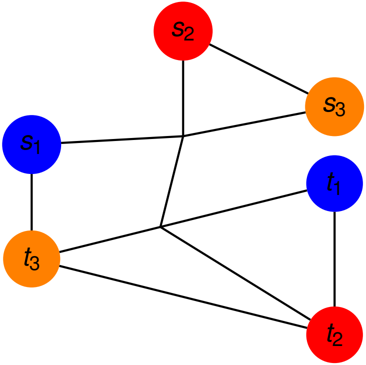

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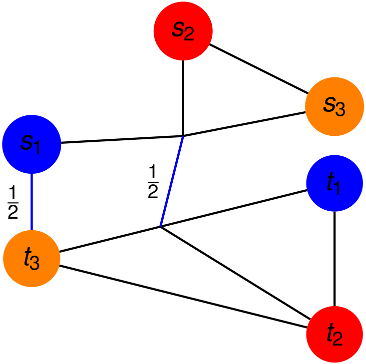

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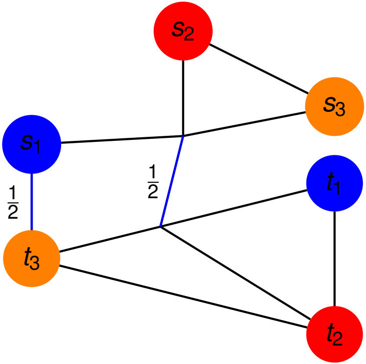

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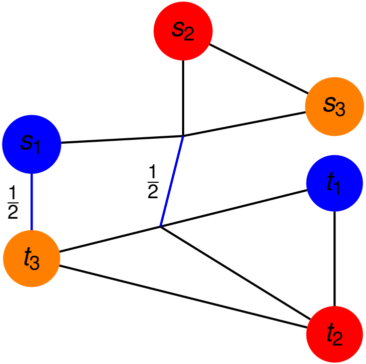

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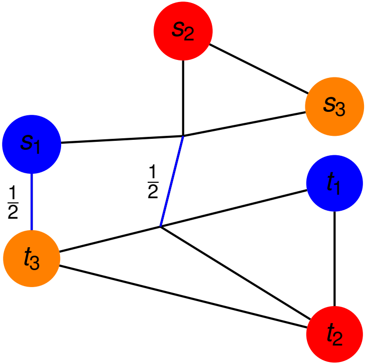

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Find routing that minimizes congestion (or maximum congestion.)

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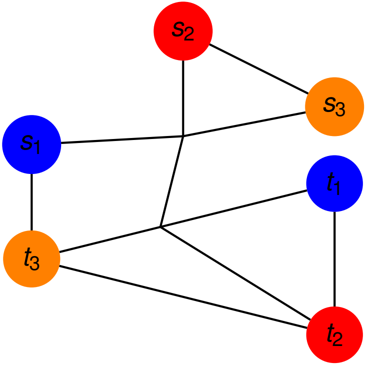

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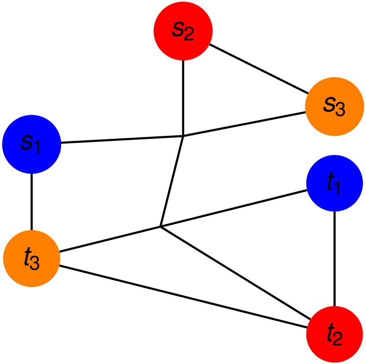

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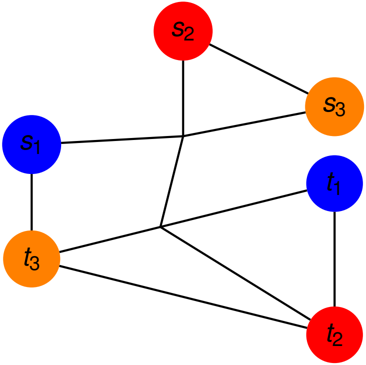

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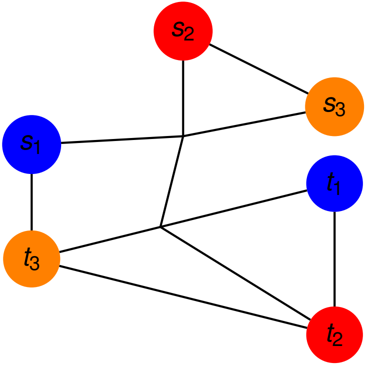

