Comments on last lecture.

Easy to come up with several Nash for non-zero-sum games.

Is the game framework only interesting in some infinite horizon?

No.

Minimize worst expected loss.

Best defense.

Any prior distribution on opponent.

Best offense.

Rational players should play this way!

"Infinite horizon" is just an assumption of rationality.
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Today

Finish Maximum Weight Matching Algorithm.
Finish Maximum Weight Matching Algorithm.
Exact algorithm with dueling players.
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Exact algorithm with dueling players.

Multiplicative Weights Framework.
Finish Maximum Weight Matching Algorithm.
   Exact algorithm with dueling players.

Multiplicative Weights Framework.
   Very general framework of toll/congestion algorithm.
Matching/Weighted Vertex Cover

Maximum Weight Matching.
Matching/Weighted Vertex Cover

Maximum Weight Matching.

Given a bipartite graph, $G = (U, V, E)$, with edge weights $w : E \rightarrow \mathbb{R}$, find a maximum weight matching.
Matching/Weighted Vertex Cover

**Maximum Weight Matching.**

Given a bipartite graph, $G = (U, V, E)$, with edge weights $w : E \rightarrow R$, find a maximum weight matching.

A matching is a set of edges where no two share an endpoint.
Matching/Weighted Vertex Cover

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Given a bipartite graph, $G = (U, V, E)$, with edge weights $w : E \rightarrow R$, find a vertex cover function of minimum total value.

A function $p : V \rightarrow R$, where for all edges, $e = (u, v)$

$p(u) + p(v) \geq w(e)$. 
Matching/Weighted Vertex Cover

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Given a bipartite graph, \( G = (U, V, E) \), with edge weights \( w : E \to R \), find a maximum weight matching.
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\( p(u) + p(v) \geq w(e) \).
Minimize \( \sum_{v \in U \cup V} p(u) \).
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Optimal solutions to both if
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Optimal solutions to both if for $e \in M$, $w(e) = p(u) + p(v)$ (Defn: tight edge.) and
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Minimize $\sum_{v \in U \cup V} p(u)$.

Optimal solutions to both if

for $e \in M$, $w(e) = p(u) + p(v)$ (Defn: tight edge.) and perfect matching.
Maximum Weight Matching
Goal: perfect matching on tight edges.
Maximum Weight Matching
Goal: perfect matching on tight edges.

**Algorithm**
Start with empty matching, feasible cover function ($\rho(\cdot)$)
Maximum Weight Matching
Goal: perfect matching on tight edges.

Algorithm
Start with empty matching, feasible cover function \((\rho(\cdot))\)
Add tight edges to matching.
Algorithm
Start with empty matching, feasible cover function ($\rho(\cdot)$)
Add tight edges to matching.
  Use alt./aug. paths of tight edges.
Maximum Weight Matching
Goal: perfect matching on tight edges.

Algorithm
Start with empty matching, feasible cover function \( (\rho(\cdot)) \)
Add tight edges to matching.
Use alt./aug. paths of tight edges.
"maximum matching algorithm."

\[
\delta = \min_{e \in (S \cup T \times V)} \left( w(e) - \rho(u) - \rho(v) \right)
\]
Maximum Weight Matching
Goal: perfect matching on tight edges.

Algorithm

Start with empty matching, feasible cover function $(\rho(\cdot))$
Add tight edges to matching.
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No augmenting path.
Maximum Weight Matching
Goal: perfect matching on tight edges.

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Add tight edges to matching.

Use alt./aug. paths of tight edges.

"maximum matching algorithm."

No augmenting path.

Cut, \((S, T)\), in directed graph of tight edges!

\[
p(u) - \delta(p(u)) + \delta(p(v))
\]
Maximum Weight Matching
Goal: perfect matching on tight edges.

Algorithm
Start with empty matching, feasible cover function ($\rho(\cdot)$)
Add tight edges to matching.
Use alt./aug. paths of tight edges.
"maximum matching algorithm."
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All edges across cut are not tight. (loose?)
Maximum Weight Matching
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Nontight edges leaving cut, go from \(S_U, T_V\).
Maximum Weight Matching
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No augmenting path.
  Cut, \((S, T)\), in directed graph of tight edges!

All edges across cut are not tight. (loose?)

Nontight edges leaving cut, go from \(S_U, T_V\).
Lower prices in \(S_U,\)
Maximum Weight Matching
Goal: perfect matching on tight edges.

Algorithm
Start with empty matching, feasible cover function \((p(\cdot))\)

Add tight edges to matching.

Use alt./aug. paths of tight edges.

"maximum matching algorithm."

No augmenting path.

Cut, \((S, T)\), in directed graph of tight edges!

All edges across cut are not tight. (loose?)

Nontight edges leaving cut, go from \(S_U, T_V\).

Lower prices in \(S_U\), raise prices in \(S_T\),
Maximum Weight Matching
Goal: perfect matching on tight edges.

Algorithm
Start with empty matching, feasible cover function \( p(\cdot) \)
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Cut, \((S, T)\), in directed graph of tight edges!
All edges across cut are not tight. (loose?)
Nontight edges leaving cut, go from \( S_U, T_V \).
Lower prices in \( S_U \), raise prices in \( S_T \),
all explored edges still tight,
backward edges still feasible
Maximum Weight Matching
Goal: perfect matching on tight edges.

Algorithm

Start with empty matching, feasible cover function \( \rho(\cdot) \)

Add tight edges to matching.

- Use alt./aug. paths of tight edges.
- "maximum matching algorithm."

No augmenting path.

- Cut, \((S, T)\), in directed graph of tight edges!
- All edges across cut are not tight. (loose?)
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... and get new tight edge!
Maximum Weight Matching
Goal: perfect matching on tight edges.

