Today

Facility Location.

Today

Facility Location.

Lagrangian Dual.

Today

Facility Location.

Lagrangian Dual. Already.

Today

Facility Location.

Lagrangian Dual. Already.

Convex Separator.

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Facility Location.

Lagrangian Dual. Already.

Convex Separator.

Farkas Lemma.

Facility location

Set of facilities: F, opening cost f_i for facility i

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Set of clients: D.

Facility location

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d_{ij} - distance between i and j.

Facility location

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(notation abuse: clients/facility confusion.)

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Triangle inequality: $d_{ij} \leq d_{ik} + d_{kj}$.

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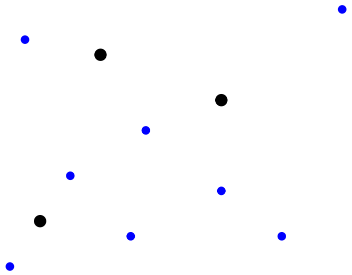

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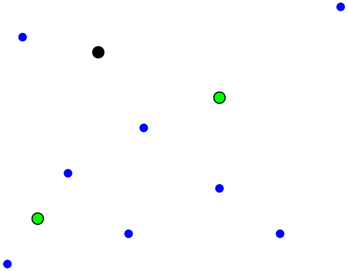

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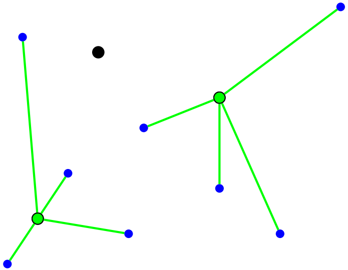

Facility Location

Linear program relaxation:

Facility Location

Linear program relaxation:

“Decision Variables”.

Facility Location

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y_i - facility i open?

Facility Location

Linear program relaxation:

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Linear program relaxation:

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y_i - facility i open?

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$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$

$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$

$$\forall i \in F, j \in D \quad x_{ij} \leq y_i,$$

$$x_{ij}, y_i \geq 0$$

Facility Location

Linear program relaxation:

"Decision Variables".

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\end{aligned}
$$

Facility opening cost.

Facility Location

Linear program relaxation:

"Decision Variables".

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Facility opening cost.

Client Connnection cost.

Facility Location

Linear program relaxation:

"Decision Variables".

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x_{ij} - client j assigned to facility i.

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$
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Facility opening cost.
Client Connnection cost.
Must connect each client.

Facility Location

Linear program relaxation:

"Decision Variables".
y_i - facility i open?
x_{ij} - client j assigned to facility i.

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$
$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$
$$\forall i \in F, j \in D \quad x_{ij} \leq y_i,$$
$$x_{ij}, y_i \geq 0$$

Facility opening cost.
Client Connnection cost.
Must connect each client.
Only connect to open facility.

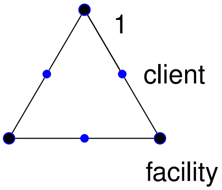
Integer Solution?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$

$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$

$$\forall i \in F, j \in D \quad x_{ij} \leq y_i,$$

$$x_{ij}, y_i \geq 0$$



$x_{ij} = \frac{1}{2}$ edges.
$y_i = \frac{1}{2}$ edges.

Integer Solution?

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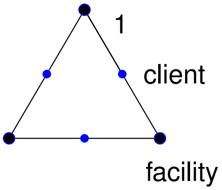

$x_{ij} = \frac{1}{2}$ edges.
$y_i = \frac{1}{2}$ edges.

Facility Cost: $\frac{3}{2}$

Integer Solution?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$

$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$

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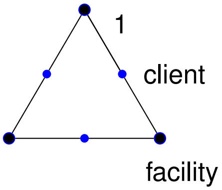


$x_{ij} = \frac{1}{2}$ edges.
$y_i = \frac{1}{2}$ edges.

Facility Cost: $\frac{3}{2}$ Connection Cost: 3

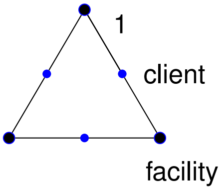
Integer Solution?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$

$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$

$$\forall i \in F, j \in D \quad x_{ij} \leq y_i,$$

$$x_{ij}, y_i \geq 0$$



$x_{ij} = \frac{1}{2}$ edges.
$y_i = \frac{1}{2}$ edges.