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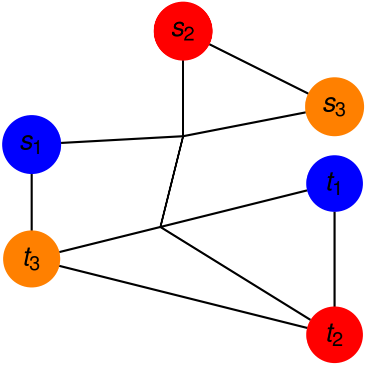

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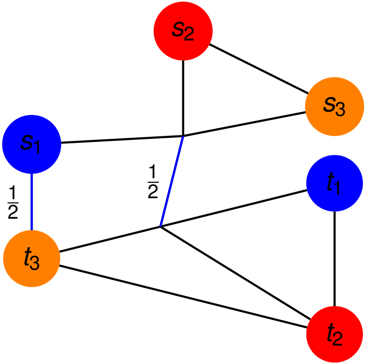

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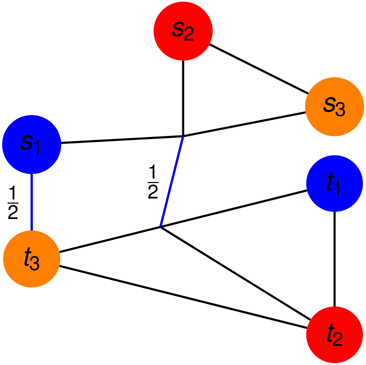

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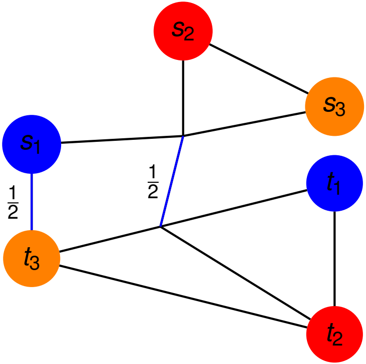

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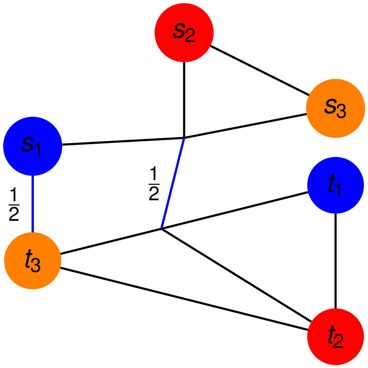

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Each path, p_i, in routing has length $d(p_i) \geq d(s_i, t_i)$.

$$\max_e c(e) \geq \sum_e c(e)d(e) = \sum_i d(p_i) \geq \sum_i d(s_i, t_i).$$

A toll solution is lower bound on any routing solution.

Toll is lower bound on Path Routing.

From before:
Max bigger than minimum weighted average:
$\max_e c(e) \geq \sum_e c(e)d(e)$
Total length is total congestion:
$\sum_e c(e)d(e) = \sum_i d(p_i)$

Each path, p_i, in routing has length $d(p_i) \geq d(s_i, t_i)$.

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A toll solution is lower bound on any routing solution.

Any routing solution is an upper bound on a toll solution.

Algorithm.

Assign tolls according to routing.

Algorithm.

Assign tolls according to routing.

How to route?

Algorithm.

Assign tolls according to routing.

How to route? Shortest paths!

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How to assign tolls? Higher tolls on congested edges.

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Toll: $d(e) =\propto 2^{c(e)}$.

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How to assign tolls? Higher tolls on congested edges.

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Equilibrium:

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"The routing is stable, the tolls are stable."

Algorithm.

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The shortest path routing has <u>has</u> $d(e) \propto 2^{c(e)}$.

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Routing: each path p_i in routing is a shortest path w.r.t $d(\cdot)$

Tolls: ...where $d(e)$ is defined w.r.t. to current routing.

Algorithm.

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How to route? Shortest paths!