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Start with empty matching, feasible cover function ($\rho(\cdot)$)
Add tight edges to matching.
- Use alt./aug. paths of tight edges.
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No augmenting path.
- Cut, $(S, T)$, in directed graph of tight edges!
All edges across cut are not tight. (loose?)
Nontight edges leaving cut, go from $S_U, T_V$.
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  all explored edges still tight,
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... and get new tight edge!
What’s delta?
Maximum Weight Matching
Goal: perfect matching on tight edges.
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Lower prices in \(S_U\), raise prices in \(S_T\),
all explored edges still tight,
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... and get new tight edge!
What’s delta? \(w(e) > p(u) + p(v) \rightarrow \delta = \min_{e \in (S_U \times T_V)} w(e) - p(u) - p(v).\)
Some details/Runtime

Add 0 value edges, so that optimal solution contains perfect matching.
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Beginning “Matcher” Solution: $M = \{\}$. 

Some details/Runtime

Add 0 value edges, so that optimal solution contains perfect matching.

Beginning “Matcher” Solution: \( M = \{ \} \).

Feasible!
Some details/Runtime

Add 0 value edges, so that optimal solution contains perfect matching.

Beginning “Matcher” Solution: $M = \{\}$. Feasible! Value $= 0$. 
Some details/Runtime

Add 0 value edges, so that optimal solution contains perfect matching.

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Beginning “Coverer” Solution: $\rho(u) =$ maximum incident edge for $u \in U$,
Some details/Runtime

Add 0 value edges, so that optimal solution contains perfect matching.

Beginning “Matcher” Solution: $M = \{\}$. Feasible! Value = 0.

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Main Work:
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Main Work:
  breadth first search from unmatched nodes finds cut.
Some details/Runtime

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Update prices (find minimum delta.)
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  Each bfs either augments or adds node to \( S \) in next cut.
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$O(n^2 m)$ time.
Example

Weight legend:
black 1, green 2, blue 3

Reachable: $S = \{u, v\}$
Blue edges soon to be tight!
Adjust prices...

Reachable: $S = \{v, w, x, a\}$
Blue edges minimally non-tight.
Adjust prices.

Some more tight edges.
And X shows a "new" nontight edge.
..and another augmentation...
..and finally: a perfect matching.

All matched edges tight.
Perfect matching.
Feasible price function.
Values the same.
Optimal!

Notice:
no weights on the right problem.
retain previous matching through price changes.
retains edges in failed search through price changes.
Example

Weight legend:
black 1, green 2, blue 3
Tight edges for initial prices.
Example

Weight legend:
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Max matching in tight edges.
dashed means matched.

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The multiplicative weights framework.
Expert’s framework.

$n$ experts.
Expert’s framework.

$n$ experts.

Every day, each offers a prediction.
$n$ experts.

Every day, each offers a prediction.

“Rain” or “Shine.”
Expert’s framework.

$n$ experts.
Every day, each offers a prediction.
“Rain” or “Shine.”
Whose advise do you follow?
Expert’s framework.

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“The one who is correct most often.”
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Sort of.
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Whose advise do you follow?

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Sort of.

How well do you do?
Infallible expert.

One of the expert’s is infallible!
Infallible expert.

One of the expert’s is infallible!

Your strategy?
Infallible expert.

One of the expert’s is infallible!

Your strategy?

Choose any expert that has not made a mistake!
Infallible expert.

One of the expert’s is infallible!

Your strategy?

Choose any expert that has not made a mistake!

How long to find perfect expert?
Infallible expert.

One of the expert’s is infallible!

Your strategy?

Choose any expert that has not made a mistake!

How long to find perfect expert?

Maybe..
Infallible expert.

One of the expert’s is infallible!

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Better model?
Infallible expert.

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Better model?

How many mistakes could you make?
Infallible expert.

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Better model?

How many mistakes could you make? Mistake Bound.
Infallible expert.

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Better model?

How many mistakes could you make? **Mistake Bound.**

(A) 1

(B) 2

(C) \( \log n \)

(D) \( n - 1 \)

Adversary designs setup to watch who you choose, and make that expert make a mistake.
Infallible expert.

One of the expert’s is infallible!

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(B) 2
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(D) \( n - 1 \)

Adversary designs setup to watch who you choose, and make that expert make a mistake.

\( n - 1! \)
Concept Alert.

Note.
Concept Alert.

Note.

Adversary:
Concept Alert.

Note.

Adversary:
  makes you want to look bad.
Concept Alert.

Note.

Adversary:
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  "You could have done so well"...
Concept Alert.

Note.

Adversary: makes you want to look bad.
"You could have done so well"... but you didn’t!
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  makes you want to look bad.
  "You could have done so well"...
  but you didn’t! ha..
Concept Alert.

Note.

Adversary:
makes you want to look bad.
”You could have done so well”...
but you didn’t! ha..ha!
Concept Alert.

Note.

Adversary:
  makes you want to look bad.
  "You could have done so well"
  but you didn’t! ha..ha!

Analysis of Algorithms: do as well as possible!
Back to mistake bound.

Infallible Experts.
Infallible Experts.

Alg: Choose one of the perfect experts.
Back to mistake bound.

Infallible Experts.
Alg: Choose one of the perfect experts.
Mistake Bound: $n - 1$
Infallible Experts.

Alg: Choose one of the perfect experts.

Mistake Bound: \( n - 1 \)

Lower bound: adversary argument.
Back to mistake bound.

Infallible Experts.

Alg: Choose one of the perfect experts.

Mistake Bound: \( n - 1 \)
- Lower bound: adversary argument.
- Upper bound:
Infallible Experts.

Alg: Choose one of the perfect experts.

**Mistake Bound: \( n - 1 \)**
- Lower bound: adversary argument.
- Upper bound: every mistake finds fallible expert.
Infallible Experts.

Alg: Choose one of the perfect experts.

**Mistake Bound: $n - 1$**
- Lower bound: adversary argument.
- Upper bound: every mistake finds fallible expert.

Better Algorithm?
Infallible Experts.

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**Mistake Bound: \( n - 1 \)**

- Lower bound: adversary argument.
- Upper bound: every mistake finds fallible expert.

Better Algorithm?

Making decision, not trying to find expert!
Infallible Experts.

Alg: Choose one of the perfect experts.