Facility Cost: $\frac{3}{2}$ Connection Cost: 3
Any one Facility:
Facility Cost: 1

Integer Solution?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$

$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$

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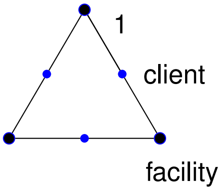

$x_{ij} = \frac{1}{2}$ edges.
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Facility Cost: $\frac{3}{2}$ Connection Cost: 3
Any one Facility:
Facility Cost: 1 Client Cost: 3.7

Integer Solution?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$

$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$

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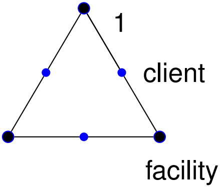

$x_{ij} = \frac{1}{2}$ edges.
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Facility Cost: $\frac{3}{2}$ Connection Cost: 3
Any one Facility:
Facility Cost: 1 Client Cost: 3.7
Make it worse?

Integer Solution?

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$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$

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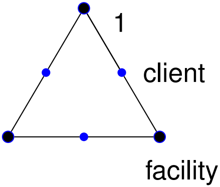

$x_{ij} = \frac{1}{2}$ edges.
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Facility Cost: $\frac{3}{2}$ Connection Cost: 3
Any one Facility:
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Make it worse? Sure.

Integer Solution?

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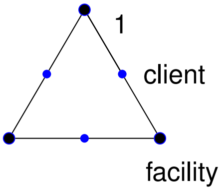

$x_{ij} = \frac{1}{2}$ edges.
$y_i = \frac{1}{2}$ edges.

Facility Cost: $\frac{3}{2}$ Connection Cost: 3
Any one Facility:
Facility Cost: 1 Client Cost: 3.7
Make it worse? Sure. Not as pretty!

Round solution?

$$\begin{aligned} \min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij} \\ \forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1 \\ \forall i \in F, j \in D \quad x_{ij} \leq y_i, \\ x_{ij}, y_i \geq 0 \end{aligned}$$

Round solution?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$
$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$
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Round independently?

Round solution?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$

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Round independently?

y_i and x_{ij} separately?

Round solution?

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Round independently?

y_i and x_{ij} separately? Assign to closed facility!

Round solution?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$

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Round x_{ij} and open facilities?

Round solution?

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Round independently?

y_i and x_{ij} separately? Assign to closed facility!

Round x_{ij} and open facilities?
Different clients force different facilities open.

Round solution?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$

$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$

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Different clients force different facilities open.

Any ideas?

Round solution?

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Round independently?

y_i and x_{ij} separately? Assign to closed facility!

Round x_{ij} and open facilities?
Different clients force different facilities open.

Any ideas?

Use Dual!

The dual.

$\min cx, Ax \geq b$

The dual.

$\min cx, Ax \geq b \leftrightarrow$

The dual.

$\min cx, Ax \geq b \leftrightarrow \max bx, y^T A \leq c$.

The dual.

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$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$
$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1 \qquad ; \ \alpha_j$$
$$\forall i \in F, j \in D \quad y_i - x_{ij} \geq 0 \qquad ; \ \beta_{ij}$$
$$x_{ij}, y_i \geq 0$$

The dual.

$\min cx, Ax \geq b \leftrightarrow \max bx, y^T A \leq c.$

$$
\begin{aligned}
\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij} \\
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\forall i \in F, j \in D \quad y_i - x_{ij} \geq 0 & \quad ; \ \beta_{ij} \\
x_{ij}, y_i \geq 0 &
\end{aligned}
$$

$$
\begin{aligned}
\max \sum_{j} \alpha_j & \\
\forall i \quad \sum_{j \in D} \beta_{ij} \leq f_i & \quad ; \ y_i \\
\forall i \in F, j \in D \quad \alpha_i - \beta_{ij} \leq d_{ij} & \quad ; \ x_{ij} \\
\beta_{ij}, \alpha_j \geq 0 &
\end{aligned}
$$

Interpretation of Dual?

$$\min \sum_{i\in F} y_i f_i + \sum_{i\in F, j\in D} x_{ij} d_{ij}$$
$$\forall j \in D \quad \sum_{i\in F} x_{ij} \geq 1$$
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$$\alpha_j, \beta_{ij} \leq 0$$

Interpretation of Dual?

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$$\alpha_j, \beta_{ij} \leq 0$$

α_j charge to client.

Interpretation of Dual?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$
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α_j charge to client.
maximize price paid by client to connect!

Interpretation of Dual?

$$\min \sum_{i\in F} y_i f_i + \sum_{i\in F, j\in D} x_{ij} d_{ij}$$
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$$\alpha_j, \beta_{ij} \leq 0$$

α_j charge to client.
maximize price paid by client to connect!
Objective: $\sum_j \alpha_j$ total payment.

Interpretation of Dual?

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α_j charge to client.
maximize price paid by client to connect!
Objective: $\sum_j \alpha_j$ total payment.
Client j travels or pays to open facility i.

