CS270: Lecture 1.

1. Overview
2. Administration
3. Dueling Subroutines: Congestion/Tolls.

Algorithms.

Undergraduate.

Algorithms.

Undergraduate.

1. Classical.

Algorithms.

Undergraduate.

1. Classical.

2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.

Algorithms.

Undergraduate.

1. Classical.
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
3. Solutions:

Algorithms.

Undergraduate.

1. Classical.
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
3. Solutions: effective

Algorithms.

Undergraduate.

1. Classical.
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
3. Solutions: effective precise bounds!

Algorithms.

Undergraduate.

1. Classical.
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
3. Solutions: effective precise bounds!
4. Techniques:

Algorithms.

Undergraduate.

1. Classical.
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
3. Solutions: effective precise bounds!
4. Techniques: Greedy

Algorithms.

Undergraduate.

1. Classical.
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3. Solutions: effective precise bounds!
4. Techniques: Greedy Dyn. Programming

Algorithms.

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2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
3. Solutions: effective precise bounds!
4. Techniques: Greedy Dyn. Programming Linear Programming.

Algorithms.

Undergraduate.

1. Classical.
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
3. Solutions: effective precise bounds!
4. Techniques: Greedy Dyn. Programming Linear Programming.
5. Techniques tend to be Combinatorial.

Algorithms.

Undergraduate.
This class.

1. Classical.
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
3. Solutions: effective precise bounds!
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Algorithms.

Undergraduate.
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1. Classical.
   Flavor of the week?
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
3. Solutions: effective precise bounds!
4. Techniques: Greedy Dyn. Programming Linear Programming.
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Algorithms.

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1. Classical.
   Modern.
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3. Solutions: effective precise bounds!
4. Techniques: Greedy Dyn. Programming Linear Programming.
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Algorithms.

Undergraduate.
This class.

1. Classical.
   Modern.
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
   Vaguely stated problems!
3. Solutions: effective precise bounds!
4. Techniques: Greedy Dyn. Programming Linear Programming.
5. Techniques tend to be Combinatorial.

Algorithms.

Undergraduate.
This class.

1. Classical.
   Modern.
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
   Address problems; messy or not.
3. Solutions: effective precise bounds!
4. Techniques: Greedy Dyn. Programming Linear Programming.
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Algorithms.

Undergraduate.
This class.

1. Classical.
   Modern.
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
   Address problems; messy or not.
3. Solutions: effective precise bounds!
   Ineffective ..imprecise!
4. Techniques: Greedy Dyn. Programming Linear Programming.
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Algorithms.

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1. Classical.
   Modern.
2. Cleanly Stated Problems. Shortest Paths, max-flow, MST.
   Address problems; messy or not.
3. Solutions: effective precise bounds!
   Analysis sometimes based on modelling world.
4. Techniques: Greedy Dyn. Programming Linear Programming.
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4. Techniques: Greedy Dyn. Programming Linear Programming.
   Heuristic, in practice.
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   Address problems; messy or not.
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   Analysis sometimes based on modelling world.
4. Techniques: Greedy Dyn. Programming Linear Programming.
   Heuristic, in practice.
5. Techniques tend to be Combinatorial.
   Probabilistic, linear algebra methods, continuous.

Example Problem: clustering.

- Points: documents, dna, preferences.
- Graphs: applications to VLSI, parallel processing, image segmentation.

Image example.

Image Segmentation

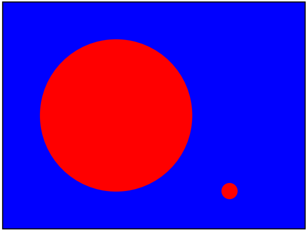
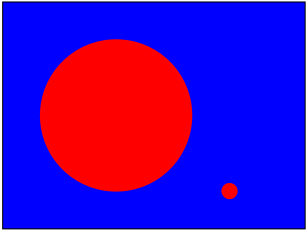
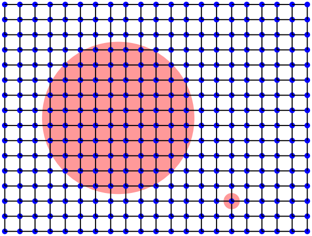


Image Segmentation





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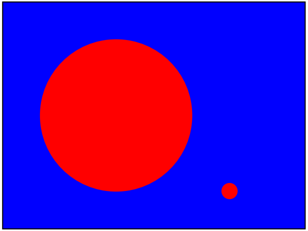
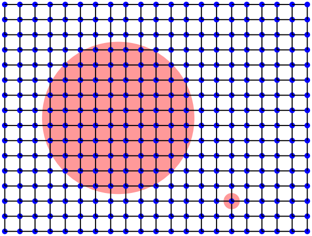
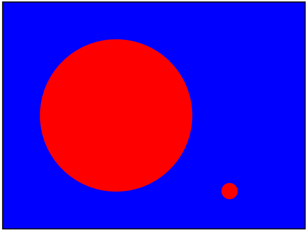
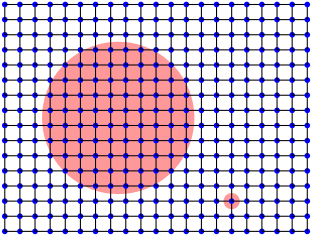




Which region?

