Welcome back...

Welcome back...

..to me.

Welcome back...

..to me.

Test out

Welcome back...

..to me.

Test out !

Welcome back...

..to me.

Test out !!

Welcome back...

..to me.

Test out !!!

Welcome back...

..to me.

Test out !!!

Don’t worry.

Welcome back...

..to me.

Test out !!!

Don't worry. Be happy.

Welcome back...

..to me.

Test out !!!

Don't worry. Be happy.

Look at instructions.

Welcome back...

..to me.

Test out !!!

Don’t worry. Be happy.

Look at instructions.
No collaboration.

Welcome back...

..to me.

Test out !!!

Don't worry. Be happy.

Look at instructions.
No collaboration.
Private message on piazza.

Welcome back...

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Look at instructions.
No collaboration.
Private message on piazza.
Note: Content can be declassified.

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Turn in by Monday.

Welcome back...

..to me.

Test out !!!

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Look at instructions.
No collaboration.
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Turn in by Monday.
Grade by Wednesday

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Turn in by Monday.
Grade by Wednesday .. night

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Turn in by Monday.
Grade by Wednesday .. night ...late

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Turn in by Monday.
Grade by Wednesday .. night ...late ..hopefully.

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Don't worry. Be happy.

Look at instructions.
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Private message on piazza.
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Turn in by Monday.
Grade by Wednesday .. night ...late ..hopefully.

Try to get it in then or soon after!

Pareto:

Pareto:

20% of pods have 80% of peas.

Pareto:

20% of pods have 80% of peas.
20% of peple have 80% of land.

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Pareto:

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City populations:

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City populations:

*i*th largest city has population $\frac{p_1}{i}$.

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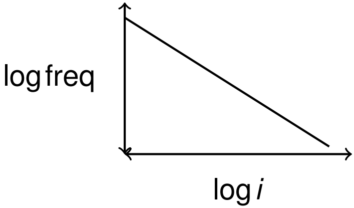

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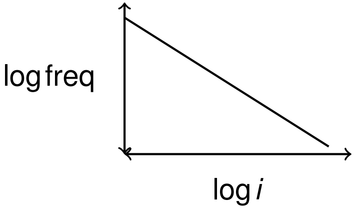

Zipf's law.

Pareto:

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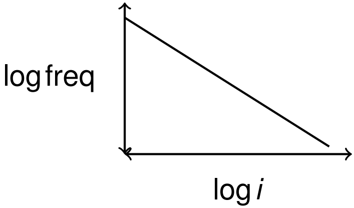

Zipf's law. Zipf's graph.

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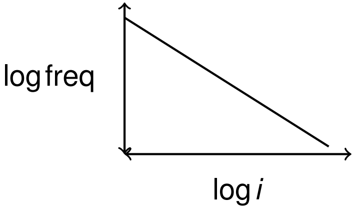

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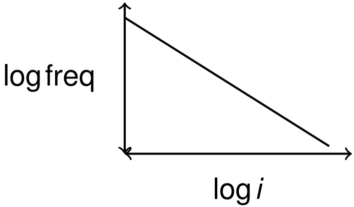

Zipf's law. Zipf's graph.

Not a distribution.

As a distribution.

Pareto.

As a distribution.

Pareto.

$\text{Income}_i \propto \frac{\text{income}_1}{i^\beta}$.

As a distribution.

Pareto.

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As a distribution.

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Bill Gates...then someone much less.

As a distribution.

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Bill Gates...then someone much less. Prelude: why?

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Bill Gates...then someone much less. Prelude: why? Rich get richer?

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Distribution:

  Pareto.

    $Pr[X \geq x] \propto x^{-\alpha+1}$.

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Distribution:

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      Survival function.

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Note: “p.d.f.”

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Note: “p.d.f.” $Pr[X = x] \propto x^{-alpha}$.

See Adamic for comment on estimating for real data.

http://www.hpl.hp.com/research/idl/papers/ranking/ranking.html

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MAKE SOME DRAWINGS.

Pareto to Zipf

Zipf:

Pareto to Zipf

Zipf:

ith guy has $C\frac{1}{i^{\beta}}$

Pareto to Zipf

Zipf:

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Pareto to Zipf

Zipf:
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N people.

Pareto to Zipf

Zipf:
ith guy has $C\frac{1}{i^\beta}$

N people.
How many people have value more than x_i?

Pareto to Zipf

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How many people have value more than x_i?