Interpretation of Dual?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$
$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$
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$$\alpha_j, \beta_{ij} \leq 0$$

α_j charge to client.
maximize price paid by client to connect!
Objective: $\sum_j \alpha_j$ total payment.
Client j travels or pays to open facility i.
Costs client d_{ij} to get to there.

Interpretation of Dual?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$
$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$
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α_j charge to client.
maximize price paid by client to connect!
Objective: $\sum_j \alpha_j$ total payment.
Client j travels or pays to open facility i.
Costs client d_{ij} to get to there.
Savings is $\alpha_j - d_{ij}$.

Interpretation of Dual?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$
$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$
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$$\forall i \in F \quad \sum_{j \in D} \beta_{ij} \leq f_i$$
$$\forall i \in F, j \in D \quad \alpha_j - \beta_{ij} \leq d_{ij} \quad x_{ij}$$
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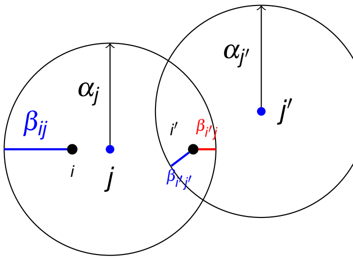
α_j charge to client.
maximize price paid by client to connect!
Objective: $\sum_j \alpha_j$ total payment.
Client j travels or pays to open facility i.
Costs client d_{ij} to get to there.
Savings is $\alpha_j - d_{ij}$.
Willing to pay $\beta_{ij} = \alpha_j - d_{ij}$.



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$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$
$$\forall i \in F, j \in D \quad x_{ij} \leq y_i,$$
$$x_{ij}, y_i \geq 0$$

$$\max \sum_j \alpha_j$$
$$\forall i \in F \quad \sum_{j \in D} \beta_{ij} \leq f_i$$
$$\forall i \in F, j \in D \quad \alpha_j - \beta_{ij} \leq d_{ij} \quad x_{ij}$$
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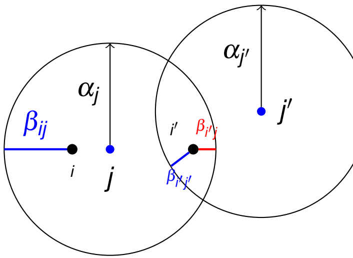
α_j charge to client.
maximize price paid by client to connect!
Objective: $\sum_j \alpha_j$ total payment.
Client j travels or pays to open facility i.
Costs client d_{ij} to get to there.
Savings is $\alpha_j - d_{ij}$.
Willing to pay $\beta_{ij} = \alpha_j - d_{ij}$.
Total payment to facility i at most f_i before opening.



Interpretation of Dual?

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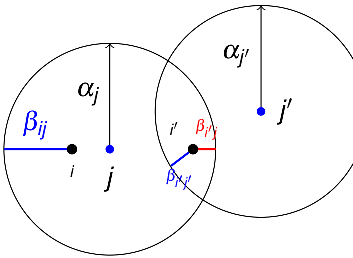
Complementary slackness:



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\begin{aligned}
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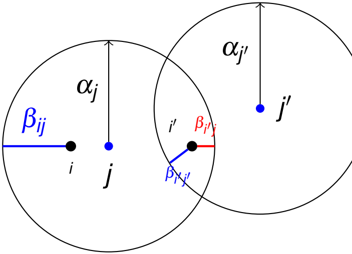

Total payment to facility i at most f_i before opening.
Complementary slackness: $x_{ij} \geq 0$ if and only if $\alpha_j \geq d_{ij}$.

Interpretation of Dual?

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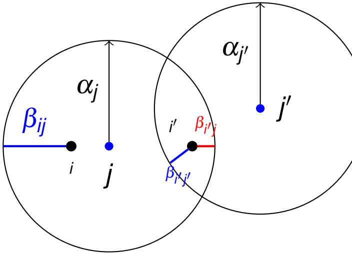
Total payment to facility i at most f_i before opening.
Complementary slackness: $x_{ij} \geq 0$ if and only if $\alpha_j \geq d_{ij}$.
  only assign client to "paid to" facilities.



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1. Find solution to primal, (x, y). and dual, (α, β).

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3. Removed assigned clients, goto 2.

Integral facility cost at most LP facility cost.

Claim: Total facility cost is at most $\sum_i f_i y_i$.