Image Segmentation





Which region? Normalized Cut: Find S, which minimizes

$$\frac{w(S,\overline{S})}{w(S) \times w(\overline{S})}.$$

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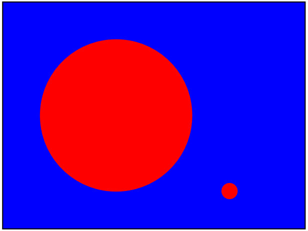
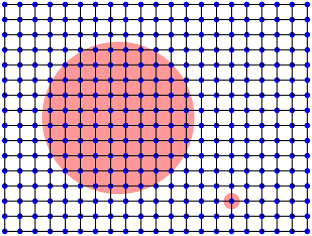




Which region? Normalized Cut: Find S, which minimizes

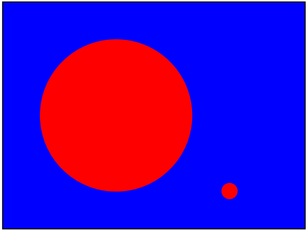
$$\frac{w(S,\overline{S})}{w(S)\times w(\overline{S})}.$$

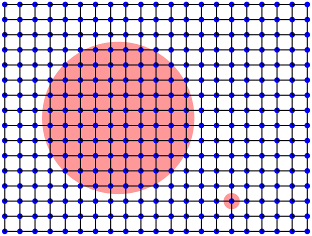
Ratio Cut: minimize

$$\frac{w(S,\overline{S})}{w(S)},$$

$w(S)$ no more than half the weight. (Minimize cost per unit weight that is removed.)

Image Segmentation





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$w(S)$ no more than half the weight. (Minimize cost per unit weight that is removed.)

Either is generally useful!

Example: recommendations.

Sarah Palin likes True Grit (the old one.)

Example: recommendations.

Sarah Palin likes True Grit (the old one.)
Sarah Palin doesn't like The Social Network.

Example: recommendations.

Sarah Palin likes True Grit (the old one.)
Sarah Palin doesn't like The Social Network.
Sarah Palin doesn't like Black Swan.

Example: recommendations.

Sarah Palin likes True Grit (the old one.)
Sarah Palin doesn't like The Social Network.
Sarah Palin doesn't like Black Swan.
Sarah Palin likes Sarah Palin on Discovery channel.

Example: recommendations.

Sarah Palin likes True Grit (the old one.)
Sarah Palin doesn't like The Social Network.
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Hillary Clinton doesn't like True Grit (the old one.)

Example: recommendations.

Sarah Palin likes True Grit (the old one.)
Sarah Palin doesn't like The Social Network.
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Hillary Clinton doesn't like True Grit (the old one.)
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Should you recommend the discovery channel to Hillary?

Example: recommendations.

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What about you?

Example: recommendations.

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Are you Hillary?

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What about you?

Are you Hillary? Are you Sarah?

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High dimensional data: dimension for each movie.

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More than three dimensions!

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Nearest neighbors.

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High dimensional data: dimension for each movie.
More than three dimensions!
Nearest neighbors. Principal Components methods.

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Nearest neighbors. Principal Components methods.
Topic Models.

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What about you?

Are you Hillary? Are you Sarah? A bit of both?

High dimensional data: dimension for each movie.
More than three dimensions!
Nearest neighbors. Principal Components methods.
Topic Models.
Reasoning about these methods.

Linear Systems.

Revolution!

Linear Systems.

Revolution!

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Revolution!

Linear Systems.

Revolution!

Linear Systems.

Revolution!

Linear Systems.

Revolution!

Physical Simulation.

Linear Systems.

Revolution!

Physical Simulation. Airflow.

Linear Systems.

Revolution!

Physical Simulation. Airflow.

Solve $Ax = b$.

Linear Systems.

Revolution!

Physical Simulation. Airflow.

Solve $Ax = b$.

How long?

Linear Systems.

Revolution!

Physical Simulation. Airflow.

Solve $Ax = b$.

How long?

$n \times n$ matrix A.

Linear Systems.

Revolution!

Physical Simulation. Airflow.

Solve $Ax = b$.

How long?

$n \times n$ matrix A.

Middle School:

Linear Systems.

Revolution!

Physical Simulation. Airflow.

Solve $Ax = b$.

How long?

$n \times n$ matrix A.

Middle School: substitution,

Linear Systems.

Revolution!

Physical Simulation. Airflow.

Solve $Ax = b$.

How long?

$n \times n$ matrix A.

Middle School: substitution, adding equations ...

Linear Systems.

Revolution!

Physical Simulation. Airflow.

Solve $Ax = b$.

How long?

$n \times n$ matrix A.

Middle School: substitution, adding equations ...
Time: $O(n^3)$.

Linear Systems.

Revolution!

Physical Simulation. Airflow.

Solve $Ax = b$.

How long?

$n \times n$ matrix A.

Middle School: substitution, adding equations ...
Time: $O(n^3)$.

Linear Systems.

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Physical Simulation. Airflow.

Solve $Ax = b$.

How long?

$n \times n$ matrix A.

Middle School: substitution, adding equations ...
Time: $O(n^3)$.

Now: $\tilde{O}(m)$.

Linear Systems.

Revolution!

Physical Simulation. Airflow.

Solve $Ax = b$.

How long?

$n \times n$ matrix A.

Middle School: substitution, adding equations ...
Time: $O(n^3)$.

Now: $\tilde{O}(m)$. Hmmm.