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How to assign tolls? Higher tolls on congested edges.

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The shortest path routing has <u>has</u> $d(e) \propto 2^{c(e)}$.

“The routing is stable, the tolls are stable.”

Routing: each path p_i in routing is a shortest path w.r.t $d(\cdot)$

Tolls: ...where $d(e)$ is defined w.r.t. to current routing.
Subtlety here due to $\sum_e d(e) = 1$.

How good is equilibrium?

Path is routed along shortest path and $d(e) = \frac{2^{c(e)}}{\sum_{e'} 2^{c(e')}}$.

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$$\begin{aligned} c_{opt} &\geq \sum_i d(s_i, t_i) = \sum_e d(e) c(e) \\ &= \sum_e \frac{2^{c(e)}}{\sum_{e'} 2^{c(e')}} c(e) \end{aligned}$$

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&\geq \frac{\sum_{e:c(e)>c_t} 2^{c(e)} c(e)}{\sum_{e:c(e)>c_t} 2^{c(e)} + \sum_{e:c(e)\leq c_t} 2^{c(e)}}
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Let $c_t = c_{max} - 2\log m$.

How good is equilibrium?

Path is routed along shortest path and $d(e) = \frac{2^{c(e)}}{\sum_{e'} 2^{c(e')}}$.

For e with $c(e) \leq c_{max} - 2\log m$; $2^{c(e)} \leq 2^{c_{\max} - 2\log m} = \frac{2^{c_{max}}}{m^2}$.

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Or $c_{max} \le (1+\frac{1}{m}) c_{opt} + 2\log m$.

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\end{aligned}
$$

Or $c_{max} \le (1 + \frac{1}{m}) c_{opt} + 2\log m$.

(Almost) within additive term of $2\log m$ of optimal!

Getting to equilibrium.

Maybe no equilibrium!

Getting to equilibrium.

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Approximate equilibrium:

Getting to equilibrium.

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Each path is routed along a path with length within a factor of 3 of the shortest path and $d(e) \propto 2^{c(e)}$.

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Lose a factor of three at the beginning.

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Lose a factor of three at the beginning.
$c_{opt} \geq \sum_i d(s_i, t_i) \geq \frac{1}{3} \sum_e d(p_i)$

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We obtain $c_{max} = 3(1 + \frac{1}{m}) c_{opt} + 2 \log m$.

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This is worse!

Getting to equilibrium.

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We obtain $c_{max} = 3(1 + \frac{1}{m}) c_{opt} + 2 \log m$.

This is worse!
What do we gain?

An algorithm!

An algorithm!

Repeat: reroute any path that is off by a factor of 3.

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(Note: $d(e)$ recomputed every rerouting.)

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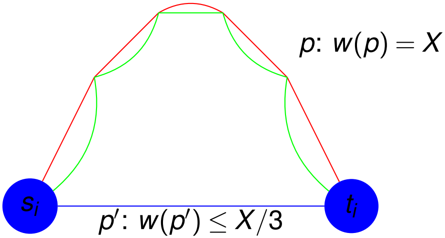

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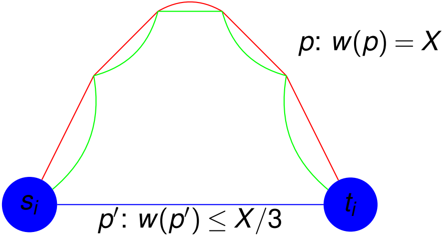

Potential function: $\sum_e w(e)$, $w(e) = 2^{c(e)}$

An algorithm!

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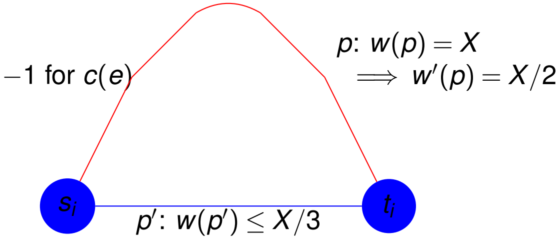

Potential function: $\sum_e w(e)$, $w(e) = 2^{c(e)}$
Moving path:

An algorithm!

