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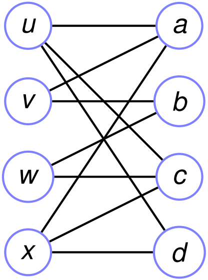

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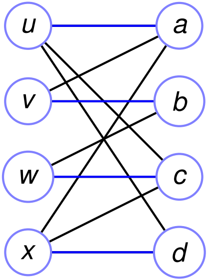

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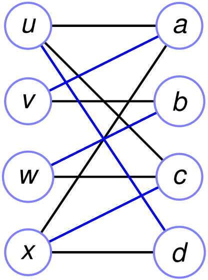

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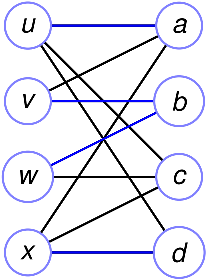

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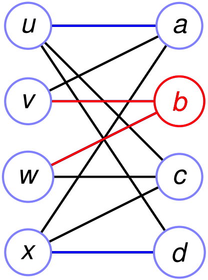

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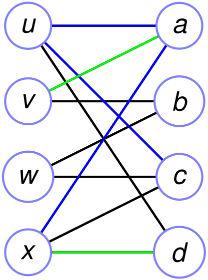

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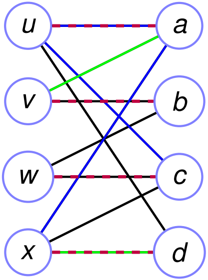

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Solution Value: 7.

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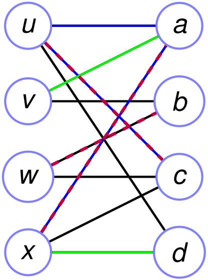

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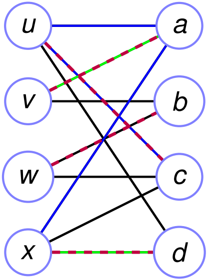

Blue – 3. Green - 2,
Black - 1, Non-edges - 0.

Solution Value: 7.

Solution Value: 7.

Solution Value: 8.

Applications

Jobs to workers.

Applications

Jobs to workers.

Teachers to classes.

Applications

Jobs to workers.

Teachers to classes.

Classes to classrooms.

Applications

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“The assignment problem”

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Min Weight Matching.

Applications

Jobs to workers.

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“The assignment problem”

Min Weight Matching.

Negate values and find maximum weight matching.

Vertex Cover

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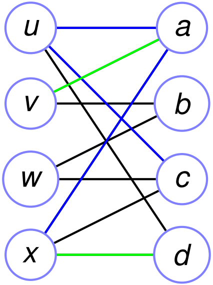
Minimize $\sum_{v \in U \cup V} p(u)$.



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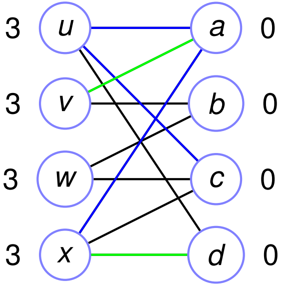

Solution Value: 12.

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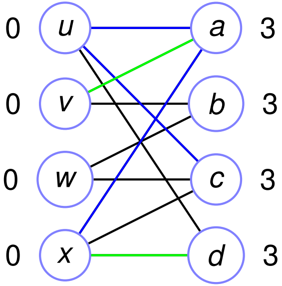

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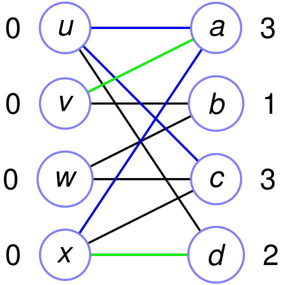

Solution Value: 12.

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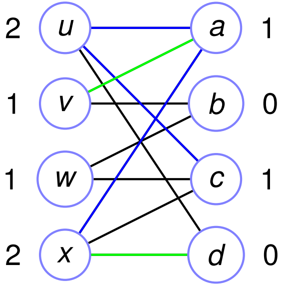

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Cover is upper bound.