Linear Systems.

Revolution!

Physical Simulation. Airflow.

Solve $Ax = b$.

How long?

$n \times n$ matrix A.

Middle School: substitution, adding equations ...
Time: $O(n^3)$.

Now: $\tilde{O}(m)$. Hmmm. What's that tilde?

Linear Systems.

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Techniques:

Linear Systems.

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Techniques:
Relate graph theory to matrix properties.

Linear Systems.

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Now: $\tilde{O}(m)$. Hmmm. What's that tilde?

Techniques:
Relate graph theory to matrix properties.
Dense matrix (graph) to sparse matrix (graph).

Linear Systems.

Revolution!

Physical Simulation. Airflow.

Solve $Ax = b$.

How long?

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Middle School: substitution, adding equations ...
Time: $O(n^3)$.

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Techniques:
Relate graph theory to matrix properties.
Dense matrix (graph) to sparse matrix (graph).
Approximating distances by trees.

Linear Systems.

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Relate graph theory to matrix properties.
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Approximating distances by trees.
Electrical networks analysis.

Linear Systems.

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Physical Simulation. Airflow.

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Techniques:
Relate graph theory to matrix properties.
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Combinatorial Applications:

Linear Systems.

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Solve $Ax = b$.

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Techniques:
Relate graph theory to matrix properties.
Dense matrix (graph) to sparse matrix (graph).
Approximating distances by trees.
Electrical networks analysis.

Combinatorial Applications: Better Max Flow!

Other Algorithmic Techiniques

Sketching:

Large stream of data: $a_1, a_2, \ldots$

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Find digest.

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Graphs: Sparse graph.

Other Algorithmic Techiniques

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Large stream of data: $a_1, a_2, \ldots$

Find digest.

Graphs: Sparse graph.

Data: average, statistics.

Other Algorithmic Techiniques

Sketching:

Large stream of data: $a_1, a_2, \ldots$

Find digest.

Graphs: Sparse graph.

Data: average, statistics.

Points: center point, k-medians, .

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High Dimensional optimization.

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High Dimensional optimization.

Gradient Descent. Convexity.

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Semidefinite Programming.

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Dueling Subroutines. Duality.

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Lower bounder, upper bounder.

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Lower bounder, upper bounder.
Upper uses lower's evidence to find better solutions.

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Lower bounder, upper bounder.
Upper uses lower's evidence to find better solutions.
Lower uses upper's evidence to prove better lower bounds.

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CS270: Administration.

1. Staff:
   Satish Rao

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   Benjamin Weitz
2. Piazza. Log in! Pay attention to “bypass email preferences” especially.
3. Assessment.
   3.1 Homeworks (40%).

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   3.3 Project (35%)
       Groups of 2 or 3.

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   3.1 Homeworks (40%).
   Homework 1 out tonight/tomorrow.
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   3.3 Project (35%)
   Groups of 2 or 3.
   Connect research to class.

CS270: Administration.

1. Staff:
   Satish Rao
   Benjamin Weitz
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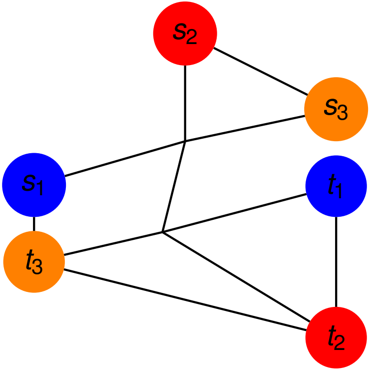
   Connect research to class.
   Or explore/digest a topic from class.
   3.4 No Discussion this week.

Path Routing.

Given $G = (V, E)$, $(s_1, t_1), \ldots, (s_k, t_k)$, find a set of k paths connecting s_i and t_i and minimize max load on any edge.



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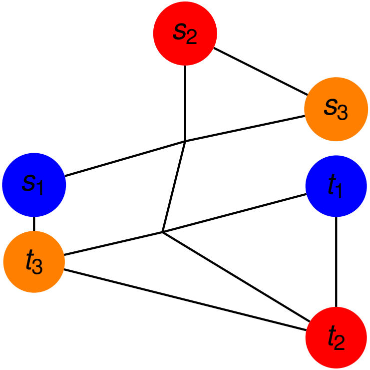

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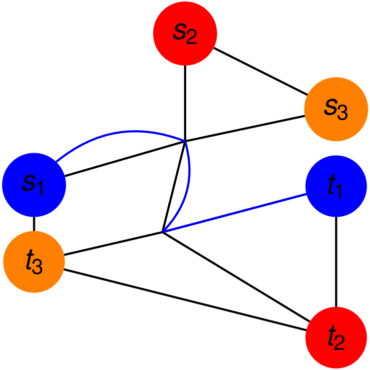

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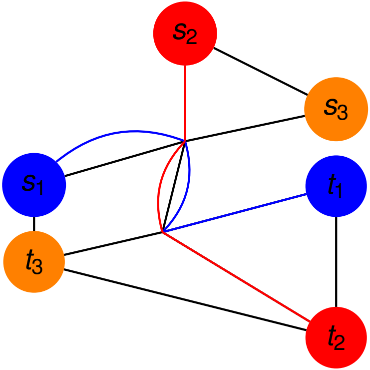