On expection?

Pareto to Zipf

Zipf:
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On expection? $NDx^{-\alpha+1}$.

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$x_i = \frac{1}{i^{1/(1-\alpha)}}$

Pareto to Zipf

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$i \approx NDx_i^{-\alpha+1}$

$x_i = \frac{1}{i^{1/(1-\alpha)}}$

Relationship: $\beta = \frac{1}{1-\alpha}$

Self similarity.

Power laws.

Self similarity.

Power laws.

No matter where you are there you are...

Self similarity.

Power laws.

No matter where you are there you are...

$x_{t+1} = x_t \times \gamma$.

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No matter where you are there you are...

$x_{t+1} = x_t \times \gamma$.

Actually $\gamma_t \approx (1 + \beta/t)$.

Self similarity.

Power laws.

No matter where you are there you are...

$x_{t+1} = x_t \times \gamma$.

Actually $\gamma_t \approx (1 + \beta/t)$.

Roughly constant for interval of wdith β.

Power law and philosophy.

Wow!

Power law and philosophy.

Wow! Power laws.

Power law and philosophy.

Wow! Power laws. Cool.

Power law and philosophy.

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Zipf: for frequency of words.

Power law and philosophy.

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Zipf: for frequency of words. For all languages!!!

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Must have something to do with the brain!

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Permute the letters at random..

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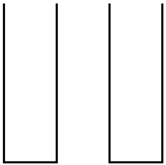
Polya Urns



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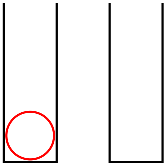

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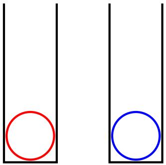

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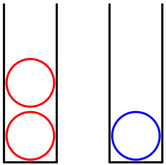

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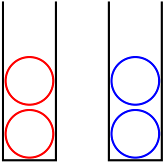

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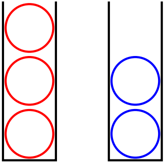
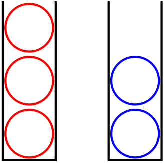


Choose bin uniformly at random.

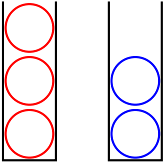
Polya Urns



Choose bin uniformly at random.

Load on red bin?

Polya Urns



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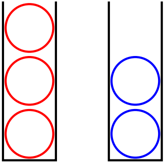
Expectation?

Polya Urns

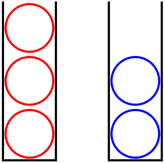


Choose bin uniformly at random.

Load on red bin?

Expectation? $n/2$

Polya Urns



Choose bin uniformly at random.

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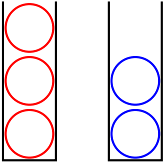
Expectation? $n/2$

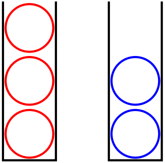
Within $n/2 \pm \sqrt{n}$ with good probability.

Polya Urns



Choose bin uniformly at random.
  Load on red bin?
  Expectation? $n/2$
  Within $n/2 \pm \sqrt{n}$ with good probability.
  Approximately Gaussian with variance $\sqrt{n}/2$

Polya Urns



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Choose red bin with probability $\frac{r+1}{r+b+2}$

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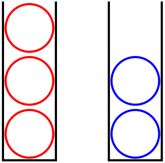

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  Expectation?

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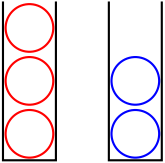

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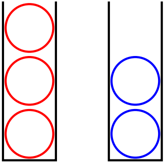

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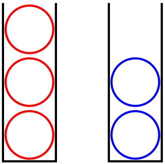
Choose red bin with probability $\frac{r+1}{r+b+2}$
  Expectation? $n/2$
  Distribution?

Polya Urns



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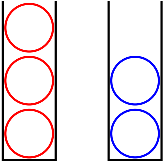
 Distribution?
 Guesses?

Polya Urns



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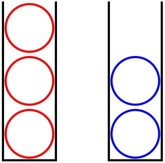
Distribution?
Guesses?
Uniform

Polya Urns



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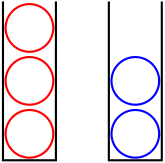
Distribution?
Guesses?
Uniform!

Polya Urns



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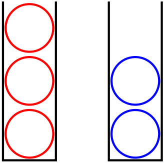
Distribution?
Guesses?
Uniform! !