Feasible $p(\cdot)$,

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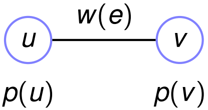

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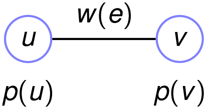
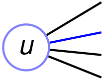


For a matching M, each u is the endpoint of at most one edge in M.



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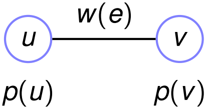

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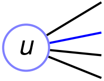

$$\sum_{e=(u,v)\in M} w(e)$$

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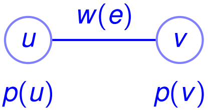

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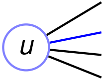

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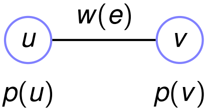

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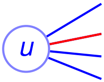

$$\sum_{e=(u,v)\in M} w(e) \leq \sum_{e=(u,v)\in M} (p(u)+p(v)) \leq \sum_{u\in U} p(u) + \sum_{v\in V} p(v)$$

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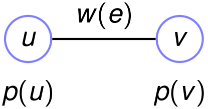

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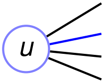

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Holds with equality if

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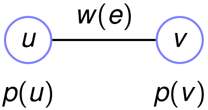

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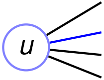

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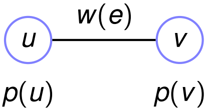

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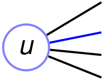

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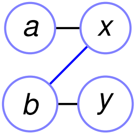
Holds with equality if

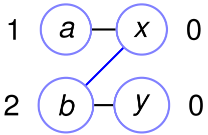
for $e \in M$, $w(e) = p(u) + p(v)$ (Defn: tight edge.) and perfect matching.

Simple example.



Blue edge – 2, Others – 1.

Simple example.



Blue edge – 2, Others – 1.

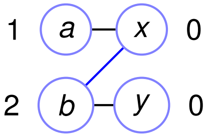
Using max incident edge.

Value: 3.
Using max incident edge.

Value: 2.
    Same as optimal matching!

Proof of optimality.

Simple example.



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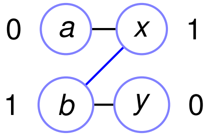

Value: 2.
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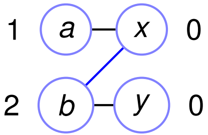

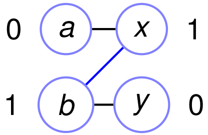

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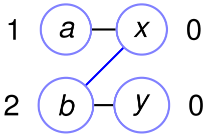

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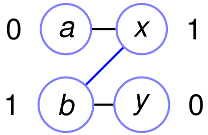

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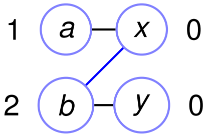

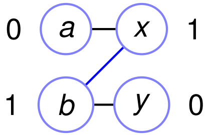

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Given a bipartite graph, $G = (U, V, E)$, find a maximum sized matching.

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Key Idea: Augmenting Alternating Paths.

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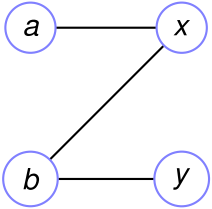

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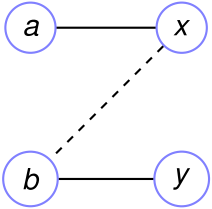

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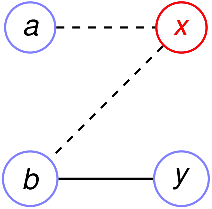

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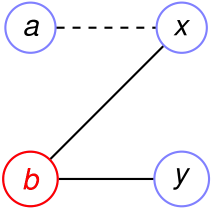

Start at unmatched node(s),
follow unmatched edge(s),
follow matched.
Repeat until an unmatched node.