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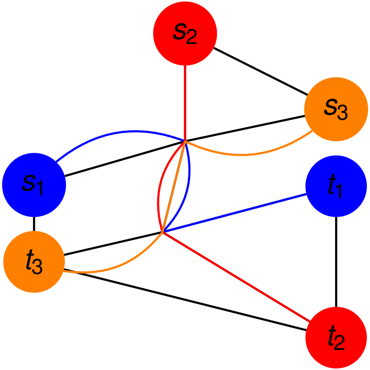

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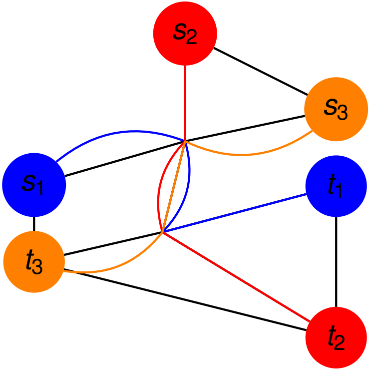

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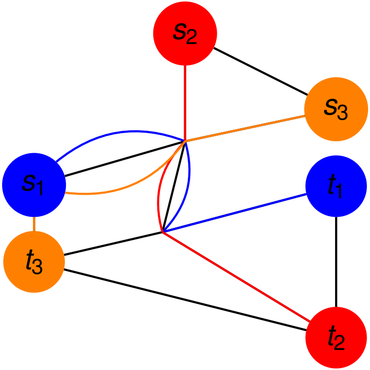

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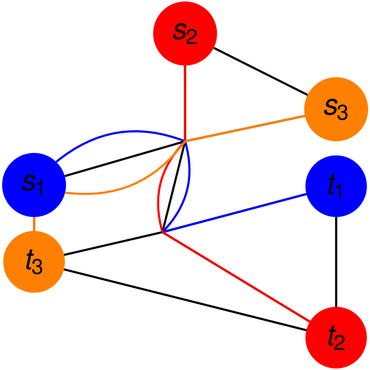

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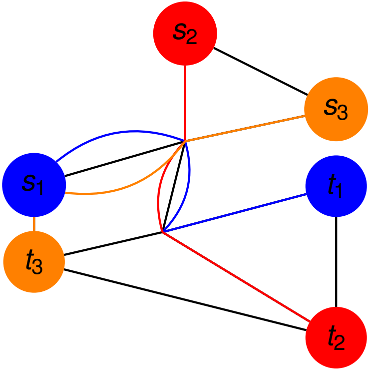

~~Value: 3~~

Value: 2

Terminology

Routing: Paths $p_1, p_2, \ldots, p_k$, p_i connects s_i and t_i.

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Congestion of edge, e: $c(e)$
number of paths in routing that contain e.

Congestion of routing: maximum congestion of any edge.

Find routing that minimizes congestion (or maximum congestion.)

Algorithms?

Route along any path.

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Feasible...

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Feasible...but is it as good as possible?

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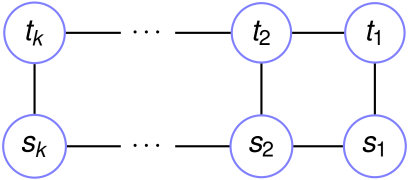

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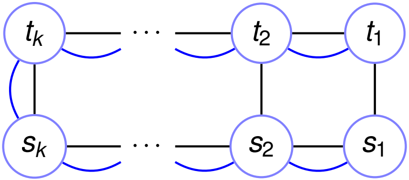

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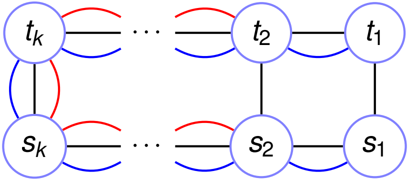

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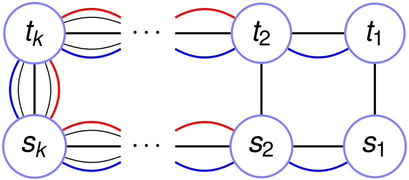

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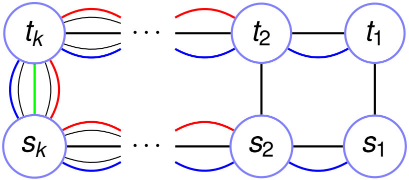

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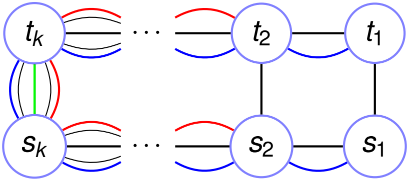

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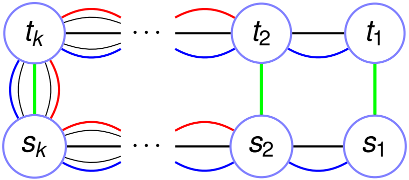

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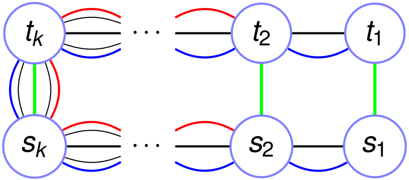

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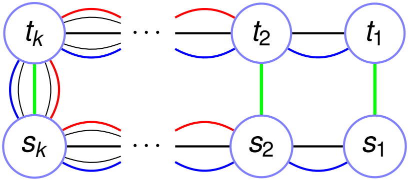