Repeat: reroute any path that is off by a factor of 3.
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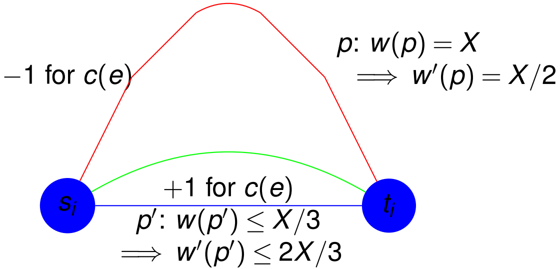

Potential function: $\sum_e w(e)$, $w(e) = 2^{c(e)}$
Moving path:
Divides $w(e)$ along long path (with $w(p)$ of X) by two.

An algorithm!

Repeat: reroute any path that is off by a factor of 3.
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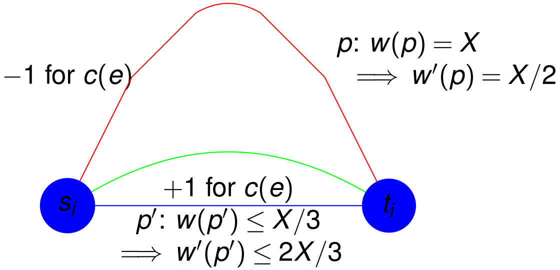
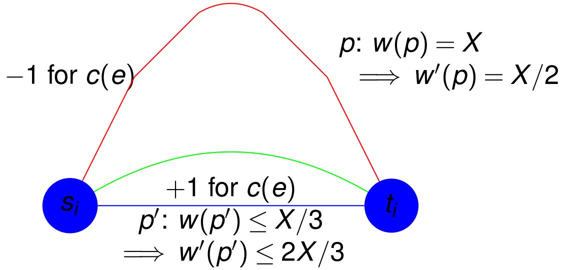


Potential function: $\sum_e w(e)$, $w(e) = 2^{c(e)}$
Moving path:
Divides $w(e)$ along long path (with $w(p)$ of X) by two.
Multiplies $w(e)$ along shorter ($w(p) \leq X/3$) path by two.

An algorithm!

Repeat: reroute any path that is off by a factor of 3.
(Note: $d(e)$ recomputed every rerouting.)



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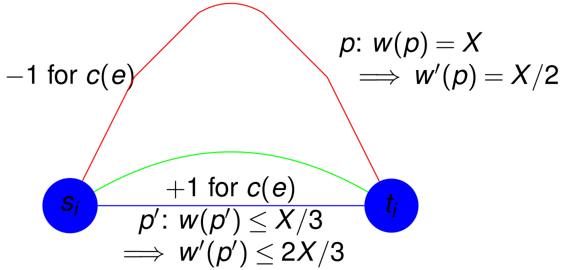
$$-\frac{X}{2} + \frac{X}{3} = -\frac{X}{6}.$$

An algorithm!

Repeat: reroute any path that is off by a factor of 3.
(Note: $d(e)$ recomputed every rerouting.)



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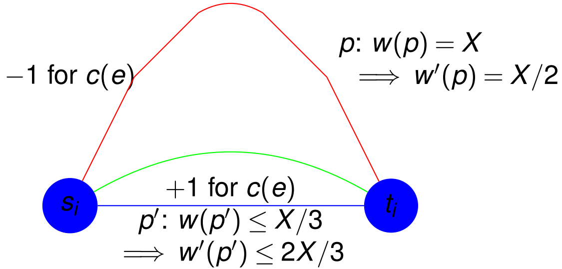
$$-\frac{X}{2} + \frac{X}{3} = -\frac{X}{6}.$$

Potential function decreases.

An algorithm!

Repeat: reroute any path that is off by a factor of 3.
(Note: $d(e)$ recomputed every rerouting.)



Potential function: $\sum_e w(e)$, $w(e) = 2^{c(e)}$
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Multiplies $w(e)$ along shorter ($w(p) \leq X/3$) path by two.

$$-\frac{X}{2} + \frac{X}{3} = -\frac{X}{6}.$$

Potential function decreases. $\implies$ termination and existence.

Tuning...

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Replace $d(e) = (1 + \varepsilon)^{c(e)}$.