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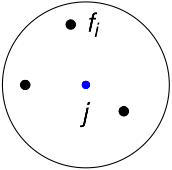

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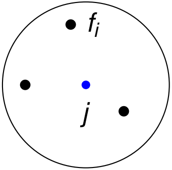

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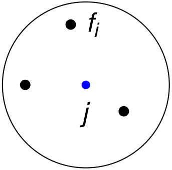

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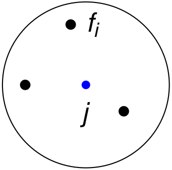

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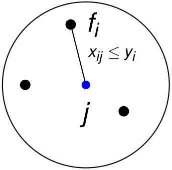

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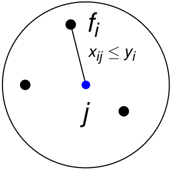

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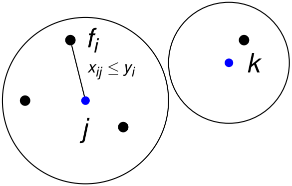

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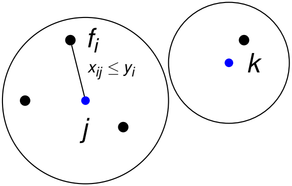

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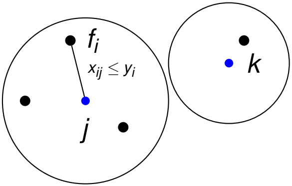

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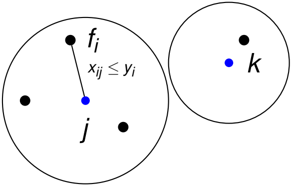

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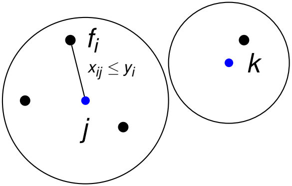

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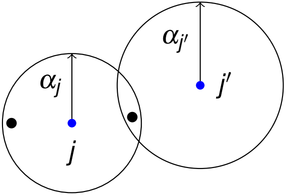

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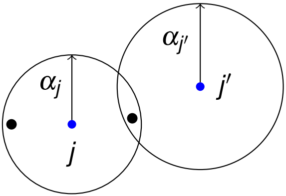

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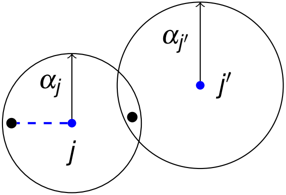

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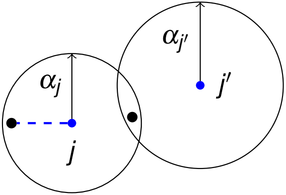

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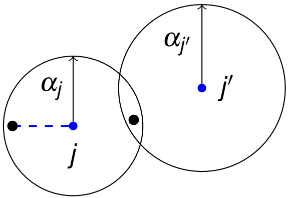

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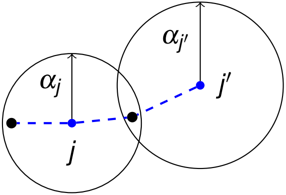

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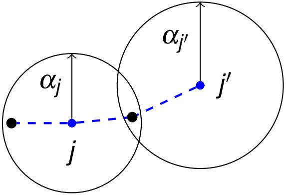

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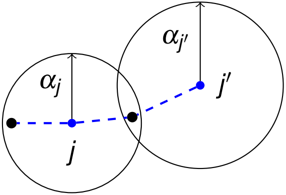

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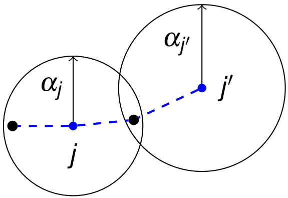
$\leq \alpha_{j'} + \alpha_j + \alpha_j \leq 3\alpha_{j'}$.

Connection Cost.

2. For smallest (remaining) α_j,
   (a) Let $N_j = \{i : x_{ij} > 0\}$.
   (b) Open cheapest facility i in N_j.
   Every client j' with $N_{j'} \cap N_j \neq \emptyset$ assigned to i.

Client j is directly connected. Clients j' are indirectly connected.



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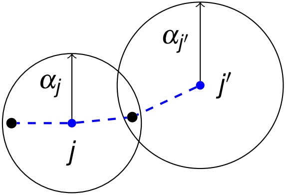

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Total connection cost:
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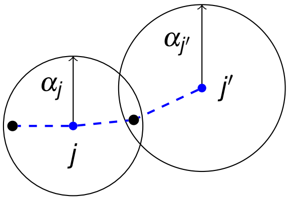

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Previous Slide: Facility cost:

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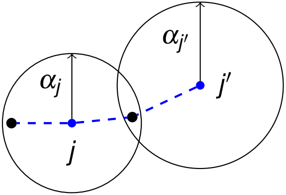

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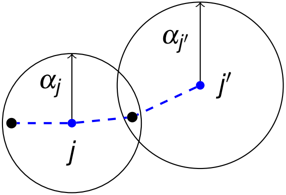

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Total Cost: 4 OPT.