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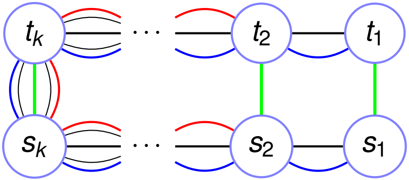

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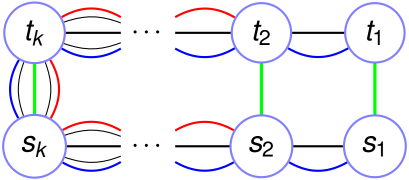

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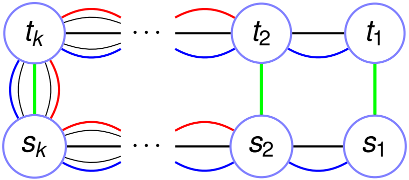

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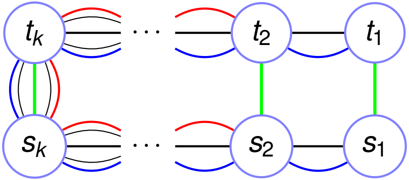

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Optimal use of “resources” ..or edges.

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Both count "uses"

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$\sum_i \ell(p_i) = \sum_i \sum_{e \in p_i} 1 = \sum_e \sum_{p_i \ni e} 1 = \sum_e c(e)$

Shortest Path Routing.

Minimizes $\sum_i \ell(p_i)$.

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Also minimizes total congestion: $\sum_e c(e)$

where $c(e)$ congestion of edge.

Why? Let $\ell(p_i)$ be the length of path p_i.

(A) $\sum_i \ell(p_i) = \sum_e c(e)$?
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Does it minimize maximum load?

One problem...

How far from optimal?

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Optimal?

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Optimal? Factor 2?

One problem...

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Optimal? Factor 2? Factor k?

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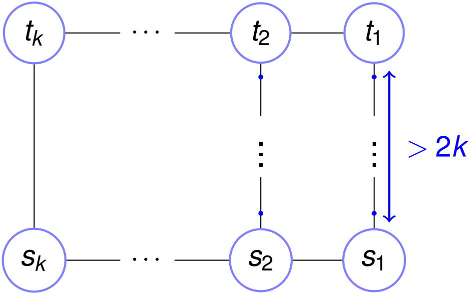

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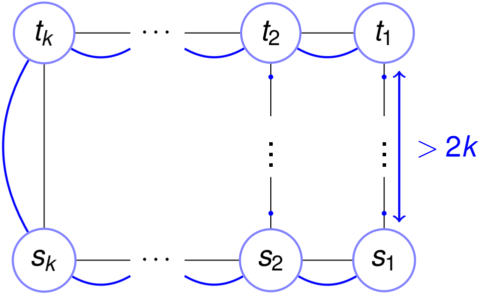

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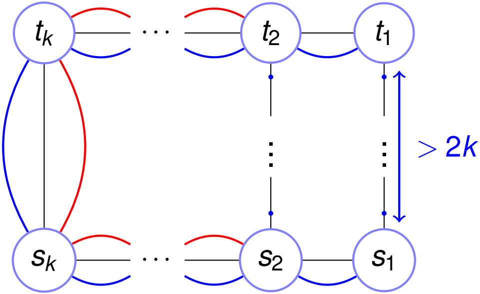

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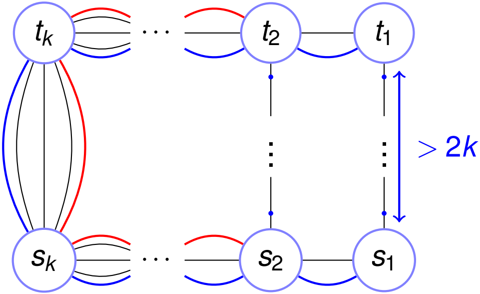

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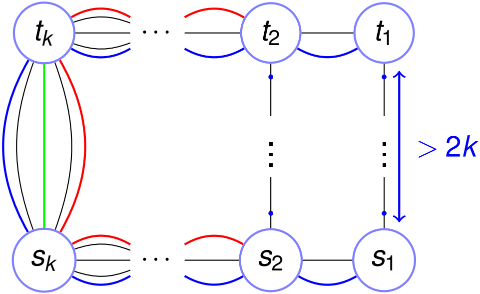

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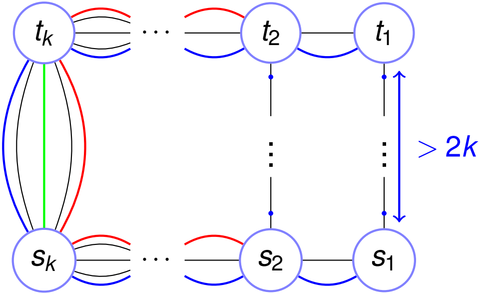

Value: k.

