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$\rightarrow$ symmetric diagonally dominant matrices by reduction.

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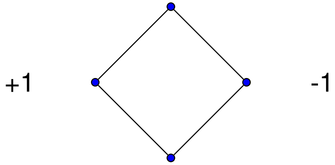

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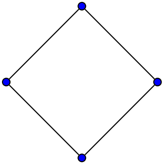
+1  -1

Claim: Minimizer is electrical flow.

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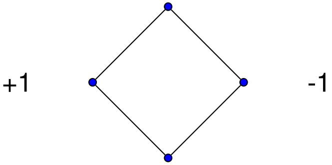

Claim: Minimizer is electrical flow.

Flow corresponds to flow induced by a set of potentials.

Some Matrices.

Given $G = (V, E)$, arbitrarily orient edges.

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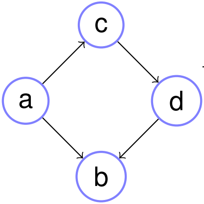

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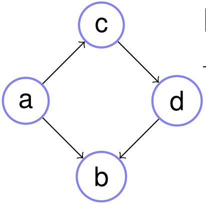

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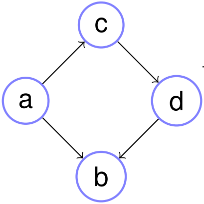

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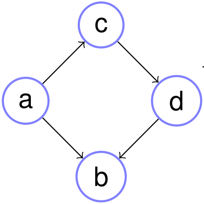

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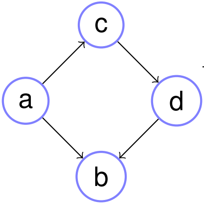

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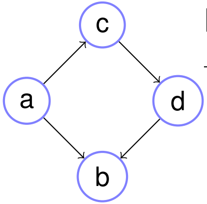

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Minimize Squared Potential differences!

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Algorithm: Work on flow and potentials.
To drive gap to 0.

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Given: χ, G

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Take a spanning tree T of G.

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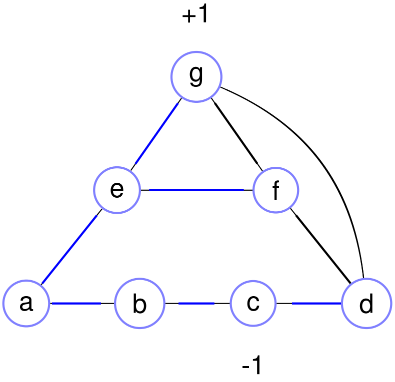
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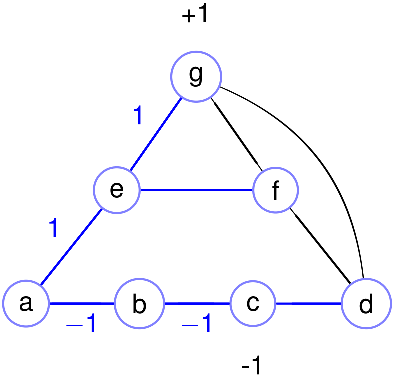

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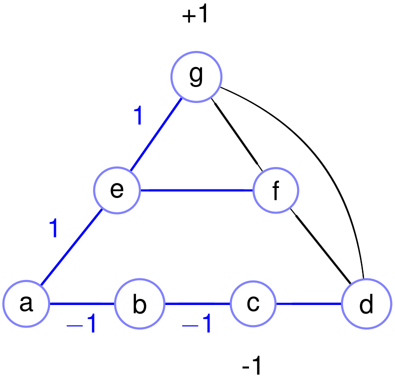

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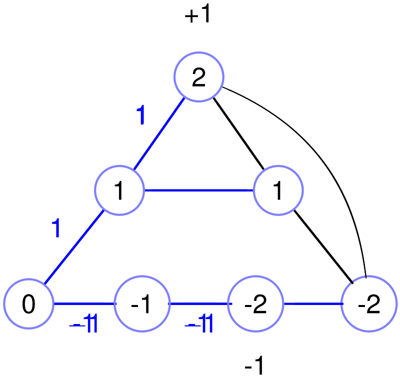