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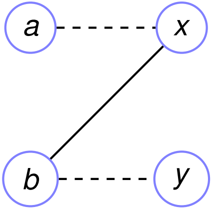

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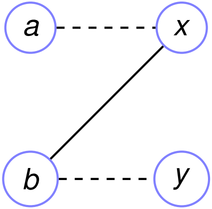

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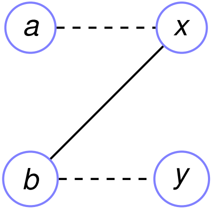

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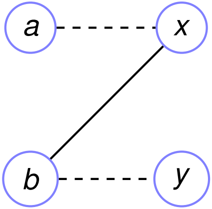


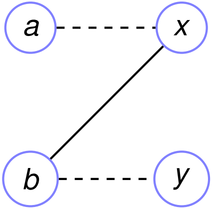
Start at unmatched node(s),
follow unmatched edge(s),
follow matched.

Maximum Matching

Given a bipartite graph, $G = (U, V, E)$, find a maximum sized matching.

Key Idea: Augmenting Alternating Paths.

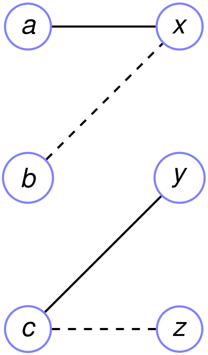
Example:



Start at unmatched node(s),
follow unmatched edge(s),
follow matched.
Repeat until an unmatched node.

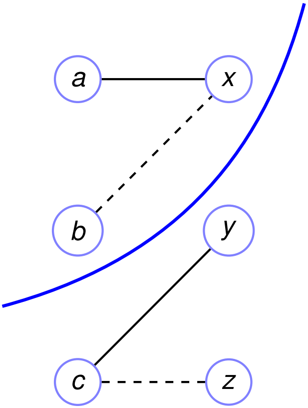
No perfect matching

No perfect matching



Can’t increase matching size.
No alternating path from (a) to (y).

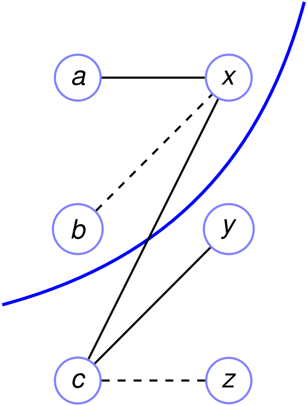
No perfect matching



Can’t increase matching size.
No alternating path from (a) to (y).

Cut!

No perfect matching

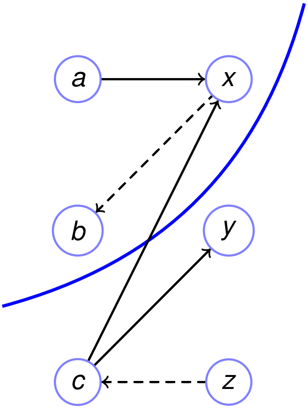


Can’t increase matching size.
No alternating path from (a) to (y).

Cut!

Still no augmenting path.
Still Cut?

No perfect matching



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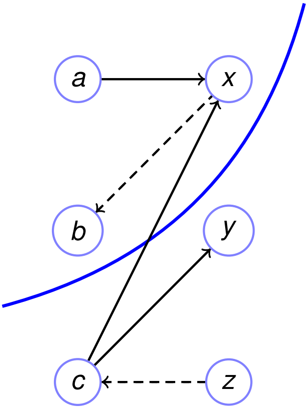
Still no augmenting path.
Still Cut?

Use directed graph!
Cut in this graph.

Algorithm:

No perfect matching



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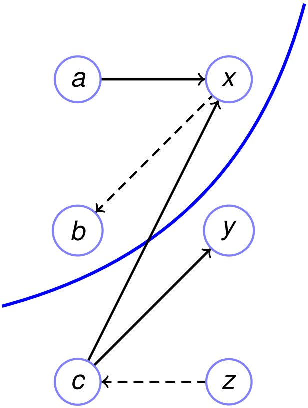
Algorithm:
Given matching.