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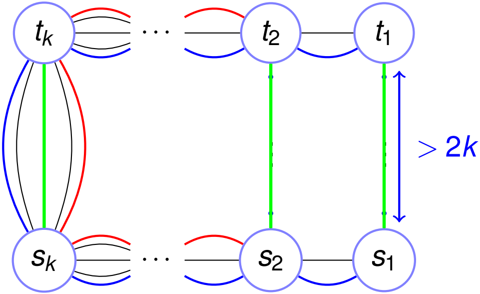

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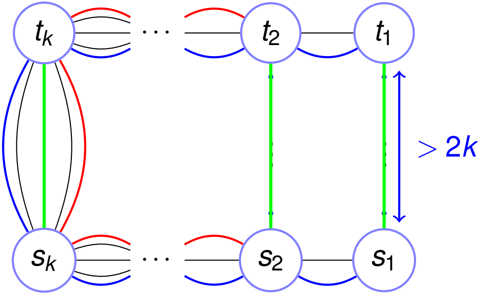

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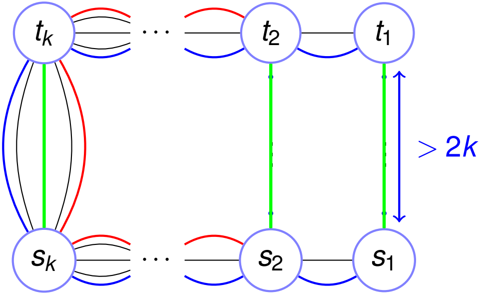

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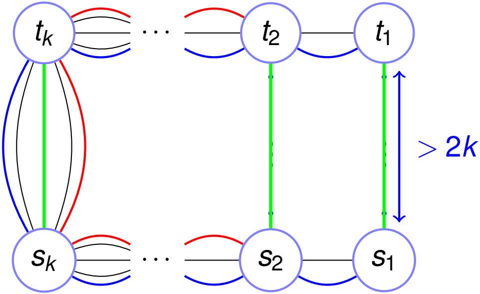

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Any suggestions?

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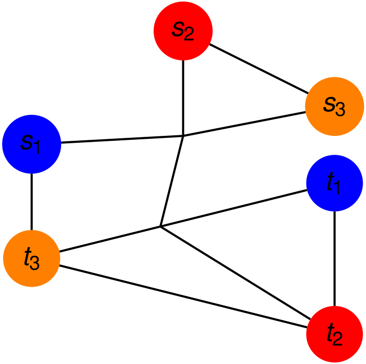

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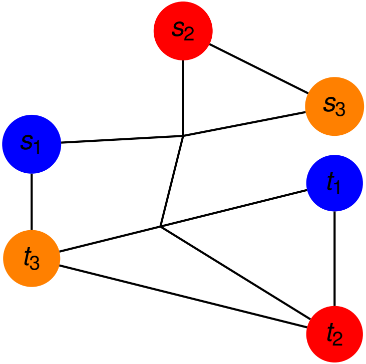

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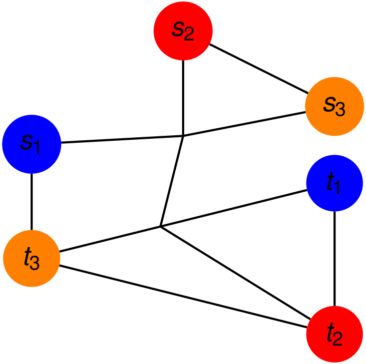

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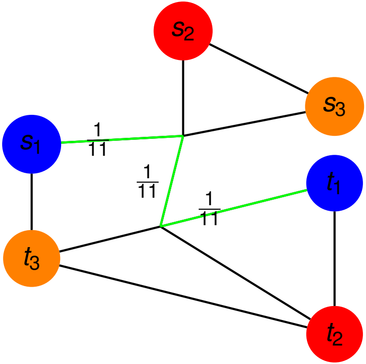

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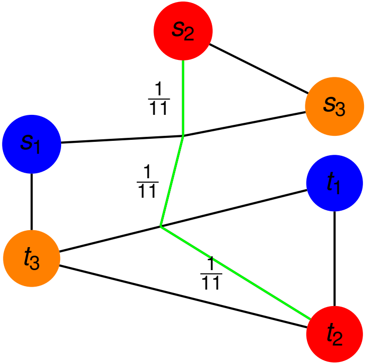

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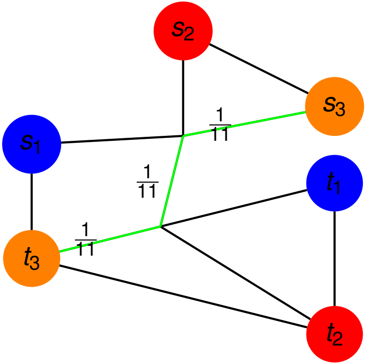

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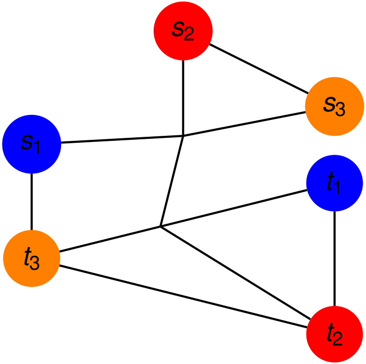

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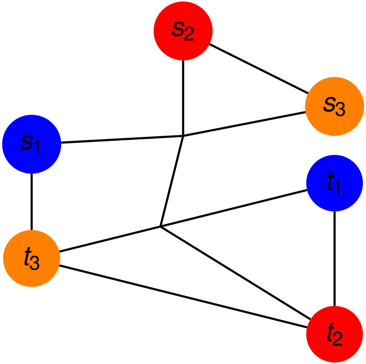