Polya Urns



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- Guesses?
- Uniform! !!

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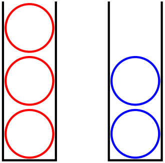

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Permutations

Choose bin with probability $\frac{r+1}{r+b+2}$.

Permutations

Choose bin with probability $\frac{r+1}{r+b+2}$.

Claim: After n balls the $Pr[i \text{ red}] = \frac{1}{n+1}$.

Permutations

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Claim: After n balls the $Pr[i \text{ red}] = \frac{1}{n+1}$.

Analyse?

Permutations

Choose bin with probability $\frac{r+1}{r+b+2}$.

Claim: After n balls the $Pr[i \text{ red}] = \frac{1}{n+1}$.

Analyse?Another process.

Permutations

Choose bin with probability $\frac{r+1}{r+b+2}$.

Claim: After n balls the $Pr[i \text{ red}] = \frac{1}{n+1}$.

Analyse?Another process.

Start with two balls, insert n more.

Permutations

Choose bin with probability $\frac{r+1}{r+b+2}$.

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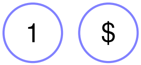

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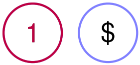

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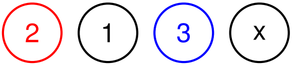

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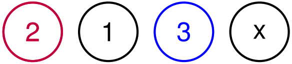

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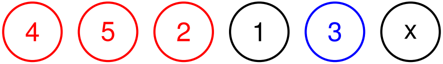

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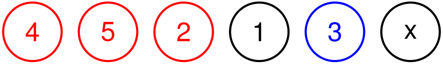

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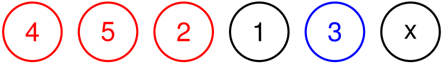

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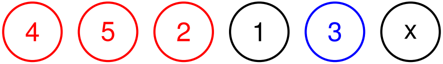

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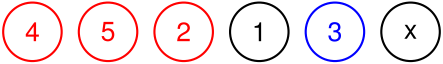

Where is ball 1? Position 4.

How many red balls? 3.

Permutations

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Claim: After n balls the $Pr[i \text{ red}] = \frac{1}{n+1}$.

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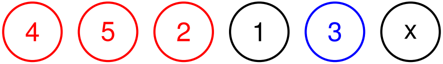

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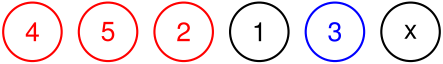

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Random permuation.

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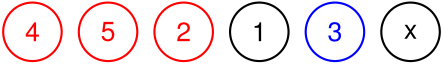

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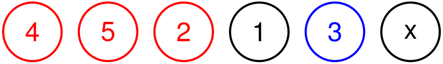

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Random permuation. Position $i \in [1, n+1]$

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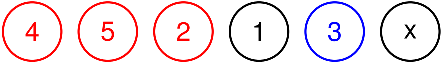

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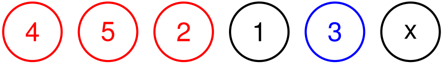

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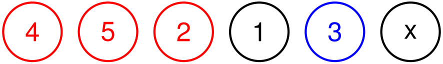

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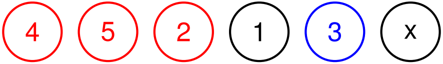

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How many red balls? $j = i - 1 \in [0, n]$

Permutations

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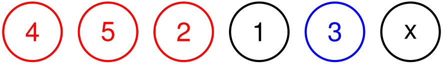

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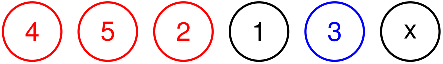

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Balls in bins?

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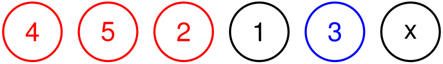

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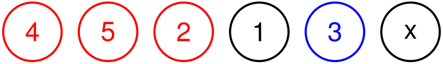

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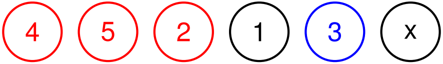

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choose one of $r+b$ balls or 2 bottoms.