No perfect matching



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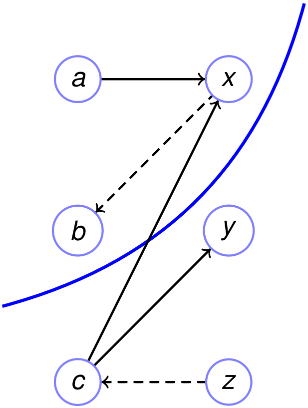
Cut in this graph.

Algorithm:
Given matching.
Direct unmatched edges U to V, matched V to U.

No perfect matching



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Cut in this graph.

Algorithm:
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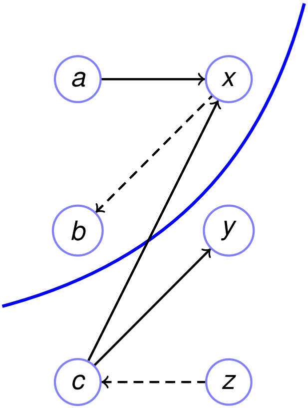
Direct unmatched edges U to V, matched V to U.
Find path between unmatched nodes on left to right. (BFS, DFS).

No perfect matching



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Algorithm:
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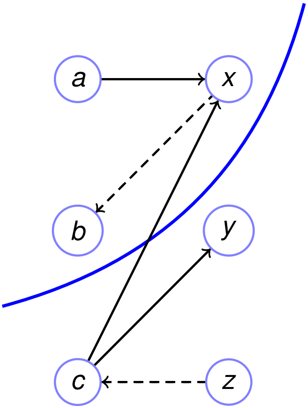
Find path between unmatched nodes on left to right. (BFS, DFS).
Until everything matched

No perfect matching



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No alternating path from (a) to (y).

Cut!

Still no augmenting path.
Still Cut?

Use directed graph!
Cut in this graph.

Algorithm:
Given matching.
Direct unmatched edges U to V, matched V to U.
Find path between unmatched nodes on left to right. (BFS, DFS).
Until everything matched ... or output a cut.

Back to Maximum Weight Matching.

Want vector cover (price function) $p(\cdot)$ and matching where.

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Optimal solutions to <u>both</u> if

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for $e \in M$, $w(e) = p(u) + p(v)$ (Defn: tight edge.) and

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Want vector cover (price function) $p(\cdot)$ and matching where.

Optimal solutions to both if

for $e \in M$, $w(e) = p(u) + p(v)$ (Defn: tight edge.) and
perfect matching.

Maximum Weight Matching

Goal: perfect matching on tight edges.

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Algorithm

Start with empty matching, feasible cover function ($p(\cdot)$)

Maximum Weight Matching

Goal: perfect matching on tight edges.

Algorithm

Start with empty matching, feasible cover function ($p(\cdot)$)

Add tight edges to matching.

Maximum Weight Matching

Goal: perfect matching on tight edges.

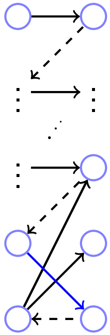
Algorithm

Start with empty matching, feasible cover function ($p(\cdot)$)

Add tight edges to matching.
  Use alt./aug. paths of tight edges.

Maximum Weight Matching

Goal: perfect matching on tight edges.



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Maximum Weight Matching

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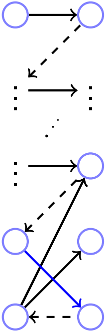

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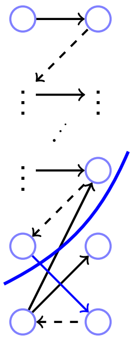

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No augmenting path.
  Cut, (S, T), in directed graph of tight edges!

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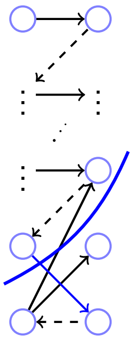
Goal: perfect matching on tight edges.



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All edges across cut are not tight. (loose?)