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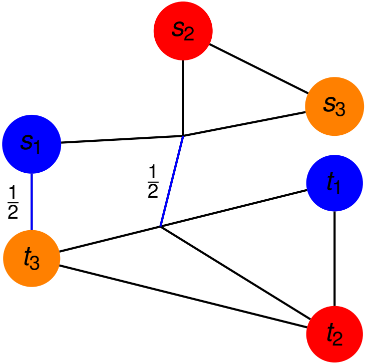

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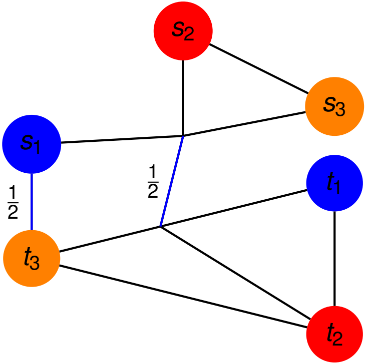

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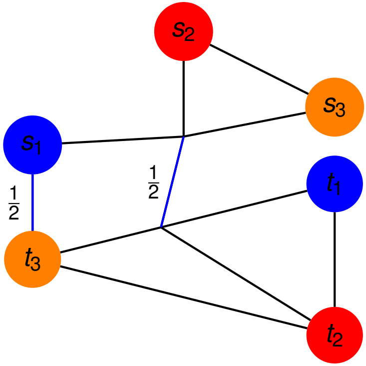

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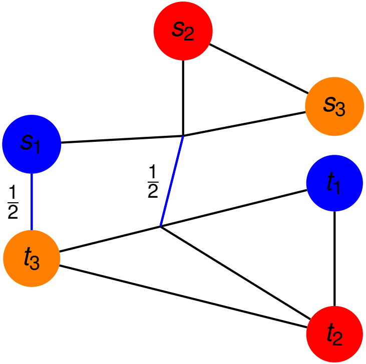

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Any toll solution value (weighted average congestion) is lower bound on path routing value (max congestion).

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$$\begin{aligned} \sum_i d(p_i) &= \sum_i \sum_{e \in p_i} d(e) \\ &= \sum_e \sum_{i : e \ni p_i} d(e) \end{aligned}$$

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Proving lower bound.

Routing solution: p_i connects (s_i, t_i) and has length $d(p_i)$.

$c(e)$ - congestion on edge e under routing.
Max $c(e)$?
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$\max_e c(e) \geq \sum_e d(e)c(e) = \sum_i d(p_i) \geq \sum_i d(s_i, t_i)$.
Routing solution cost $\geq$ Any toll solution cost.

Toll is lower bound.

From before:
Max bigger than minimum weighted average:
$\max_e c(e) \geq \sum_e c(e)d(e)$
Total length is total congestion: $\sum_e c(e)d(e) = \sum_i d(p_i)$

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A toll solution is lower bound on any routing solution.
Any routing solution is an upper bound on a toll solution.

Shall we continue?

Algorithm.

Assign tolls.

Algorithm.

Assign tolls.
How to route?

Algorithm.

Assign tolls.

How to route? Shortest paths!

Algorithm.

Assign tolls.
How to route? Shortest paths!
Assign routing.

Algorithm.

Assign tolls.
How to route? Shortest paths!
Assign routing.
How to assign tolls?

Algorithm.

Assign tolls.
How to route? Shortest paths!
Assign routing.
How to assign tolls? Higher tolls on congested edges.

Algorithm.

Assign tolls.
How to route? Shortest paths!
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Toll: $d(e) \propto 2^{c(e)}$.

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How to route? Shortest paths!
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Equilibrium:

Algorithm.

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Equilibrium:
The shortest path routing has $d(e) \propto 2^{c(e)}$.

Algorithm.

Assign tolls.
How to route? Shortest paths!
Assign routing.
How to assign tolls? Higher tolls on congested edges.
Toll: $d(e) \propto 2^{c(e)}$.

Equilibrium:
The shortest path routing has $d(e) \propto 2^{c(e)}$.

The routing does not change, the tolls do not change.

How good is equilibrium?

Path is routed along shortest path and $d(e) \propto 2^{c(e)}$.

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Path is routed along shortest path and $d(e) \propto 2^{c(e)}$.

$$c_{opt} \quad \geq \quad \sum_i d(s_i, t_i) = \sum_e d(e)c(e)$$

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Path is routed along shortest path and $d(e) \propto 2^{c(e)}$.

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&\geq \frac{\sum_{e:c(e)>c_t} 2^{c(e)} c(e)}{\sum_{e:c(e)>c_t} 2^{c(e)} + \sum_{e:c(e)\leq c_t} 2^{c(e)}}
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Path is routed along shortest path and $d(e) \propto 2^{c(e)}$.

For e with $c(e) \leq c_{max} - 2\log m$; $2^{c(e)} \leq 2^{c_{\max} - 2\log m} = \frac{2^{c_{max}}}{m^2}$.

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Or $c_{max} \leq (1 + \frac{1}{m}) c_{opt} + 2\log m$.

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Let $c_t = c_{max} - 2\log m$.

Or $c_{max} \leq (1+\frac{1}{m}) c_{opt} + 2\log m$.
(Almost) within $2\log m$ of optimal!

The end: sort of.

Getting to equilibrium.

Maybe no equilibrium!

Getting to equilibrium.

Maybe no equilibrium!