Twist on randomized rounding.

Client j:

Twist on randomized rounding.

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Probability distribution!

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Client j: $\sum_i x_{ij} = 1$, $x_{ij} \geq 0$.
Probability distribution! $\rightarrow$ Choose from distribution, x_{ij}, in step 2.

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Expected opening cost:

Twist on randomized rounding.

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$\sum_{i \in N_j} x_i j f_i \leq \sum_{i \in N_j} y_i f_i$.

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and separate balls implies total $\leq \sum_i y_i f_i$.

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$D_j = \sum_i x_{ij} d_{ij}$ Connection cost of primal for j.

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Expected connection cost j' $\alpha_j + \alpha_{j'} + D_j$.

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In step 2: pick in increasing order of $\alpha_j + D_j$.

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$\rightarrow$ Expected cost is $(2\alpha_{j'} + D_{j'})$. Connection cost: $2\sum_j \alpha_j + \sum_j D_j$.
$2OPT(D)$ plus connection cost or primal.

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Connection cost at most $2OPT$ + connection cost of prmal.

$\rightarrow$ at most $3OPT$.

Primal dual algorithm.

1. Feasible integer solution.

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1. Feasible integer solution.
2. Feasible dual solution.
3. Cost of integer solution $\leq \alpha$ times dual value.

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Just did it. Used linear program.

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  Begin with feasible dual.
  Raise dual variables until tight constraint.

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  Set corresponding primal variable to an integer.

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Recall Dual:

Primal dual algorithm.

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3. Cost of integer solution $\leq \alpha$ times dual value.

Just did it. Used linear program. Faster?

Typically.
Begin with feasible dual.
Raise dual variables until tight constraint.
Set corresponding primal variable to an integer.

Recall Dual:

$$\max \sum_j \alpha_j$$

$$\forall i \in F \quad \sum_{j \in D} \beta_{ij} \leq f_i$$

$$\forall i \in F, j \in D \quad \alpha_j - \beta_{ij} \leq d_{ij}$$

$$\alpha_j, \beta_{ij} \leq 0$$

Facility location primal dual.

Phase 1:

Facility location primal dual.

Phase 1: 1. Initially $\alpha_j, \beta_{ij} = 0$.

Facility location primal dual.

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When $\alpha_j = d_{ij}$ for some i
raise β_{ij} at same rate

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Stop when $\sum_i \beta_{ij} = f_i$.

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Why? Dual: $\sum_i \beta_{ij} \leq f_i$
Intution: facility paid for.

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Temporarily open i.

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Temporarily open i.

Connect all tight ji clients j to i.

Facility location primal dual.

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3. Continue until all clients connected.

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Phase 2:

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Phase 2:
Connect facilities that were paid by same client.

Facility location primal dual.

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Phase 2:
Connect facilities that were paid by same client.
Permanently open an independent set of facilities.

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For client j, connected facility i is opened.

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Temporarily open i.
Connect all tight ji clients j to i.

3. Continue until all clients connected.

Phase 2:
Connect facilities that were paid by same client.
Permanently open an independent set of facilities.

For client j, connected facility i is opened. Good.

Facility location primal dual.

Phase 1: 1. Initially $\alpha_j, \beta_{ij} = 0$.
2. Raise α_j for every (unconnected) client.
When $\alpha_j = d_{ij}$ for some i
raise β_{ij} at same rate   Why? Dual: $\alpha_j - \beta_{ij} \le d_{ij}$.
Intution:Paying β_{ij} to open i.
Stop when $\sum_i \beta_{ij} = f_i$.
Why? Dual: $\sum_i \beta_{ij} \le f_i$
Intution: facility paid for.
Temporarily open i.
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Connect facilities that were paid by same client.
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Connected facility not open

Facility location primal dual.

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3. Continue until all clients connected.

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Connect facilities that were paid by same client.
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For client j, connected facility i is opened. Good.
Connected facility not open
$\rightarrow$ exists client j' paid i and connected to open facility.

Facility location primal dual.

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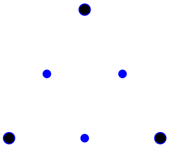
3. Continue until all clients connected.

Phase 2:
Connect facilities that were paid by same client.
Permanently open an independent set of facilities.

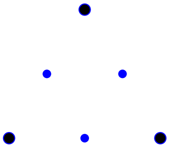
For client j, connected facility i is opened. Good.
Connected facility not open
$\rightarrow$ exists client j' paid i and connected to open facility.
Connect j to j''s open facility.

Constraints for dual.



Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$



Constraints for dual.