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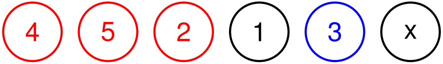

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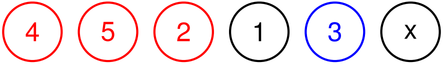

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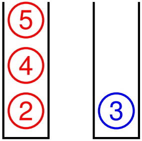

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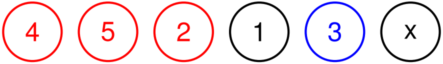

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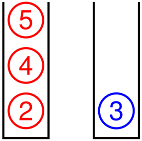

Red balls have same distribution in two processes.

More bins.

m bins.

More bins.

m bins.

More bins.

m bins.

Uniformly at random.

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Max load: $\frac{n}{m} + \sqrt{\frac{n}{m} \log n}$

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Analysis: random permutation with m separators.

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Analyse min and max size of interval.

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Roughly: $(1/m)$ probability of stopping at any point.

More bins.

m bins.

Uniformly at random.

Max load: $\frac{n}{m} + \sqrt{\frac{n}{m} \log n}$

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Router Graph:

Router Graph:

- Average degree: 4
- Max Degree?

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- Max Degree?   Uniformly random

Router Graph:

Average degree: 4

Max Degree? Uniformly random $\Longrightarrow$ $Pr[\text{degree } \geq 20] \approx 10^{-4}$.

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5% of nodes have degree greater than 20.

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Internet graph:

Router Graph:

Average degree: 4

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Internet graph:

Average degree: 12.

Router Graph:

Average degree: 4

Max Degree? Uniformly random $\implies$ Pr[degree $\geq 20] \approx 10^{-4}$.

Actual high degree nodes more common:

5% of nodes have degree greater than 20.

Internet graph:

Average degree: 12.

Degree ≥ 100 with prob. $\leq 10^{-6}$.

Actual:

Router Graph:
  Average degree: 4
  Max Degree? Uniformly random $\Longrightarrow$ $Pr[\text{degree } \geq 20] \approx 10^{-4}$.
Actual high degree nodes more common:
  5% of nodes have degree greater than 20.

Internet graph:
  Average degree: 12.
  Degree $\geq$ 100 with prob. $\leq 10^{-6}$.
  Actual: 1% greater than 100.
  Some very large.

Router Graph:

Average degree: 4

Max Degree? Uniformly random $\implies$ $Pr[\text{degree } \geq 20] \approx 10^{-4}$.

Actual high degree nodes more common:

5% of nodes have degree greater than 20.

Internet graph:

Average degree: 12.

Degree ≥ 100 with prob. $\leq 10^{-6}$.

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Some very large.

Processes?

Router Graph:

Average degree: 4

Max Degree? Uniformly random $\implies$ Pr[degree $\geq 20] \approx 10^{-4}$.

Actual high degree nodes more common:

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Internet graph:

Average degree: 12.

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Processes?

Preferential Attachment.

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Internet graph:

Average degree: 12.

Degree ≥ 100 with prob. $\leq 10^{-6}$.

Actual: 1% greater than 100.

Some very large.

Processes?

Preferential Attachment.

For routers?

Router Graph:

Average degree: 4

Max Degree? Uniformly random $\implies$ *Pr*[degree $\geq 20] \approx 10^{-4}$.

Actual high degree nodes more common:

5% of nodes have degree greater than 20.

Internet graph:

Average degree: 12.

Degree ≥ 100 with prob. $\leq 10^{-6}$.

Actual: 1% greater than 100.

Some very large.

Processes?

Preferential Attachment.

For routers?

Connect at random.

Router Graph:

Average degree: 4

Max Degree? Uniformly random $\implies$ $Pr[\text{degree } \geq 20] \approx 10^{-4}$.

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Average degree: 12.

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Processes?

Preferential Attachment.

For routers?

Connect at random. Not!

For the internet graph?

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Processes?

Preferential Attachment.

For routers?

Connect at random. Not!

For the internet graph?

Degrees too large for even that.

Internet: copy links.

Surf.

Internet: copy links.

Surf. Cool page.

Internet: copy links.

Surf. Cool page. Link for mine.

Internet: copy links.

Surf. Cool page. Link for mine.

Model:

Internet: copy links.

Surf. Cool page. Link for mine.

Model:
  Pick a random neighbor.

Internet: copy links.

Surf. Cool page. Link for mine.

Model:
  Pick a random neighbor.
  Copy all links.

Internet: copy links.

Surf. Cool page. Link for mine.

Model:
  Pick a random neighbor.
  Copy all links.

Random Graph with average degree 4.

Internet: copy links.