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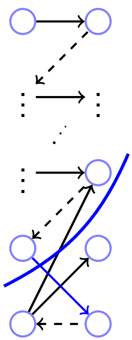

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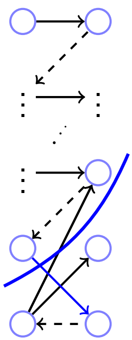
Nontight edges leaving cut, go from S_U, T_V.

Maximum Weight Matching

Goal: perfect matching on tight edges.



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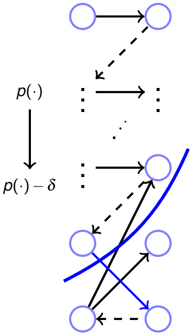

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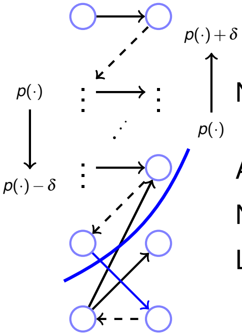
Nontight edges leaving cut, go from S_U, T_V.

Lower prices in S_U,

Maximum Weight Matching

Goal: perfect matching on tight edges.



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Add tight edges to matching.
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Lower prices in S_U, raise prices in S_V,

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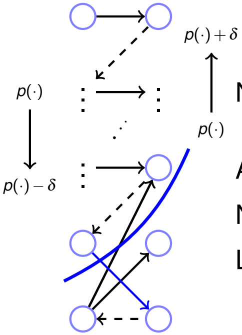

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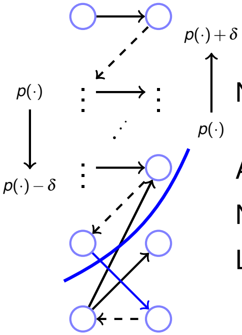
Nontight edges leaving cut, go from S_U, T_V.

Lower prices in S_U, raise prices in S_V,
  all explored edges still tight,
  backward edges still feasible

Maximum Weight Matching

Goal: perfect matching on tight edges.



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... and get new tight edge!

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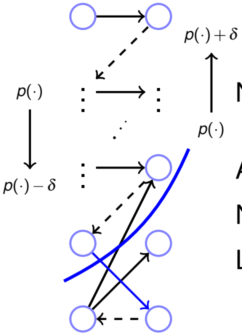

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What’s delta?

Maximum Weight Matching

Goal: perfect matching on tight edges.

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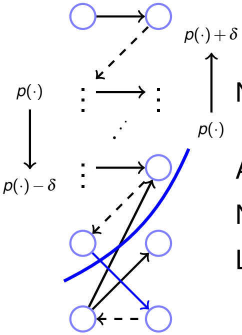

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... and get new tight edge!
What's delta? $w(e) < p(u) + p(v) \rightarrow$
$\delta = min_{e \in (S_U \times T_V)} p(u) + p(v) - w(e)$.

Lecture 2 ended in the middle of the previous slide.
A question was asked why don’t we just drop prices around the blue (loose) edge in the figure. Why not?

Some details

Add 0 value edges, so that optimal solution contains perfect matching.

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Beginning “Matcher” Solution: $M = \{\}$.

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Feasible!

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Feasible! Value = 0.

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Main Work:

Some details

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Main Work:
breadth first search from unmatched nodes finds cut.

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Main Work:
breadth first search from unmatched nodes finds cut.
Update prices (find minimum delta.)

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  Each bfs either augments or adds node to S in next cut.

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$O(n)$ iterations per augmentation.

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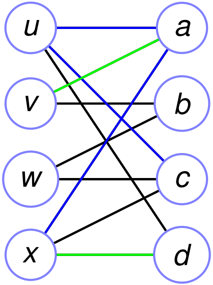
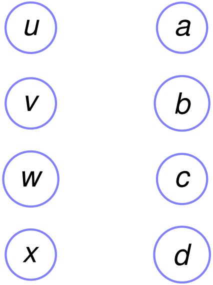
$O(n)$ iterations per augmentation.
$O(n)$ augmentations.