**Mistake Bound:** $n - 1$
- Lower bound: adversary argument.
- Upper bound: every mistake finds fallible expert.

Better Algorithm?

Making decision, not trying to find expert!

Algorithm: Go with the majority of previously correct experts.
Infallible Experts.

Alg: Choose one of the perfect experts.

**Mistake Bound: $n - 1$**
- Lower bound: adversary argument.
- Upper bound: every mistake finds fallible expert.

Better Algorithm?

Making decision, not trying to find expert!

Algorithm: Go with the majority of previously correct experts.

What you would do anyway!
Alg 2: find majority of the perfect

How many mistakes could you make?
Alg 2: find majority of the perfect

How many mistakes could you make?
(A) 1
(B) 2
(C) \( \log n \)
(D) \( n - 1 \)
Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1
(B) 2
(C) \( \log n \)
(D) \( n - 1 \)

At most \( \log n \)!
Alg 2: find majority of the perfect

How many mistakes could you make?
(A) 1
(B) 2
(C) $\log n$
(D) $n - 1$

At most $\log n$!

When alg makes a mistake, |“perfect” experts| drops by a factor of two.
Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1
(B) 2
(C) $\log n$
(D) $n - 1$

At most $\log n$!

When alg makes a mistake, 
|“perfect” experts| drops by a factor of two.

Initially $n$ perfect experts
Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1
(B) 2
(C) $\log n$
(D) $n - 1$

At most $\log n$!

When alg makes a mistake,

$|\text{"perfect" experts}|$ drops by a factor of two.

Initially $n$ perfect experts $\rightarrow \leq n/2$ perfect experts
Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1
(B) 2
(C) $\log n$
(D) $n - 1$

At most $\log n!$

When alg makes a mistake,

$|\text{"perfect" experts}|$ drops by a factor of two.

Initially $n$ perfect experts

mistake $\rightarrow \leq n/2$ perfect experts

mistake $\rightarrow \leq n/4$ perfect experts
Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1
(B) 2
(C) log \(n\)
(D) \(n - 1\)

At most log \(n\)!

When alg makes a mistake, the number of “perfect” experts drops by a factor of two.

Initially \(n\) perfect experts

- mistake \(\rightarrow\) \(\leq n/2\) perfect experts
- mistake \(\rightarrow\) \(\leq n/4\) perfect experts
- :
Alg 2: find majority of the perfect

How many mistakes could you make?
(A) 1
(B) 2
(C) $\log n$
(D) $n - 1$

At most $\log n$!

When alg makes a mistake, $|\text{“perfect” experts}|$ drops by a factor of two.

Initially $n$ perfect experts mistake $\rightarrow \leq n/2$ perfect experts
mistake $\rightarrow \leq n/4$ perfect experts

$\vdots$

mistake $\rightarrow \leq 1$ perfect expert
Alg 2: find majority of the perfect

How many mistakes could you make?

(A) 1
(B) 2
(C) $\log n$
(D) $n - 1$

At most $\log n$!

When alg makes a mistake, $|\text{"perfect" experts}|$ drops by a factor of two.

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mistake $\rightarrow$ $\leq n/2$ perfect experts

mistake $\rightarrow$ $\leq n/4$ perfect experts

$\vdots$

mistake $\rightarrow$ $\leq 1$ perfect expert
Alg 2: find majority of the perfect

How many mistakes could you make?
(A) 1
(B) 2
(C) $\log n$
(D) $n - 1$

At most $\log n$!

When alg makes a mistake, $|\text{"perfect" experts}|$ drops by a factor of two.

Initially $n$ perfect experts mistake $\rightarrow \leq \frac{n}{2}$ perfect experts
mistake $\rightarrow \leq \frac{n}{4}$ perfect experts

$\vdots$
mistake $\rightarrow \leq 1$ perfect expert

$\geq 1$ perfect expert
Alg 2: find majority of the perfect

How many mistakes could you make?
(A) 1
(B) 2
(C) $\log n$
(D) $n - 1$

At most $\log n$!

When alg makes a mistake, 
|“perfect” experts| drops by a factor of two.
Initially $n$ perfect experts mistake $\rightarrow \leq n/2$ perfect experts 
mistake $\rightarrow \leq n/4$ perfect experts 
$\vdots$

mistake $\rightarrow \leq 1$ perfect expert 

$\geq 1$ perfect expert $\rightarrow$ at most $\log n$ mistakes!
Imperfect Experts

Goal?

Algorithm.
Suggestions?
Go with majority?
Penalize inaccurate experts?
Best expert is penalized the least.

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i / 2$ if wrong.
Imperfect Experts

Goal?
Do as well as the best expert!
Imperfect Experts

Goal?

Do as well as the best expert!

Algorithm.

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Analysis: weighted majority

Initially:

1. $w_i = 1$.

2. Predict with weighted majority of experts.

3. $w_i \rightarrow w_i / 2$ if wrong.

Goal: Best expert makes $m$ mistakes.

Potential function: $\sum_i w_i$.

Initially $n$. For best expert, $b$, $w_b \geq \frac{1}{2} m$.

Each mistake: total weight of incorrect experts reduced by $-\frac{1}{2}$ factor of $\frac{1}{2}$.

Each incorrect expert weight multiplied by $\frac{1}{2}$! total weight decreases by factor of $\frac{3}{4}$.

Mistake $\rightarrow \geq$ half weight with incorrect experts.

Mistake $\rightarrow$ potential function decreased by $\frac{3}{4}$.

We have $\frac{1}{2} m \leq \sum_i w_i \leq (\frac{3}{4})^M n$, where $M$ is number of algorithm mistakes.
Analysis: weighted majority

1. Initially: \( w_i = 1 \).
2. Predict with weighted majority of experts.
3. \( w_i \rightarrow w_i/2 \) if wrong.

Goal: Best expert makes \( m \) mistakes.

Potential function:
\[
\sum_i w_i.
\]

Initially \( n \).

For best expert, \( b \), \( w_b \geq 1/2 m \).