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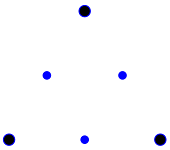
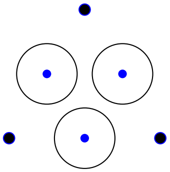


Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

$\alpha_i - \beta_{ij} \leq d_{ij}$.

Grow α_j.



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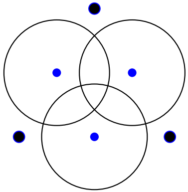


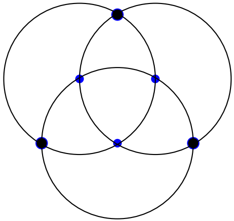
Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

$\alpha_i - \beta_{ij} \leq d_{ij}$.

Grow α_j.

$\alpha_j = d_{ij}$!



Constraints for dual.

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$\alpha_i - \beta_{ij} \leq d_{ij}$.

Grow α_j.

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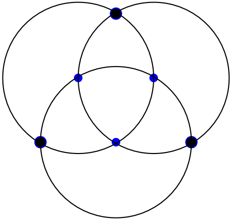
Tight constraint:



Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

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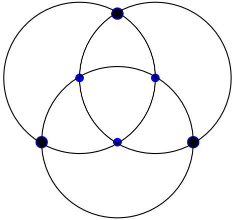

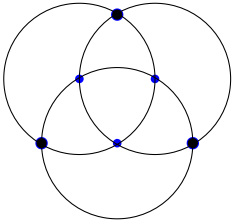

Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

$\alpha_i - \beta_{ij} \leq d_{ij}$.

Grow α_j.

$\alpha_j = d_{ij}$!

Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

Grow β_{ij} (and α_j).

Constraints for dual.

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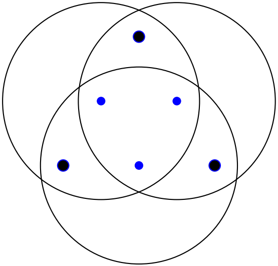

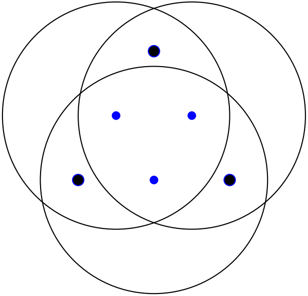

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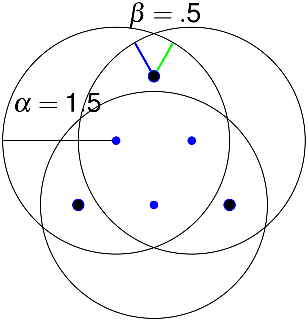

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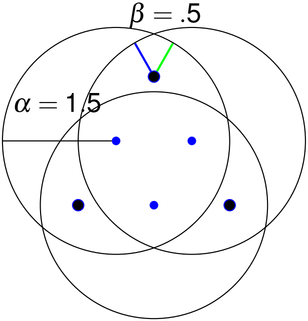
Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

Grow β_{ij} (and α_j).

$\sum_j \beta_{ij} = f_i$ for all facilities.



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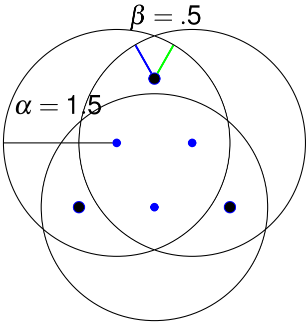

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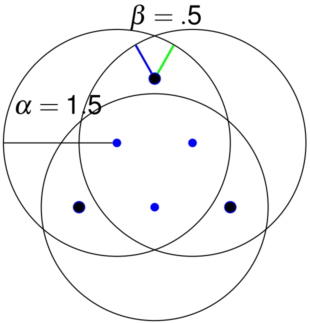

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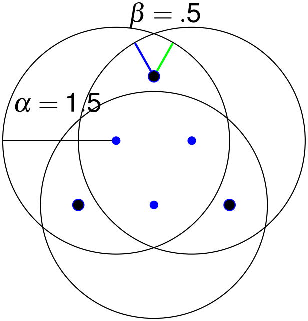
Tight: $\sum_j \beta_{ij} \leq f_i$

LP Cost: $\sum_j \alpha_j$



Constraints for dual.

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$\alpha_j = d_{ij}$!

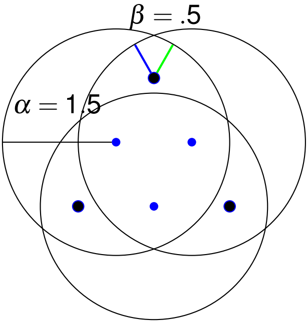
Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

Grow β_{ij} (and α_j).