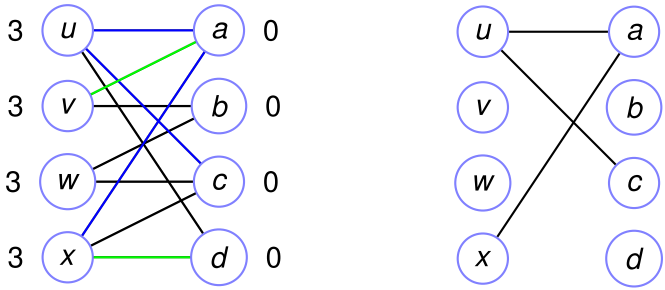
$O(n^2 m)$ time.

Example





Example



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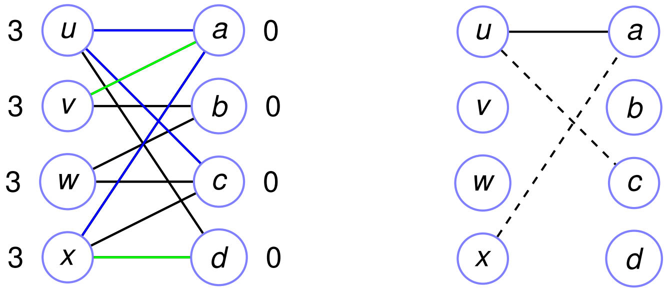

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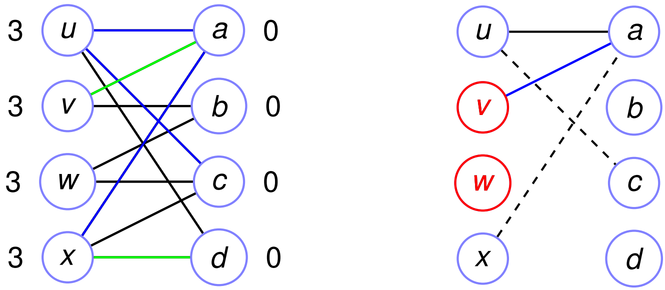

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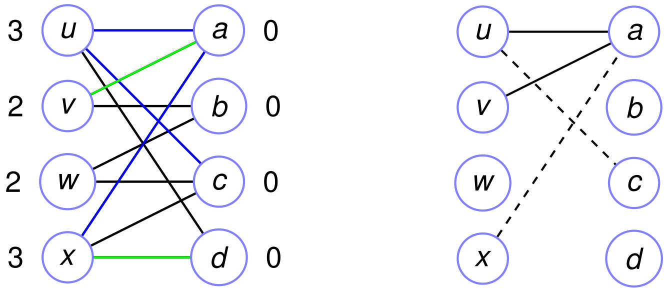

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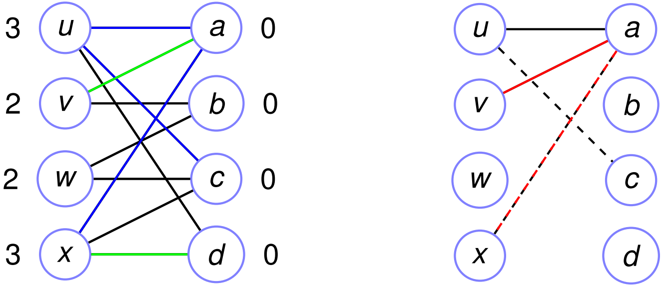

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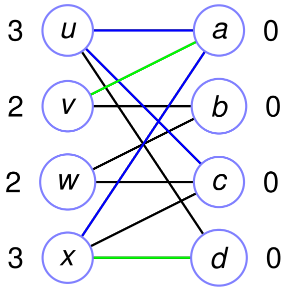

