Welcome back.

Projects comments available on Glookup!

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Turn in homework!

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I am away April 15-20.

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Midterm out when I get back.

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- Few days takehome.
- Shiftable.

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Have handle on projects before that.

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Turn in homework!

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- Few days takehome.
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Have handle on projects before that.

Progress report due Monday.

Two populations.

DNA data:

Two populations.

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human1: A $\cdots$ C $\cdots$ T $\cdots$ A

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DNA data:

human1: A $\cdots$ C $\cdots$ T $\cdots$ A
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Single Nucleotide Polymorphism.

Two populations.

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Same population?

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Same population?

Model: same populution breeds.

Two populations.

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Same population?

Model: same populution breeds.

Population 1: snp 843: Pr[A] = .4 , Pr[T] = .6
Population 2: snp 843: Pr[A] = .6 , Pr[T] = .4

Two populations.

DNA data:

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Individual: $x_1, x_2, x_3.., x_n$.

Two populations.

DNA data:

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Which population?

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Which population?

Comment: snps could be movie preferences, populations could be types.

Two populations.

DNA data:

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Which population?

Comment: snps could be movie preferences, populations could be types.

E.g., republican/democrat, shopper/saver.

Which population?

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Population 1: snp i: Pr[$x_i = 1$] = $p_i^{(1)}$
Population 2: snp i: Pr[$x_i = 0$] = $p_i^{(2)}$

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Simpler Calculation:

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Simpler Calculation:

Population 1: Gaussion with mean $\mu_1 \in R^d$, variance σ in each dim.
Population 2: Gaussion with mean $\mu_2 \in R^d$, variance σ in each dim.

Which population?

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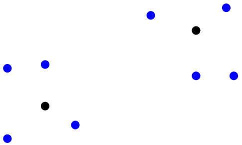

Gaussians

Population 1: Gaussion with mean $\mu_1 \in R^d$, std deviation σ in each dim.

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Difference between humans σ per snp.
Difference between populations ε per snp.

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How many snps to collect to determine population for individual x?

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$E[(x-\mu_1)^2] = d\sigma^2$

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If $(\mu_1-\mu_2)^2 = d\varepsilon^2 >> \sigma^2$, then different.

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Variance of estimator?

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Variance of estimator?
Roughly $d\sigma^4$.

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Roughly $d\sigma^4$.
Signal is difference between expecations.

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Roughly $d\sigma^4$.
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roughly $d\varepsilon^2$
Signal $>>$ Noise. $\leftrightarrow d\varepsilon^2 >> \sqrt{d}\sigma^2$.

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Variance of estimator?
  Roughly $d\sigma^4$.
  Signal is difference between expecations.
    roughly $d\varepsilon^2$
  Signal $>>$ Noise. $\leftrightarrow d\varepsilon^2 >> \sqrt{d}\sigma^2$.

Need $d >> \sigma^4/\varepsilon^4$

Projection

Population 1: Gaussion with mean $\mu_1 \in R^d$, variance σ in each dim.
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Project x onto unit vector v in direction $\mu_2 - \mu_1$.

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$E[((x - \mu_1) \cdot v)^2] = 0$ if x is population 1.

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Std deviation is σ^2!

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Std deviation is σ^2! versus $\sqrt{d}\sigma^2$!

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Std deviation is σ^2! versus $\sqrt{d}\sigma^2$!

No loss in signal!

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$d\varepsilon^2 >> \sigma^2$.

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$d\varepsilon^2 >> \sigma^2$.
$\rightarrow d >> \sigma^2/\varepsilon^2$

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$d\varepsilon^2 >> \sigma^2$.
$\rightarrow d >> \sigma^2/\varepsilon^2$

Versus $d >> \sigma^4/\varepsilon^4$.

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Std deviation is σ^2! versus $\sqrt{d}\sigma^2$!

No loss in signal!

$d\varepsilon^2 >> \sigma^2$.
$\rightarrow d >> \sigma^2/\varepsilon^2$

Versus $d >> \sigma^4/\varepsilon^4$.

A quadratic difference in amount of data!

Don’t know much about...

Don’t know μ_1 or μ_2?

Without the means?

Sample of n people.

Without the means?

Sample of n people.

Some (say half) from population 1,

Without the means?

Sample of n people.

Some (say half) from population 1,

some from population 2.

Without the means?

Sample of n people.

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Which are which?

Without the means?

Sample of n people.

Some (say half) from population 1,

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Which are which?

Near Neighbors Approach

Without the means?

Sample of *n* people.

Some (say half) from population 1,

some from population 2.

Which are which?

Near Neighbors Approach

Compute Euclidean distance squared.

Without the means?

Sample of n people.
Some (say half) from population 1,
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Which are which?

Near Neighbors Approach

Compute Euclidean distance squared.
Cluster using threshold.

Without the means?

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Some (say half) from population 1,
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Near Neighbors Approach

Compute Euclidean distance squared.
Cluster using threshold.

Signal $E[d(x_1, x_2)] - E[d(x_1, y_1)]$

Without the means?

Sample of n people.
  Some (say half) from population 1,
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Which are which?

Near Neighbors Approach

Compute Euclidean distance squared.
  Cluster using threshold.