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Replace factor of 3 by $(1+2\varepsilon)$

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$c_{max} \leq (1+2\varepsilon)c_{opt} + 2\log m/\varepsilon$.. (Roughly)

Tuning...

Replace $d(e) = (1+\varepsilon)^{c(e)}$.

Replace factor of 3 by $(1+2\varepsilon)$

$c_{max} \leq (1+2\varepsilon)c_{opt} + 2\log m/\varepsilon$.. (Roughly)

Fractional paths?

Revisit Equilibrium.

Solution Pair: $(\{p_i\}, d())$.

Toll Solution Value: $\sum_i d(s_i, t_i)$. Path Routing Value: $\max_e c(e)$.

Revisit Equilibrium.

Solution Pair: $(\{p_i\}, d())$.

Toll Solution Value: $\sum_i d(s_i, t_i)$. Path Routing Value: $\max_e c(e)$.

Toll player assigns toll on congested edges.

Revisit Equilibrium.

Solution Pair: $(\{p_i\}, d())$.

Toll Solution Value: $\sum_i d(s_i, t_i)$. Path Routing Value: $\max_e c(e)$.

Toll player assigns toll on congested edges.
Routing player routes on only cheap paths.

Revisit Equilibrium.

Solution Pair: $(\{p_i\}, d())$.

Toll Solution Value: $\sum_i d(s_i, t_i)$. Path Routing Value: $\max_e c(e)$.

Toll player assigns toll on only maximally congested edges.
Routing player routes on only cheapest paths.

$$\sum_i d(s_i, t_i) \quad =$$

Revisit Equilibrium.

Solution Pair: $(\{p_i\}, d())$.

Toll Solution Value: $\sum_i d(s_i, t_i)$. Path Routing Value: $\max_e c(e)$.

Toll player assigns toll on only maximally congested edges.
Routing player routes on only cheapest paths.

Routing R uses shortest paths.

$$\sum_i d(s_i, t_i) \quad = \quad \sum_i d(p_i)$$

Revisit Equilibrium.

Solution Pair: $(\{p_i\}, d())$.

Toll Solution Value: $\sum_i d(s_i, t_i)$. Path Routing Value: $\max_e c(e)$.

Toll player assigns toll on only maximally congested edges.

Routing player routes on only cheapest paths.

Routing R uses shortest paths. Summation Switch

$$\begin{aligned}\sum_i d(s_i, t_i) &= \sum_i d(p_i) \\ &= \sum_e c(e) d(e)\end{aligned}$$

Revisit Equilibrium.

Solution Pair: $(\{p_i\}, d())$.

Toll Solution Value: $\sum_i d(s_i, t_i)$. Path Routing Value: $\max_e c(e)$.

Toll player assigns toll on only maximally congested edges.
Routing player routes on only cheapest paths.

Routing R uses shortest paths. Summation Switch

$d(e) \geq 0$.

$$\begin{aligned}\sum_i d(s_i, t_i) &= \sum_i d(p_i) \\ &= \sum_e c(e)d(e) \\ &= \sum_{e:d(e)>0} c(e)d(e)\end{aligned}$$

Revisit Equilibrium.

Solution Pair: $(\{p_i\}, d())$.

Toll Solution Value: $\sum_i d(s_i,t_i)$. Path Routing Value: $\max_e c(e)$.

Toll player assigns toll on only maximally congested edges.
Routing player routes on only cheapest paths.

Routing R uses shortest paths. Summation Switch

$d(e) \geq 0$. Only Toll on max congestion.

$$\begin{aligned}
\sum_i d(s_i,t_i) &= \sum_i d(p_i) \\
&= \sum_e c(e)d(e) \\
&= \sum_{e:d(e)>0} c(e)d(e) \\
&= \sum_{e:d(e)>0} d(e)(\max_e c(e))
\end{aligned}$$

Revisit Equilibrium.

Solution Pair: $(\{p_i\}, d())$.

Toll Solution Value: $\sum_i d(s_i, t_i)$. Path Routing Value: $\max_e c(e)$.

Toll player assigns toll on only maximally congested edges.
Routing player routes on only cheapest paths.