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-2
-1
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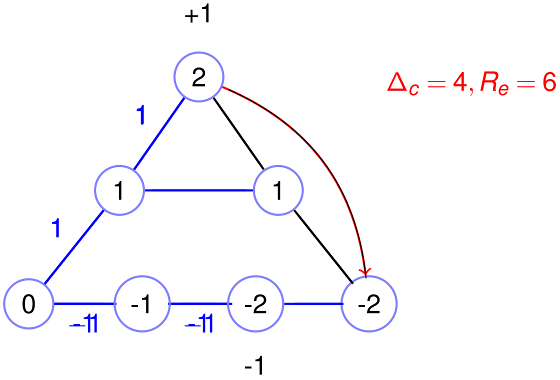
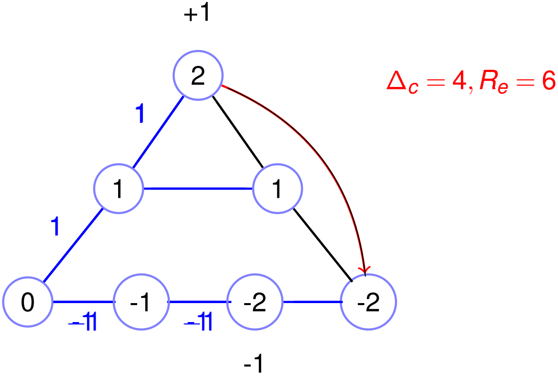
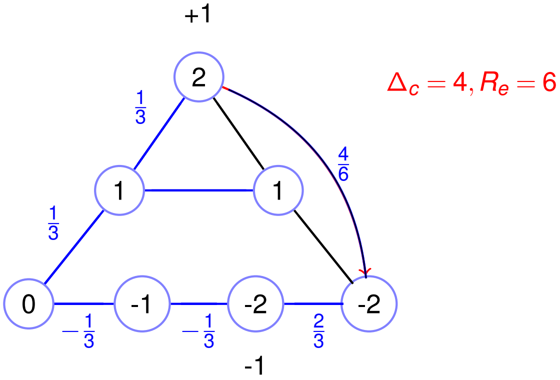
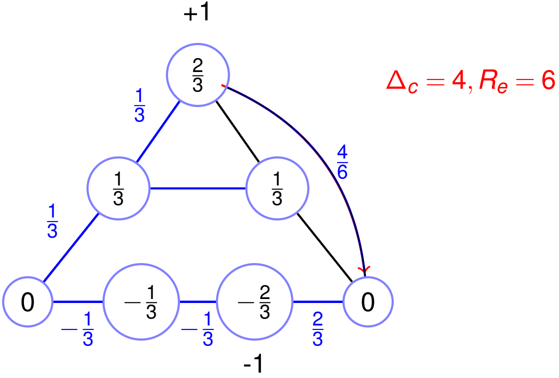


+1
2
$\Delta_c = 4, R_e = 6$
1
1
1
1
0
-1
-2
-2
-1
-1
-1



+1
2
1
1
1
1
0
-1
-2
-2
-1
$\Delta_c = 4, R_e = 6$


+1
2
1/3
$\Delta_c = 4, R_e = 6$
4/6
1
1
1/3
0
-1
-2
-2
$-\frac{1}{3}$
$-\frac{1}{3}$
$\frac{2}{3}$
-1



+1
2/3
1/3
1/3
1/3
1/3
4/6
0
−1/3
−2/3
0
−1/3
−1/3
2/3
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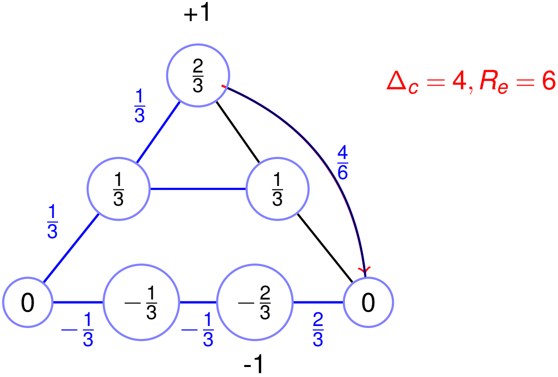

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1/3
1/3
1/3
1/3
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Energy reduction.

Given T,$e = (u,v)$, let $R_e = l_T(u,v) + 1$.

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(assume e' are oriented around cycle.)

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Difference in energy from f and f'.

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$\sum_{e' \in C_e} (f(e') - \delta)^2 - (f(e'))^2 = \sum_{e' \in C_e} -2f(e')\delta + \delta^2$

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Fix $1/R_e$ of a cycle violation!

Duality Gap?

Algorithm maintains feasible ϕ, f, $(B^T f = \chi)$

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Primal value: $|f|^2$.
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Total distance from optimal is cycle violations!

Claim: $E[\text{change in energy}|\mathit{Gap}] = \frac{\mathit{Gap}}{\tau}$

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Duality Gap: $\sum_{e \notin T} \Delta_{C_e}(f)^2$

Choose edge e reduce energy by $-\frac{\Delta^2_{C_e}}{R_e}$.

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Expected reduction $-\sum_e \frac{R_e}{\tau} \frac{\Delta^2_{c_e}}{R_e} = -\sum_e \frac{\Delta^2_{c_e}}{\tau}$ □

Duality Gap reduces by $(1 - 1/\tau)$ every iteration on expectation.

$O(\tau \log(n/\varepsilon))$ iterations gives $(1+\varepsilon)$ approximation.

$\tau = O(m \log n \log\log n)$ ...

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See you ...

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