$\sum_j \beta_{ij} = f_i$ for all facilities.

Tight: $\sum_j \beta_{ij} \leq f_i$

LP Cost: $\sum_j \alpha_j = 4.5$



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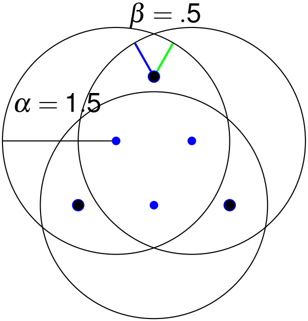
LP Cost: $\sum_j \alpha_j = 4.5$

Temporarily open all facilities.



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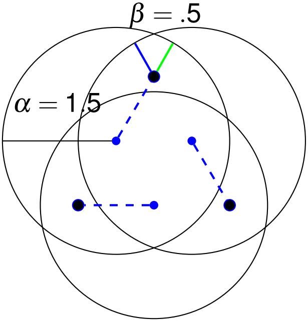

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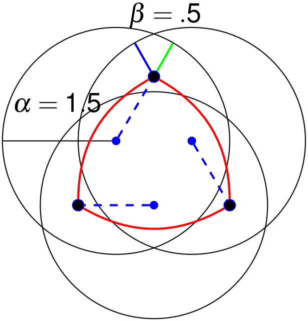
LP Cost: $\sum_j \alpha_j = 4.5$

Temporarily open all facilities.

Assign Clients to "paid to" open facility.



Constraints for dual.

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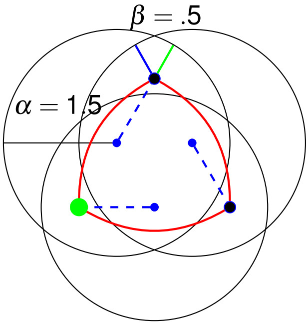
LP Cost: $\sum_j \alpha_j = 4.5$

<u>Temporarily open all facilities.</u>

Assign Clients to “paid to” open facility.

Connect facilities with client that pays both.



Constraints for dual.

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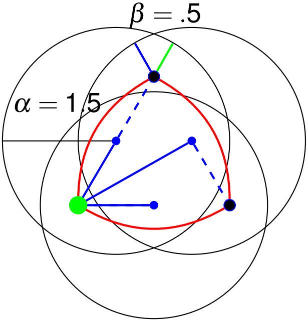
LP Cost: $\sum_j \alpha_j = 4.5$

Temporarily open all facilities.

Assign Clients to “paid to” open facility.

Connect facilities with client that pays both.

Open independent set.



Constraints for dual.

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Grow α_j.

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Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

Grow β_{ij} (and α_j).

$\sum_j \beta_{ij} = f_i$ for all facilities.

Tight: $\sum_j \beta_{ij} \leq f_i$

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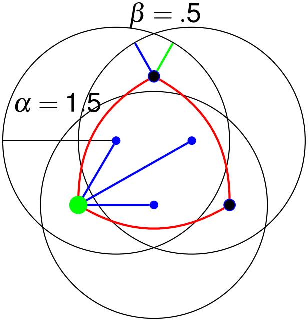
<u>Temporarily open all facilities.</u>

Assign Clients to “paid to” open facility.

Connect facilities with client that pays both.

Open independent set.

Connect to “killer” client’s facility.



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Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

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Temporarily open all facilities.

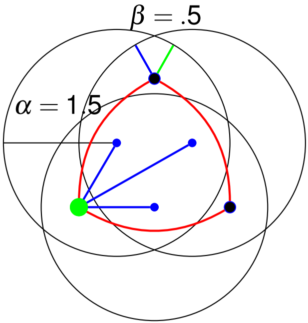
Assign Clients to “paid to” open facility.

Connect facilities with client that pays both.

Open independent set.

Connect to “killer” client’s facility.

Cost: 1 + 3.7



Constraints for dual.

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Grow α_j.

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Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

Grow β_{ij} (and α_j).

$\sum_j \beta_{ij} = f_i$ for all facilities.

Tight: $\sum_j \beta_{ij} \leq f_i$

LP Cost: $\sum_j \alpha_j = 4.5$

Temporarily open all facilities.

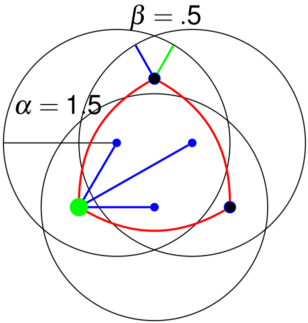
Assign Clients to “paid to” open facility.

Connect facilities with client that pays both.

Open independent set.

Connect to “killer” client’s facility.

Cost: 1 + 3.7 = 4.7.