Each mistake: total weight of incorrect experts reduced by \( -1/2 \) factor of \( 1/2 \) each incorrect expert weight multiplied by \( 1/2 \! \) total weight decreases by \( 3/4 \) factor of \( 3/4 \).

Mistake \( \rightarrow \geq \) half weight with incorrect experts.

Mistake \( \rightarrow \) potential function decreased by \( 3/4 \).

We have \( 1/2 m \leq \sum_i w_i \leq (3/4)^m \).

where \( M \) is number of algorithm mistakes.
Analysis: weighted majority

Goal: Best expert makes $m$ mistakes.

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \to w_i/2$ if wrong.

Potential function: $\sum_i w_i$.

Initially $n$. For best expert, $b$, $w_b \geq \frac{1}{2} m$.

Each mistake: total weight of incorrect experts reduced by $\frac{1}{2}$ factor of each incorrect expert weight multiplied by $\frac{1}{2}$! total weight decreases by factor of $\frac{3}{4}$.

Mistake $\to \geq$ half weight with incorrect experts.

Mistake $\to$ potential function decreased by $\frac{3}{4}$.

We have $\frac{1}{2} m \leq \sum_i w_i \leq \left( \frac{3}{4} \right)^m M n$.

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1. Initially: $w_i = 1$.
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Goal: Best expert makes $m$ mistakes.
Potential function: $\sum_i w_i$. Initially $n$.
For best expert, $b$, $w_b \geq \frac{1}{2^m}$.

1. Initially: $w_i = 1$.
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Potential function: \( \sum_i w_i \). Initially \( n \).

For best expert, \( b \), \( w_b \geq \frac{1}{2^m} \).

Each mistake:

1. Initially: \( w_i = 1 \).
2. Predict with weighted majority of experts.
3. \( w_i \rightarrow w_i/2 \) if wrong.
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Goal: Best expert makes $m$ mistakes.

Potential function: $\sum_i w_i$. Initially $n$.

For best expert, $b$, $w_b \geq \frac{1}{2^m}$.

Each mistake:
- total weight of incorrect experts reduced by $\frac{1}{2}$?

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i/2$ if wrong.
Goal: Best expert makes $m$ mistakes.

Potential function: $\sum_i w_i$. Initially $n$.

For best expert, $b$, $w_b \geq \frac{1}{2^m}$.

Each mistake:
  - total weight of incorrect experts reduced by $-1$?

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i/2$ if wrong.
Analysis: weighted majority

Goal: Best expert makes $m$ mistakes.

Potential function: $\sum_i w_i$. Initially $n$.

For best expert, $b$, $w_b \geq \frac{1}{2^m}$.

Each mistake:
- total weight of incorrect experts reduced by $-1? - 2?$

1. Initially: $w_i = 1$.

2. Predict with weighted majority of experts.

3. $w_i \rightarrow w_i/2$ if wrong.
Analysis: weighted majority

Goal: Best expert makes $m$ mistakes.

Potential function: $\sum_i w_i$. Initially $n$.

For best expert, $b$, $w_b \geq \frac{1}{2^m}$.

Each mistake:
- total weight of incorrect experts reduced by $-1? - 2? \times \text{factor of } \frac{1}{2}$?

1. Initially: $w_i = 1$.
2. Predict with weighted majority of experts.
3. $w_i \rightarrow w_i/2$ if wrong.
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Each mistake:
- total weight of incorrect experts reduced by $-1? -2? \text{ factor of } \frac{1}{2}$?
- each incorrect expert weight multiplied by $\frac{1}{2}$!

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Potential function: $\sum_i w_i$. Initially $n$.

For best expert, $b$, $w_b \geq \frac{1}{2^m}$.

Each mistake:
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  - each incorrect expert weight multiplied by $\frac{1}{2}$
- total weight decreases by

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Goal: Best expert makes \( m \) mistakes.

Potential function: \( \sum_i w_i \). Initially \( n \).

For best expert, \( b \), \( w_b \geq \frac{1}{2^m} \).

Each mistake:
- total weight of incorrect experts reduced by \(-1\)? \(-2\)? factor of \( \frac{1}{2} \)?
- each incorrect expert weight multiplied by \( \frac{1}{2} \)
- total weight decreases by factor of \( \frac{1}{2} \)? factor of \( \frac{3}{4} \)?

1. Initially: \( w_i = 1 \).
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Goal: Best expert makes $m$ mistakes.

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Each mistake:
- total weight of incorrect experts reduced by $-1$? $-2$? factor of $\frac{1}{2}$?
- each incorrect expert weight multiplied by $\frac{1}{2}$!
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- mistake $\rightarrow \geq$ half weight with incorrect experts.

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Goal: Best expert makes $m$ mistakes.

Potential function: $\sum_i w_i$. Initially $n$.

For best expert, $b$, $w_b \geq \frac{1}{2^m}$.

Each mistake:
- total weight of incorrect experts reduced by $-1$? $-2$? factor of $\frac{1}{2}$?
- each incorrect expert weight multiplied by $\frac{1}{2}$!
- total weight decreases by factor of $\frac{1}{2}$? factor of $\frac{3}{4}$?
- mistake $\rightarrow \geq$ half weight with incorrect experts.

Mistake $\rightarrow$ potential function decreased by $\frac{3}{4}$.