Routing R uses shortest paths. Summation Switch

$d(e) \geq 0$. Only Toll on max congestion. $\sum_e d(e) = 1$

$$\begin{aligned}
\sum_i d(s_i, t_i) &= \sum_i d(p_i) \\
&= \sum_e c(e)d(e) \\
&= \sum_{e:d(e)>0} c(e)d(e) \\
&= \sum_{e:d(e)>0} d(e)(\max_e c(e)) \\
&= \max_e c(e)
\end{aligned}$$

Revisit Equilibrium.

Solution Pair: $(\{p_i\}, d())$.

Toll Solution Value: $\sum_i d(s_i, t_i)$. Path Routing Value: $\max_e c(e)$.

Toll player assigns toll on only maximally congested edges.
Routing player routes on only cheapest paths.

Routing R uses shortest paths. Summation Switch

$d(e) \geq 0$. Only Toll on max congestion. $\sum_e d(e) = 1$

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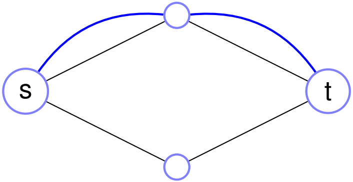
Why all the mess before? To get an algorithm!

Algorithm:exact?



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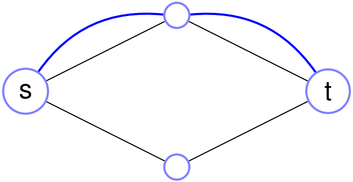

Not shortest when tolls on top.

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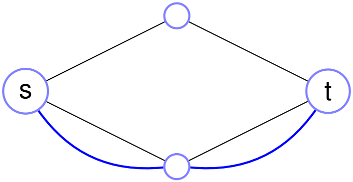

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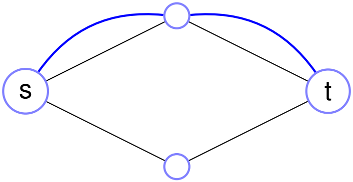

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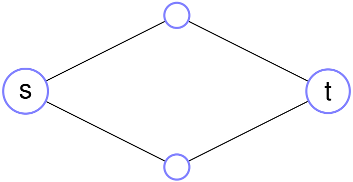
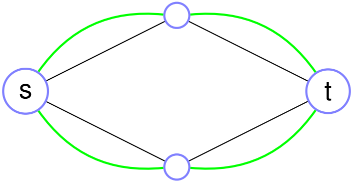


Not shortest when tolls on top.
Hmmm...
Uh oh?
Route half a unit on both!

Algorithm:exact?



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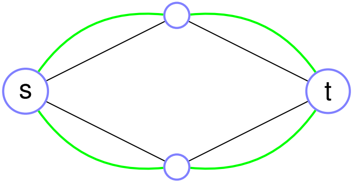
Hey! Fractional!

Algorithm:exact?



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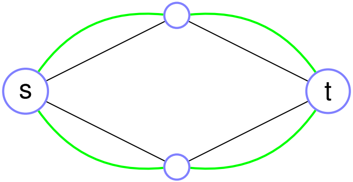

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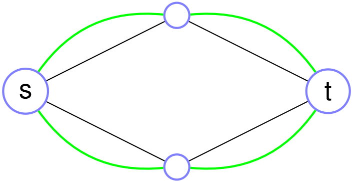

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Optimality: $(3)C_{\max} + 2\log m/2$.

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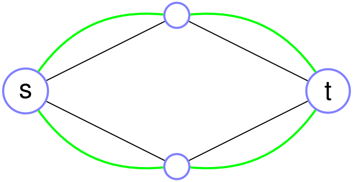

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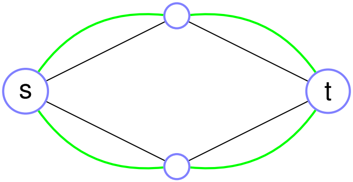

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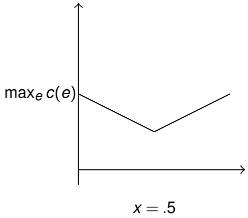
Additive factor shrinking!
The 3 can be made $(1+\varepsilon)$ using different base!

Geometrical view.