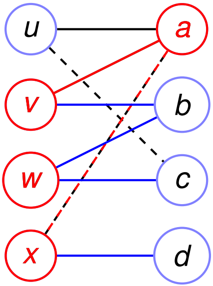

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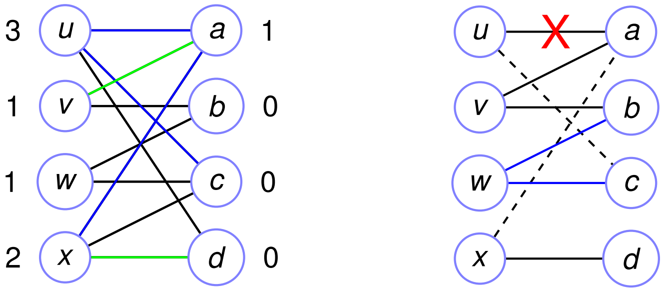

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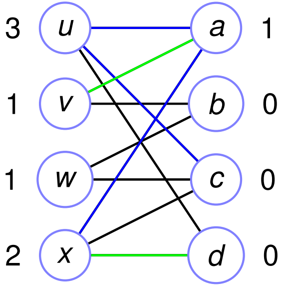

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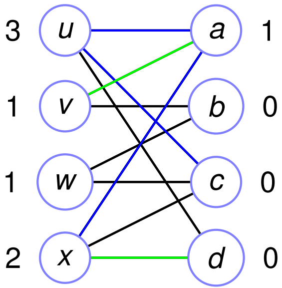

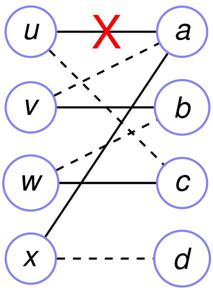

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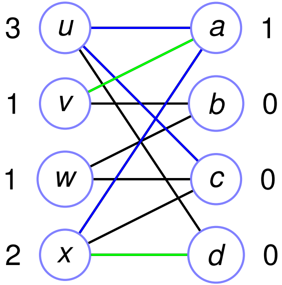
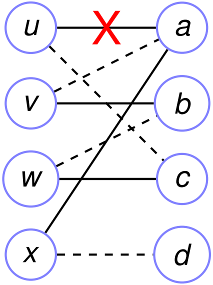


All matched edges tight.



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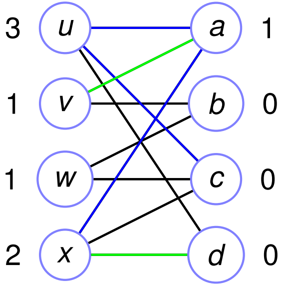
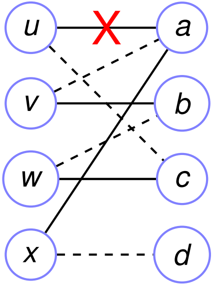
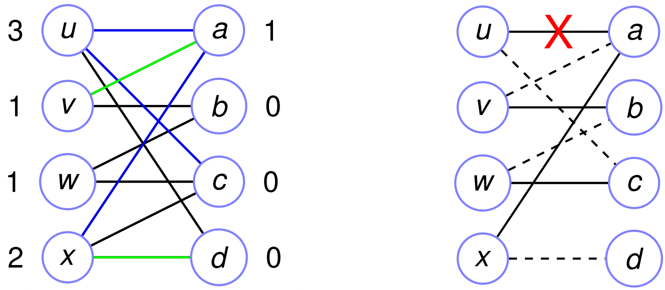


All matched edges tight.
Perfect matching.



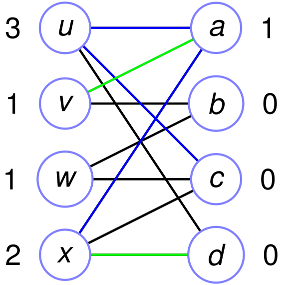
Example

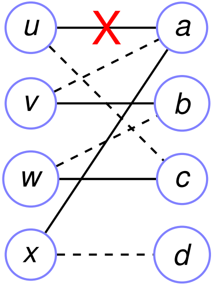


All matched edges tight.

Perfect matching. Feasible price function.