We have

$$\frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n.$$
Analysis: continued.

\[
\frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n.
\]
Analysis: continued.

\[ \frac{1}{2^m} \leq \sum_i w_i \leq \left( \frac{3}{4} \right)^M n. \]

\( m \) - best expert mistakes
Analysis: continued.

\[ \frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n. \]

\( m \) - best expert mistakes \( M \) algorithm mistakes.
Analysis: continued.

\[ \frac{1}{2^m} \leq \sum_i w_i \leq \left( \frac{3}{4} \right)^M n. \]

\( m \) - best expert mistakes \( M \) algorithm mistakes.

\[ \frac{1}{2^m} \leq \left( \frac{3}{4} \right)^M n. \]
Analysis: continued.

\[ \frac{1}{2^m} \leq \sum_i w_i \leq \left( \frac{3}{4} \right)^M n. \]

\( m \) - best expert mistakes\quad \( M \) algorithm mistakes.

\[ \frac{1}{2^m} \leq \left( \frac{3}{4} \right)^M n. \]

Take log of both sides.

\[ \log_{\frac{3}{4}} \left( \frac{1}{2^m} \right) \leq M \log_{\frac{3}{4}} n. \]

Solve for \( M \).

\[ M \leq \frac{m + \log n}{\log_{\frac{3}{4}} 2} \leq 2.4 \tag{1-2/3} \]

Multiple by \( 1 - \epsilon \) for incorrect experts...

\[ (1 - \epsilon) \frac{1}{2^m} \leq (1 - \epsilon) \left( \frac{3}{4} \right)^M n. \]

\[ M \leq 2 \left( 1 + \epsilon \right) m + 2 \ln n \epsilon. \]

Approaches a factor of two of best expert performance!
Analysis: continued.

\[ \frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n. \]

\(m\) - best expert mistakes \(M\) algorithm mistakes.

\[ \frac{1}{2^m} \leq \left(\frac{3}{4}\right)^M n. \]

Take log of both sides.

\[-m \leq -M \log(4/3) + \log n.\]
Analysis: continued.

\[ \frac{1}{2^m} \leq \sum_i w_i \leq \left( \frac{3}{4} \right)^M n. \]

1. \( m \) - best expert mistakes
2. \( M \) - algorithm mistakes.

\[ \frac{1}{2^m} \leq \left( \frac{3}{4} \right)^M n. \]

Take log of both sides.

\[ -m \leq -M \log(4/3) + \log n. \]

Solve for \( M \).

\[ M \leq \frac{m + \log n}{\log(4/3)} \]
Analysis: continued.

\[
\frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n.
\]

\(m\) - best expert mistakes \(M\) algorithm mistakes.

Take log of both sides.

\[-m \leq -M \log(4/3) + \log n.\]

Solve for \(M\).

\[M \leq (m + \log n) / \log(4/3) \leq 2.4(m + \log n)\]
Analysis: continued.

\[
\frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n.
\]

- \(m\) - best expert mistakes
- \(M\) algorithm mistakes.

\[
\frac{1}{2^m} \leq \left(\frac{3}{4}\right)^M n.
\]

Take log of both sides.

\[-m \leq -M \log(4/3) + \log n.\]

Solve for \(M\).

\[M \leq (m + \log n)/\log(4/3) \leq 2.4(m + \log n)\]

Multiple by \(1 - \varepsilon\) for incorrect experts...
Analysis: continued.

\[ \frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n. \]

\( m \) - best expert mistakes \( M \) algorithm mistakes.

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Take log of both sides.

\[ -m \leq -M \log(4/3) + \log n. \]

Solve for \( M \).

\[ M \leq \frac{m + \log n}{\log(4/3)} \leq 2.4(m + \log n) \]

Multiple by \( 1 - \varepsilon \) for incorrect experts...

\[ (1 - \varepsilon)^m \leq \left(1 - \frac{\varepsilon}{2}\right)^M n. \]
Analysis: continued.

\[ \frac{1}{2^m} \leq \sum_i w_i \leq \left( \frac{3}{4} \right)^M n. \]

\(m\) - best expert mistakes  \(M\) algorithm mistakes.

\[ \frac{1}{2^m} \leq \left( \frac{3}{4} \right)^M n. \]

Take log of both sides.

\[ -m \leq -M \log(4/3) + \log n. \]

Solve for \(M\).

\[ M \leq (m + \log n)/\log(4/3) \leq 2.4(m + \log n) \]

Multiple by \(1 - \varepsilon\) for incorrect experts...

\[ (1 - \varepsilon)^m \leq (1 - \frac{\varepsilon}{2})^M n. \]

Massage...
Analysis: continued.

\[ \frac{1}{2^m} \leq \sum_i w_i \leq \left(\frac{3}{4}\right)^M n. \]

- \( m \) - best expert mistakes
- \( M \) algorithm mistakes.

\[ \frac{1}{2^m} \leq \left(\frac{3}{4}\right)^M n. \]

Take log of both sides.

\[ -m \leq -M \log(4/3) + \log n. \]

Solve for \( M \).

\[ M \leq \frac{m + \log n}{\log(4/3)} \leq 2.4(m + \log n) \]

Multiple by \( 1 - \varepsilon \) for incorrect experts...

\[ (1 - \varepsilon)^m \leq (1 - \frac{\varepsilon}{2})^M n. \]

Massage...

\[ M \leq 2(1 + \varepsilon)m + \frac{2\ln n}{\varepsilon} \]
Analysis: continued.

\[ \frac{1}{2^m} \leq \sum_i w_i \leq \left( \frac{3}{4} \right)^M n. \]

\( m \) - best expert mistakes  \( M \) algorithm mistakes.

\[ \frac{1}{2^m} \leq \left( \frac{3}{4} \right)^M n. \]

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\[ -m \leq -M \log(4/3) + \log n. \]

Solve for \( M \).

\( M \leq (m + \log n) / \log(4/3) \leq 2.4(m + \log n) \)

Multiple by \( 1 - \varepsilon \) for incorrect experts...

\[ (1 - \varepsilon)^m \leq (1 - \frac{\varepsilon}{2})^M n. \]

Massage...

\[ M \leq 2(1 + \varepsilon)m + \frac{2\ln n}{\varepsilon} \]

Approaches a factor of two of best expert performance!
Best Analysis?

Two experts: A, B
Best Analysis?

Two experts: A, B
Bad example?

(A) A right on even, B right on odd.
(B) A right first half of days, B right second

Best expert performance: \(T/2\) mistakes.

Pattern (A): \(T - 1\) mistakes.
Factor of (almost) two worse!
Best Analysis?

Two experts: A, B

Bad example?
Which is worse?