Approximate equilibrium:

Getting to equilibrium.

Maybe no equilibrium!

Approximate equilibrium:

Each path is routed along a path with length
within a factor of 3 of the shortest path and $d(e) \propto 2^{c(e)}$.

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Lose a factor of three at the beginning.

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Lose a factor of three at the beginning.
$c_{opt} \geq \sum_i d(s_i, t_i) \geq \frac{1}{3} \sum_e d(p_i)$.

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We obtain $c_{max} = 3(1 + \frac{1}{m})c_{opt} + 2 \log m$.

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This is worse!
What do we gain?

An algorithm!

Algorithm: reroute paths that are off by a factor of three.
(Note: $d(e)$ recomputed every rerouting.)

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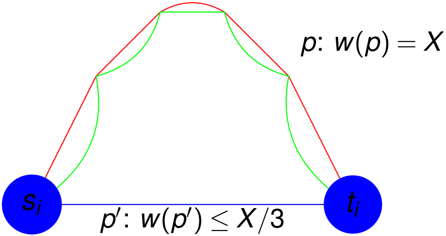

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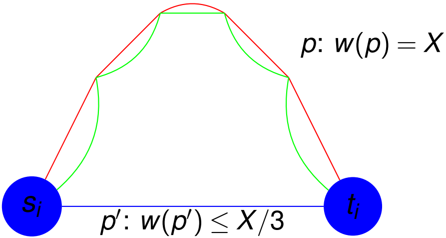

Potential function: $\sum_e w(e)$, $w(e) = 2^{c(e)}$

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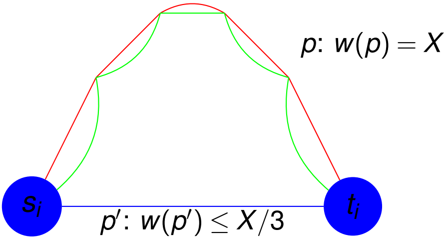

Potential function: $\sum_e w(e)$, $w(e) = 2^{c(e)}$
Moving path:

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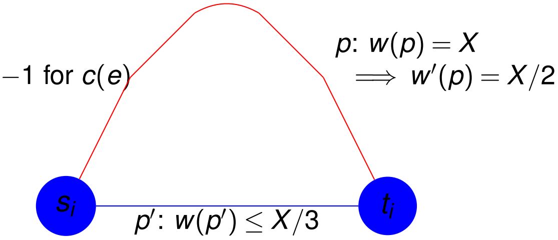

Potential function: $\sum_e w(e)$, $w(e) = 2^{c(e)}$
Moving path:
Divides $w(e)$ along long path (with $w(p)$ of X) by two.

An algorithm!

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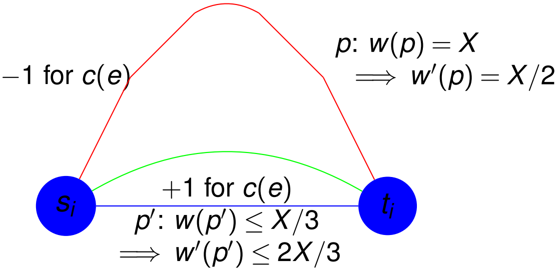
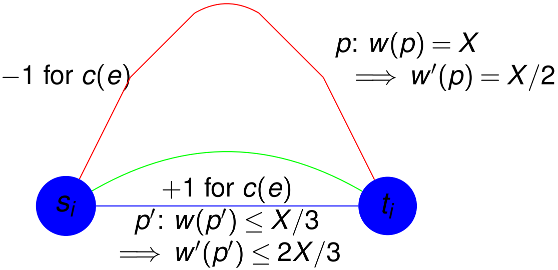


Potential function: $\sum_e w(e)$, $w(e) = 2^{c(e)}$
Moving path:
Divides $w(e)$ along long path (with $w(p)$ of X) by two.
Multiplies $w(e)$ along shorter ($w(p) \leq X/3$) path by two.

An algorithm!

Algorithm: reroute paths that are off by a factor of three.
(Note: $d(e)$ recomputed every rerouting.)



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Moving path:
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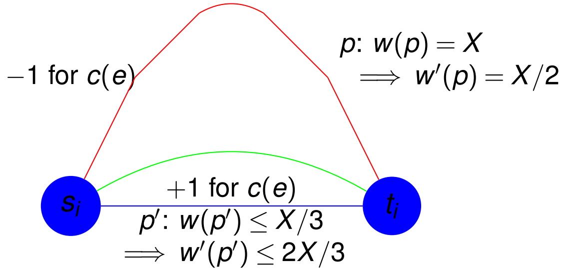

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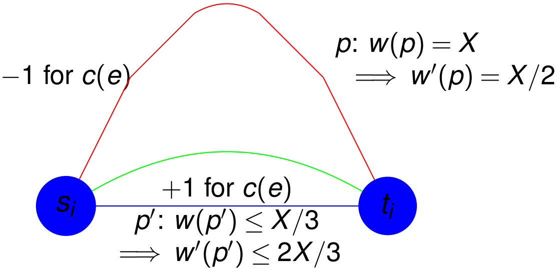

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Potential function decreases. $\implies$ termination and existence.

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Fractional paths?

Wrap up.

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A lower bound is “necessary” (natural),
and helpful (mysterious?)!

Done for the day.....

...see you on Thursday.