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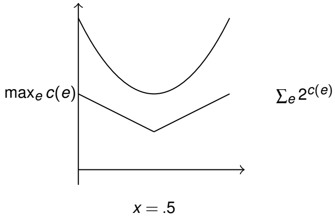
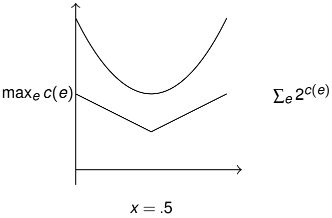


Smooth: use $\sum_e 2^{c(e)}$ as a proxy for $\max_e c(e)$.

Geometrical view.



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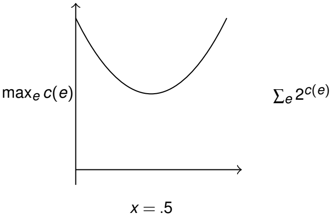
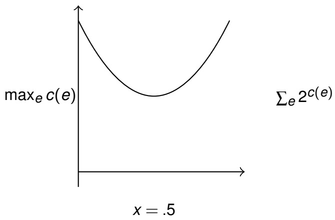


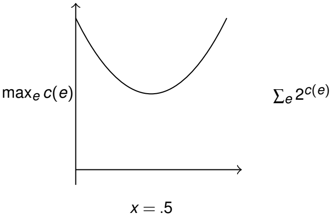
Smooth: use $\sum_e 2^{c(e)}$ as a proxy for $\max_e c(e)$.
Minimize new function.

Geometrical view.



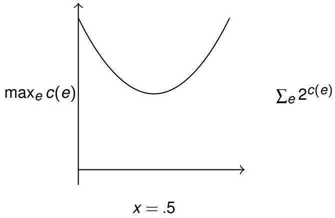
Smooth: use $\sum_e 2^{c(e)}$ as a proxy for $\max_e c(e)$.
Minimize new function.
Gradient descent.

Geometrical view.



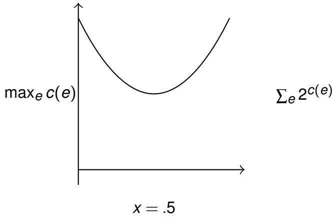
Smooth: use $\sum_e 2^{c(e)}$ as a proxy for $\max_e c(e)$.
Minimize new function.
Gradient descent.
Stepsize=1.

Geometrical view.



Smooth: use $\sum_e 2^{c(e)}$ as a proxy for $\max_e c(e)$.
Minimize new function.
Gradient descent.
Stepsize=1. Back and forth!

Geometrical view.



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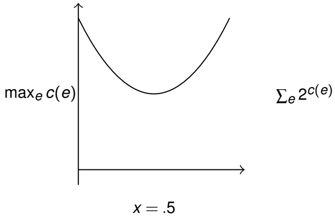
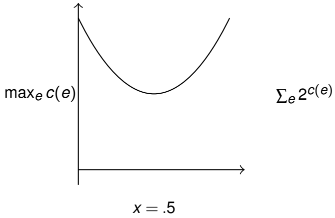


Smooth: use $\sum_e 2^{c(e)}$ as a proxy for $\max_e c(e)$.
Minimize new function.
Gradient descent.
Stepsize=1. Back and forth!
Stepsize=.5.

Geometrical view.



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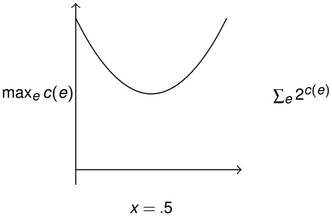
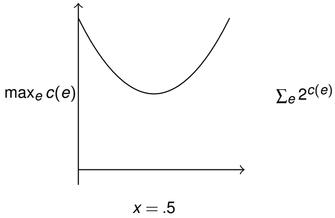
Stepsize=.5. Back and forth

Geometrical view.



Smooth: use $\sum_e 2^{c(e)}$ as a proxy for $\max_e c(e)$.
Minimize new function.
Gradient descent.
Stepsize=1. Back and forth!
Stepsize=.5. Back and forth ...but closer to minimum.

Geometrical view.



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Wrap up.

Dueling players:

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Toll player raises tolls on congested edges.

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Geometric View: Smooth. Gradient Descent. Stepsize.