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Factor of (almost) two worse!
Randomization

Better approach?
Randomization

Better approach?
Use?
Randomization!!!!

Better approach?
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Better approach?
Use?
    Randomization!
That is, choose expert $i$ with prob $\propto w_i$
Randomization!!!!

Better approach?
Use?

Randomization!
That is, choose expert $i$ with prob $\propto w_i$
Bad example: A,B,A,B,A...
Better approach?
Use?

Randomization!
That is, choose expert $i$ with prob $\propto w_i$

Bad example: A,B,A,B,A...

After a bit, A and B make nearly the same number of mistakes.
Randomization!!!!

Better approach?
Use?

Randomization!
That is, choose expert $i$ with prob $\propto w_i$
Bad example: A,B,A,B,A...
After a bit, A and B make nearly the same number of mistakes.
Choose each with approximately the same probability.
Better approach?
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Better approach?
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Best expert makes $T/2$ mistakes.
Randomization!!!!

Better approach?
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After a bit, A and B make nearly the same number of mistakes.
Choose each with approximately the same probability.
Make a mistake around $1/2$ of the time.
Best expert makes $T/2$ mistakes.
Roughly
Randomization!!!!

Better approach? 
Use? 

Randomization!

That is, choose expert $i$ with prob $\propto w_i$

Bad example: A,B,A,B,A...

After a bit, A and B make nearly the same number of mistakes.

Choose each with approximately the same probability.

Make a mistake around 1/2 of the time.

Best expert makes $T/2$ mistakes.

Roughly optimal!
Randomized analysis.

Some formulas:
Randomized analysis.

Some formulas:
For $\varepsilon \leq 1$, $x \in [0, 1]$,
Randomized analysis.

Some formulas:
For $\varepsilon \leq 1, x \in [0, 1],$

$(1 + \varepsilon)^x \leq (1 + \varepsilon x)$
$(1 - \varepsilon)^x \leq (1 - \varepsilon x)$
Randomized analysis.

Some formulas:

For $\varepsilon \leq 1$, $x \in [0, 1]$,

\[
(1 + \varepsilon)^x \leq (1 + \varepsilon x) \leq (1 - \varepsilon)^x \leq (1 - \varepsilon x)
\]

For $\varepsilon \in [0, \frac{1}{2}]$,
Randomized analysis.

Some formulas:
For $\varepsilon \leq 1$, $x \in [0, 1]$,
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(1 + \varepsilon)^x \leq (1 + \varepsilon x) \\
(1 - \varepsilon)^x \leq (1 - \varepsilon x)
\]
For $\varepsilon \in [0, \frac{1}{2}]$,
\[
-\varepsilon - \varepsilon^2 \leq \ln(1 - \varepsilon) \leq -\varepsilon \\
\varepsilon - \varepsilon^2 \leq \ln(1 + \varepsilon) \leq \varepsilon
\]
Randomized analysis.

Some formulas:

For $\varepsilon \leq 1, x \in [0, 1]$,

$$(1 + \varepsilon)^x \leq (1 + \varepsilon x)$$

$$(1 - \varepsilon)^x \leq (1 - \varepsilon x)$$

For $\varepsilon \in [0, \frac{1}{2}]$,

$$-\varepsilon - \varepsilon^2 \leq \ln(1 - \varepsilon) \leq -\varepsilon$$

$$\varepsilon - \varepsilon^2 \leq \ln(1 + \varepsilon) \leq \varepsilon$$

Proof Idea: $\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$
Randomized algorithm

Losses in $[0, 1]$. 

1. Initially $w_i = 1$ for expert $i$.
2. Choose expert $i$ with prob $w_i \frac{W}{W} = \sum_i w_i$.
3. $w_i \leftarrow w_i (1 - \varepsilon) \ell_{ti} W(t)$ sum of $w_i$ at time $t$.

Best expert, $b$, loses $L^*$ total. $\rightarrow W(T) \geq w_b \geq (1 - \varepsilon) L^*$.

$L_t = \sum_i w_i \ell_{ti}$ expected loss of alg. in time $t$.

Claim: $W(t+1) \leq W(t) (1 - \varepsilon L_t)$

Proof: $W(t+1) \leq \sum_i (1 - \varepsilon \ell_{ti}) w_i = \sum_i w_i - \varepsilon \sum_i w_i \ell_{ti} = \sum_i w_i (1 - \varepsilon \sum_i w_i \ell_{ti} \sum_i w_i) = W(t) (1 - \varepsilon L_t)$.
Randomized algorithm

Losses in $[0, 1]$.

Expert $i$ loses $\ell_i^t \in [0, 1]$ in round $t$. 

1. Initially $w_i = 1$ for expert $i$.
2. Choose expert $i$ with prob $w_i / W$, $W = \sum_i w_i$.
3. $w_i \leftarrow w_i \left(1 - \varepsilon \ell_i^t W(t)$ sum of $w_i$ at time $t$.

Best expert, $b$, loses $L^*$ total. $W(t) \geq w_b \geq (1 - \varepsilon) L^*$.

$L^t = \sum_i w_i \ell_i^t$ expected loss of alg. in time $t$.

Claim: $W(t+1) \leq W(t)(1 - \varepsilon L^t)$

Loss $\rightarrow$ weight loss.

Proof: $W(t+1) \leq \sum_i (1 - \varepsilon \ell_i^t) w_i = \sum_i w_i - \varepsilon \sum_i w_i \ell_i^t \sum_i w_i = W(t) (1 - \varepsilon \frac{L^t}{W}) = W(t)(1 - \varepsilon L^t)$.
Randomized algorithm

Losses in $[0, 1]$.
Expert $i$ loses $\ell_i^t \in [0, 1]$ in round $t$.

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Randomized algorithm

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3. $w_i \leftarrow w_i (1 - \varepsilon)^{\ell^t_i}$
Randomized algorithm

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Expert $i$ loses $\ell_i^t \in [0, 1]$ in round $t$.