Example

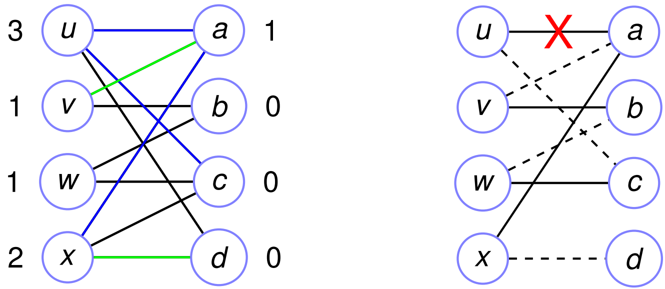




All matched edges tight.

Perfect matching. Feasible price function. Values the same.

Example

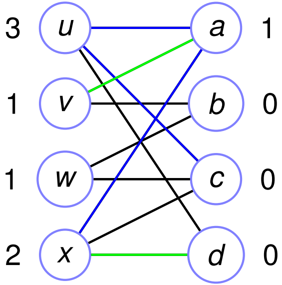


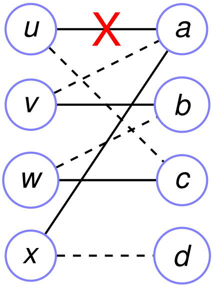
All matched edges tight.

Perfect matching. Feasible price function. Values the same.

Optimal!

Example





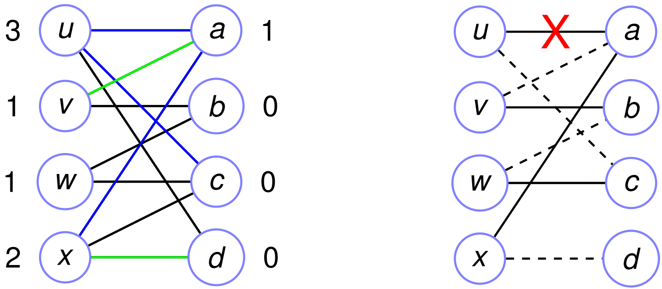
All matched edges tight.

Perfect matching. Feasible price function. Values the same.

Optimal!

Notice:

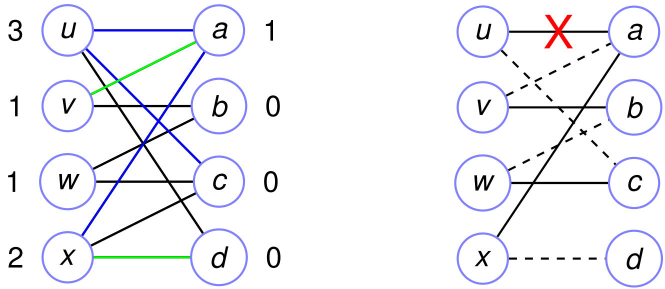
Example



All matched edges tight.
Perfect matching. Feasible price function. Values the same.
Optimal!

Notice:
no weights on the right problem.

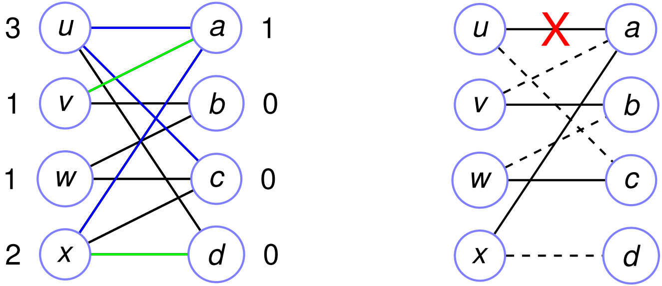
Example



All matched edges tight.
Perfect matching. Feasible price function. Values the same.
Optimal!

Notice:
- no weights on the right problem.
- retain previous matching through price changes.

Example



All matched edges tight.

Perfect matching. Feasible price function. Values the same.

Optimal!

Notice:

- no weights on the right problem.
- retain previous matching through price changes.
- retains edges in failed search through price changes.