Surf. Cool page. Link for mine.

Model:
  Pick a random neighbor.
  Copy all links.

Random Graph with average degree 4.
  Plus Copy process

Internet: copy links.

Surf. Cool page. Link for mine.

Model:
  Pick a random neighbor.
  Copy all links.

Random Graph with average degree 4.
  Plus Copy process $\rightarrow \sqrt{n}$

Routers.

Connection Game.

Routers.

Connection Game.

Process Distance:

Routers.

Connection Game.

Process Distance:
  Arrive randomly at point on unit square.

Routers.

Connection Game.

Process Distance:
 Arrive randomly at point on unit square.
 Connect to closest node.

Routers.

Connection Game.

Process Distance:
  Arrive randomly at point on unit square.
  Connect to closest node.

Generate tree with average degree 1.

Routers.

Connection Game.

Process Distance:
  Arrive randomly at point on unit square.
  Connect to closest node.

Generate tree with average degree 1.
  Max degree?

Routers.

Connection Game.

Process Distance:
  Arrive randomly at point on unit square.
  Connect to closest node.

Generate tree with average degree 1.
  Max degree? $O(\log n)$.

Routers.

Connection Game.

Process Distance:
  Arrive randomly at point on unit square.
  Connect to closest node.

Generate tree with average degree 1.
  Max degree? $O(\log n)$.

Process Hops:

Routers.

Connection Game.

Process Distance:
Arrive randomly at point on unit square.
Connect to closest node.

Generate tree with average degree 1.
Max degree? $O(\log n)$.

Process Hops:
Arrive randomly at point on unit square.

Routers.

Connection Game.

Process Distance:
  Arrive randomly at point on unit square.
  Connect to closest node.

Generate tree with average degree 1.
  Max degree? $O(\log n)$.

Process Hops:
  Arrive randomly at point on unit square.
  Connect to first node.

Routers.

Connection Game.

Process Distance:
  Arrive randomly at point on unit square.
  Connect to closest node.

Generate tree with average degree 1.
  Max degree? $O(\log n)$.

Process Hops:
  Arrive randomly at point on unit square.
  Connect to first node.

Max degree?

Routers.

Connection Game.

Process Distance:
Arrive randomly at point on unit square.
Connect to closest node.

Generate tree with average degree 1.
Max degree? $O(\log n)$.

Process Hops:
Arrive randomly at point on unit square.
Connect to first node.

Max degree? $n - 1$.

Routers.

Connection Game.

Process Distance:
  Arrive randomly at point on unit square.
  Connect to closest node.

Generate tree with average degree 1.
  Max degree? $O(\log n)$.

Process Hops:
  Arrive randomly at point on unit square.
  Connect to first node.

Max degree? $n-1$.

Process Distance/Hops:

Routers.

Connection Game.

Process Distance:
  Arrive randomly at point on unit square.
  Connect to closest node.

Generate tree with average degree 1.
  Max degree? $O(\log n)$.

Process Hops:
  Arrive randomly at point on unit square.
  Connect to first node.

Max degree? $n-1$.

Process Distance/Hops:
  Arrive randomly at point on unit square.

Routers.

Connection Game.

Process Distance:
  Arrive randomly at point on unit square.
  Connect to closest node.

Generate tree with average degree 1.
  Max degree? $O(\log n)$.

Process Hops:
  Arrive randomly at point on unit square.
  Connect to first node.

Max degree? $n-1$.

Process Distance/Hops:
  Arrive randomly at point on unit square.
  Connect to node with $\min_{j<i} \alpha d_{ij} + h_j$.

Routers.

Connection Game.

Process Distance:
  Arrive randomly at point on unit square.
  Connect to closest node.

Generate tree with average degree 1.
  Max degree? $O(\log n)$.

Process Hops:
  Arrive randomly at point on unit square.
  Connect to first node.

Max degree? $n-1$.

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Power law if $c \leq \alpha \leq \sqrt{n}$,

Routers.

Connection Game.

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 Arrive randomly at point on unit square.
 Connect to closest node.

Generate tree with average degree 1.
 Max degree? $O(\log n)$.

Process Hops:
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Process Distance/Hops:
 Arrive randomly at point on unit square.
 Connect to node with $\min_{j<i} \alpha d_{ij} + h_j$.

Power law if $c \leq \alpha \leq \sqrt{n}$, $\rightarrow$ power law!

See you ...

Thursday.