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$W(t)$ sum of $w_i$ at time $t$. 
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Best expert, $b$, loses $L^*$ total. $\rightarrow W(T) \geq w_b \geq (1 - \varepsilon)^{L^*}$.

$L_t = \sum_i \frac{w_i \ell_i^t}{W}$ expected loss of alg. in time $t$. 
Randomized algorithm

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Expert $i$ loses $\ell_i^t \in [0, 1]$ in round $t$.

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2. Choose expert $i$ with prob $\frac{w_i}{W}$, $W = \sum_i w_i$.
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$L_t = \sum_i \frac{w_i\ell_i^t}{W}$ expected loss of alg. in time $t$.

Claim: $W(t+1) \leq W(t)(1 - \varepsilon L_t)$
Randomized algorithm

Losses in \([0, 1]\).

Expert \(i\) loses \(\ell_i^t \in [0, 1]\) in round \(t\).

1. Initially \(w_i = 1\) for expert \(i\).
2. Choose expert \(i\) with prob \(\frac{w_i}{W}\), \(W = \sum_i w_i\).
3. \(w_i \leftarrow w_i (1 - \varepsilon) \ell_i^t\)

\(W(t)\) sum of \(w_i\) at time \(t\). \(W(0) = n\)

Best expert, \(b\), loses \(L^*\) total. \(\rightarrow W(T) \geq w_b \geq (1 - \varepsilon) L^*\).

\(L_t = \sum_i \frac{w_i \ell_i^t}{W}\) expected loss of alg. in time \(t\).

Claim: \(W(t+1) \leq W(t)(1 - \varepsilon L_t)\) Loss \(\rightarrow\) weight loss.
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$L_t = \sum_i \frac{w_i \ell_i^t}{W}$ expected loss of alg. in time $t$.

Claim: $W(t + 1) \leq W(t)(1 - \varepsilon L_t)$ Loss $\rightarrow$ weight loss.

Proof:
Randomized algorithm

Losses in $[0,1]$.

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Claim: $W(t+1) \leq W(t)(1 - \varepsilon L_t)$ Loss $\rightarrow$ weight loss.

Proof:

$$W(t+1) \leq \sum_i (1 - \varepsilon \ell_i^t)w_i$$
Randomized algorithm

Losses in $[0, 1]$.  
Expert $i$ loses $\ell_i^t \in [0, 1]$ in round $t$.

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$W(t)$ sum of $w_i$ at time $t$. $W(0) = n$

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Proof: 
$W(t+1) \leq \sum_i (1 - \varepsilon \ell_i^t)w_i = \sum_i w_i - \varepsilon \sum_i w_i \ell_i^t$
Randomized algorithm

Losses in $[0, 1]$. 

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W(t + 1) \leq \sum_i (1 - \varepsilon \ell_i^t) w_i = \sum_i w_i - \varepsilon \sum_i w_i \ell_i^t
\]

\[
= \sum_i w_i \left(1 - \varepsilon \frac{\sum_i w_i \ell_i^t}{\sum_i w_i}\right)
\]
Randomized algorithm

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Proof:

$$W(t + 1) \leq \sum_i (1 - \varepsilon \ell_i^t)w_i = \sum_i w_i - \varepsilon \sum_i w_i \ell_i^t$$

$$= \sum_i w_i \left(1 - \varepsilon \frac{\sum_i w_i \ell_i^t}{\sum_i w_i}\right)$$

$$= W(t)(1 - \varepsilon L_t)$$
Analysis

\[(1 - \varepsilon)^{L^*} \leq \mathcal{W}(T) \leq n \prod_t (1 - \varepsilon L_t)\]
Analysis

\[(1 - \varepsilon)^{L^*} \leq W(T) \leq n \prod_t (1 - \varepsilon L_t)\]

Take logs
\[(L^*) \ln(1 - \varepsilon) \leq \ln n + \sum \ln(1 - \varepsilon L_t)\]

\[\sum_t L_t \text{ is total expected loss of algorithm.}\]

\[(1 + \varepsilon) \text{ is of the best expert!}\]

\[\text{No factor of 2 loss!}\]
Analysis

\[(1 - \varepsilon)^{L^*} \leq W(T) \leq n \prod_t(1 - \varepsilon L_t)\]

Take logs

\[(L^*) \ln(1 - \varepsilon) \leq \ln n + \sum \ln(1 - \varepsilon L_t)\]

Use \(-\varepsilon - \varepsilon^2 \leq \ln(1 - \varepsilon) \leq -\varepsilon\)
Analysis

\[(1 - \varepsilon)^{L^*} \leq W(T) \leq n \prod_t (1 - \varepsilon L_t)\]

Take logs
\[\left(\begin{array}{c}
L^* \\
\end{array}\right) \ln(1 - \varepsilon) \leq \ln n + \sum \ln(1 - \varepsilon L_t)\]

Use \(-\varepsilon - \varepsilon^2 \leq \ln(1 - \varepsilon) \leq -\varepsilon\)

\[\left(\begin{array}{c}
L^* \\
\end{array}\right) (\varepsilon + \varepsilon^2) \leq \ln n - \varepsilon \sum L_t\]
Analysis

\[(1 - \varepsilon)^{L^*} \leq W(T) \leq n \prod_{t}(1 - \varepsilon L_t)\]

Take logs

\[(L^*) \ln(1 - \varepsilon) \leq \ln n + \sum \ln(1 - \varepsilon L_t)\]

Use \(-\varepsilon - \varepsilon^2 \leq \ln(1 - \varepsilon) \leq -\varepsilon\)

\[-(L^*) (\varepsilon + \varepsilon^2) \leq \ln n - \varepsilon \sum L_t\]

And

\[\sum L_t \leq (1 + \varepsilon)L^* + \ln n \varepsilon.\]

\[\sum L_t\] is total expected loss of algorithm. Within \((1 + \varepsilon)\) is of the best expert! No factor of 2 loss!
(1 - \varepsilon)^{L^*} \leq W(T) \leq n \prod_t (1 - \varepsilon L_t)