Signal $E[d(x_1, x_2)] - E[d(x_1, y_1)]$
  should be larger than noise in $d(x, y)$

Without the means?

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Where x's from one population, y's from other.

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Signal is proportional $d\varepsilon^2$.

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$d >> \sigma^4/\varepsilon^4 \quad \rightarrow$ same type people closer to each other.

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$d >> \sigma^4/\varepsilon^4$ $\rightarrow$ same type people closer to each other.

$d >> (\sigma^4/\varepsilon^4)\log n$ suffices for threshold clustering.

Without the means?

Sample of n people.
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Near Neighbors Approach

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$d >> \sigma^4/\varepsilon^4 \quad \rightarrow$ same type people closer to each other.

$d >> (\sigma^4/\varepsilon^4) \log n$ suffices for threshold clustering.

$\log n$ factor for union bound over $\binom{n}{2}$ pairs.

Without the means?

Sample of n people.
Some (say half) from population 1,
some from population 2.

Which are which?

Near Neighbors Approach

Compute Euclidean distance squared.
Cluster using threshold.

Signal $E[d(x_1,x_2)] - E[d(x_1,y_1)]$
should be larger than noise in $d(x,y)$
Where x's from one population, y's from other.

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Best one can do?

Principal components analysis.

Remember Projection!

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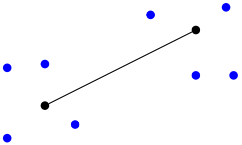

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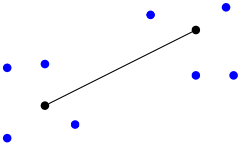

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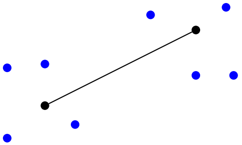

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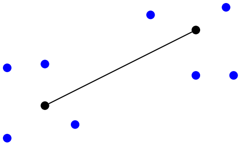

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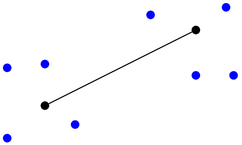

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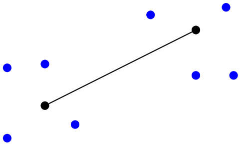

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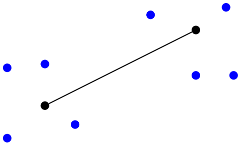

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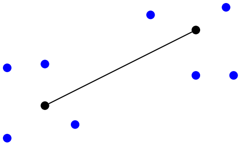

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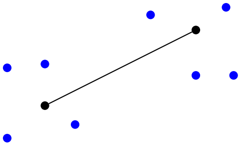

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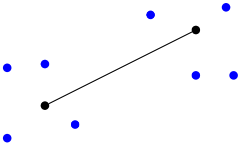

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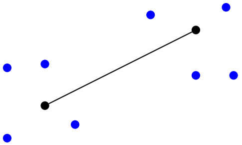

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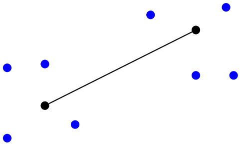

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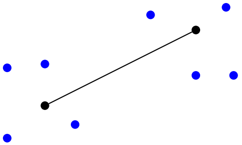

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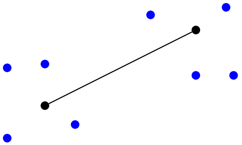

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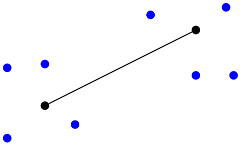

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Principal components analysis.

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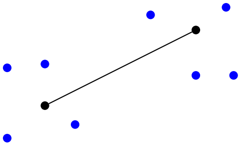

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Infinity and beyond!

Nets

“δ - Net”.

Nets

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Set $\mathscr{D}$ of directions

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where all others, v, are close to $x \in \mathscr{D}$.

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PCA can reduce d to "knowing centers" case, with reasonable number of sample points.

PCA calculation.

Matrix A where rows are points.

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$\rightarrow vBv \geq (av + x)B(av + x)$ for unit v, $av + x$.

Computing eigenvalues.

Power method:

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  Choose random x.

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Cluster Algorithm:

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Cluster Algorithm:
Choose random partition.

Computing eigenvalues.

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Repeat: Let $x = Bx$. Scale x to unit vector.

$x = a_1 v_1 + a_2 v_2 + \cdots$

$x_t \propto B^t x = a_1 \lambda_1^t v_1 + a_2 \lambda_2^t v_2 + \cdots$

Mostly v_1 after a while since $\lambda_1^t >> \lambda_2^t$.

Cluster Algorithm:
Choose random partition.
Repeat: Compute means of partition. Project, cluster.

Choose random $+1/-1$ vector. Multiply by A^T (direction between means), multiply by A (project points), cluster (round to +1/-1 vector.)

Computing eigenvalues.

Power method:
  Choose random x.
  Repeat: Let $x = Bx$. Scale x to unit vector.

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Sort of repeatedly multiplying by AA^T. Power method.

Sum up.

Clustering mixture of gaussians.

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Near Neighbor works with sufficient data.

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Power method computes principal component.

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Principal compent analysis can find subspace of means.

Power method computes principal component.

Generic clustering algorithm is rounded version of power method.

See you on Thursday.