Take logs

\((L^*) \ln (1 - \varepsilon) \leq \ln n + \sum \ln (1 - \varepsilon L_t)\)

Use \(-\varepsilon - \varepsilon^2 \leq \ln (1 - \varepsilon) \leq -\varepsilon\)

\(-(L^*) (\varepsilon + \varepsilon^2) \leq \ln n - \varepsilon \sum L_t\)

And

\[\sum_t L_t \leq (1 + \varepsilon) L^* + \frac{\ln n}{\varepsilon}\]
Analysis

\[(1 - \varepsilon)^{L^*} \leq W(T) \leq n \prod_t (1 - \varepsilon L_t)\]

Take logs
\[(L^*) \ln(1 - \varepsilon) \leq \ln n + \sum \ln(1 - \varepsilon L_t)\]

Use \(-\varepsilon - \varepsilon^2 \leq \ln(1 - \varepsilon) \leq -\varepsilon\)
\[-(L^*)(\varepsilon + \varepsilon^2) \leq \ln n - \varepsilon \sum L_t\]

And
\[\sum_t L_t \leq (1 + \varepsilon)L^* + \frac{\ln n}{\varepsilon}\]

\(\sum_t L_t\) is total expected loss of algorithm.
Analysis

\[(1 - \varepsilon)^{L^*} \leq W(T) \leq n \prod_t (1 - \varepsilon L_t)\]

Take logs
\[(L^*) \ln(1 - \varepsilon) \leq \ln n + \sum \ln(1 - \varepsilon L_t)\]

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\(\sum_t L_t\) is total expected loss of algorithm.

Within \((1 + \varepsilon)\)
Analysis

\[(1 - \varepsilon)^{L^*} \leq W(T) \leq n \prod_t (1 - \varepsilon L_t)\]

Take logs
\[(L^*) \ln (1 - \varepsilon) \leq \ln n + \sum \ln (1 - \varepsilon L_t)\]

Use \[-\varepsilon - \varepsilon^2 \leq \ln (1 - \varepsilon) \leq -\varepsilon\]
\[-(L^*)(\varepsilon + \varepsilon^2) \leq \ln n - \varepsilon \sum L_t\]

And
\[\sum_t L_t \leq (1 + \varepsilon)L^* + \frac{\ln n}{\varepsilon} .\]

\[\sum_t L_t\] is total expected loss of algorithm.

Within \((1 + \varepsilon)\) ish
Analysis

\[(1 - \varepsilon)^{L^*} \leq \mathcal{W}(T) \leq n \prod_t (1 - \varepsilon L_t)\]

Take logs
\[(L^*) \ln (1 - \varepsilon) \leq \ln n + \sum \ln (1 - \varepsilon L_t)\]

Use \[-\varepsilon - \varepsilon^2 \leq \ln (1 - \varepsilon) \leq -\varepsilon\]
\[-(L^*) (\varepsilon + \varepsilon^2) \leq \ln n - \varepsilon \sum L_t\]

And
\[\sum_t L_t \leq (1 + \varepsilon) L^* + \frac{\ln n}{\varepsilon}\]

\(\sum_t L_t\) is total expected loss of algorithm.
Within \((1 + \varepsilon)\) ish of the best expert!
Analysis

\[(1 - \varepsilon)^{L^*} \leq W(T) \leq n \prod_t (1 - \varepsilon L_t)\]

Take logs
\[ (L^*) \ln(1 - \varepsilon) \leq \ln n + \sum \ln(1 - \varepsilon L_t) \]

Use \[-\varepsilon - \varepsilon^2 \leq \ln(1 - \varepsilon) \leq -\varepsilon\]
\[ -(L^*)(\varepsilon + \varepsilon^2) \leq \ln n - \varepsilon \sum L_t \]

And
\[ \sum_t L_t \leq (1 + \varepsilon)L^* + \frac{\ln n}{\varepsilon}. \]

\[ \sum_t L_t \] is total expected loss of algorithm.

Within \((1 + \varepsilon)\) ish of the best expert!

No factor of 2 loss!
Gains.

Why so negative?
Gains.

Why so negative?
Each day, each expert gives gain in $[0, 1]$. 

$$G \geq (1 - \epsilon) G^* - \log n \epsilon$$ 

Scaling: Not $[0, 1]$, say $[0, \rho]$. 

$$L \leq (1 + \epsilon) L^* + \rho \log n \epsilon$$
Gains.

Why so negative?
Each day, each expert gives gain in $[0, 1]$.
Multiplicative weights with $(1 + \varepsilon)g_i^t$.
Gains.

Why so negative?
Each day, each expert gives gain in $[0, 1]$.
Multiplicative weights with $(1 + \varepsilon)^{g_i^t}$.

$$G \geq (1 - \varepsilon) G^* - \frac{\log n}{\varepsilon}$$

where $G^*$ is payoff of best expert.
Gains.

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Gains.

Why so negative?
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Multiplicative weights with \((1 + \varepsilon)^{g_i^t}\).

\[
\frac{G}{\varepsilon} \geq (1 - \varepsilon)G^* - \frac{\log n}{\varepsilon}
\]

where \(G^*\) is payoff of best expert.

Scaling:
Not \([0, 1]\), say \([0, \rho]\).

\[
\frac{L}{\varepsilon} \leq (1 + \varepsilon)L^* + \frac{\rho \log n}{\varepsilon}
\]
Summary: multiplicative weights.
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Framework: $n$ experts, each loses different amount every day.
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Perfect Expert: $\log n$ mistakes.
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Framework: $n$ experts, each loses different amount every day. 
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Summary: multiplicative weights.

Framework: $n$ experts, each loses different amount every day.
Perfect Expert: $\log n$ mistakes.
Imperfect Expert: best makes $m$ mistakes.
Deterministic Strategy: $2(1 + \varepsilon)m + \frac{\log n}{\varepsilon}$
Summary: multiplicative weights.

Framework: $n$ experts, each loses different amount every day.

Perfect Expert: $\log n$ mistakes.

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Multiplicative weights framework!
Applications next!