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Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

Grow β_{ij} (and α_j).

$\sum_j \beta_{ij} = f_i$ for all facilities.

Tight: $\sum_j \beta_{ij} \leq f_i$

LP Cost: $\sum_j \alpha_j = 4.5$

Temporarily open all facilities.

Assign Clients to "paid to" open facility.

Connect facilities with client that pays both.

Open independent set.

Connect to "killer" client's facility.

Cost: 1 + 3.7 = 4.7.

A bit more than the LP cost.

Analysis

Claim: Client only pays one facility.

Analysis

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Independent set of facilities.

Analysis

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Independent set of facilities.

Claim: S_i - directly connected clients to open facility i.

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$f_i + \sum_{j \in S_i} d_{ij} \leq \sum_j \alpha_j$.

Analysis

Claim: Client only pays one facility.

Independent set of facilities.

Claim: S_i - directly connected clients to open facility i.
$f_i + \sum_{j \in S_i} d_{ij} \leq \sum_j \alpha_j$.

Proof:

Analysis

Claim: Client only pays one facility.

Independent set of facilities.

Claim: S_i - directly connected clients to open facility i.
$f_i + \sum_{j \in S_i} d_{ij} \leq \sum_j \alpha_j$.

Proof:
$f_i = \sum_{j \in S_i} \beta_{ij}$

Analysis

Claim: Client only pays one facility.

Independent set of facilities.

Claim: S_i - directly connected clients to open facility i.
$f_i + \sum_{j \in S_i} d_{ij} \leq \sum_j \alpha_j$.

Proof:
$f_i = \sum_{j \in S_i} \beta_{ij} = \sum_{j \in S_i} \alpha_j - d_{ij}$.
Since directly connected: $\beta_{ij} = \alpha_j - d_{ij}$. □

Analysis.

Claim: Client j is indirectly connected to i

Analysis.

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$\rightarrow d_{ij} \leq 3\alpha_j$.

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Directly connected to (temp open) i'

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Directly connected to (temp open) i'
conflicts with i.

Analysis.

Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.

Directly connected to (temp open) i'
conflicts with i.
exists j' with $\alpha_{j'} \geq d_{ij'}$ and $\alpha_j \geq d_{i'j'}$.

Analysis.

Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.

Directly connected to (temp open) i'
conflicts with i.
exists j' with $\alpha_{j'} \geq d_{ij'}$ and $\alpha_j \geq d_{i'j'}$.
When i' opens, stops both α_j and α'_j.

Analysis.

Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.

Directly connected to (temp open) i'
conflicts with i.
exists j' with $\alpha_{j'} \geq d_{ij'}$ and $\alpha_j \geq d_{i'j'}$.
When i' opens, stops both α_j and α_j'.
α_j' stopped no later

Analysis.

Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.

Directly connected to (temp open) i'
conflicts with i.
exists j' with $\alpha_{j'} \geq d_{ij'}$ and $\alpha_j \geq d_{i'j'}$.
When i' opens, stops both α_j and α_j'.
α_j' stopped no later (..maybe earlier..)

Analysis.

Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.

Directly connected to (temp open) i'
conflicts with i.
exists j' with $\alpha_{j'} \geq d_{ij'}$ and $\alpha_j \geq d_{i'j'}$.
When i' opens, stops both α_j and α_j'.
α_j' stopped no later (..maybe earlier..)
$\alpha_j \leq \alpha_j'$.

Analysis.

Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.

Directly connected to (temp open) i'
conflicts with i.
exists j' with $\alpha_{j'} \geq d_{ij'}$ and $\alpha_j \geq d_{i'j'}$.
When i' opens, stops both α_j and α'_j.
α'_j stopped no later (..maybe earlier..)
$\alpha_j \leq \alpha'_j$.
Total distance from j to i'.

Analysis.

Claim: Client j is indirectly connected to i $\rightarrow d_{ij} \leq 3\alpha_j$.

Directly connected to (temp open) i'
conflicts with i.
exists j' with $\alpha_{j'} \geq d_{ij'}$ and $\alpha_j \geq d_{i'j'}$.
When i' opens, stops both α_j and α_j'.
α_j' stopped no later (..maybe earlier..)
$\alpha_j \leq \alpha_j'$.
Total distance from j to i'.
$d_{ij} +$

Analysis.

Claim: Client j is indirectly connected to i $\rightarrow d_{ij} \leq 3\alpha_j$.

Directly connected to (temp open) i' conflicts with i.

exists j' with $\alpha_{j'} \geq d_{ij'}$ and $\alpha_j \geq d_{i'j'}$.

When i' opens, stops both α_j and α_j'.

α_j' stopped no later (..maybe earlier..)

$\alpha_j \leq \alpha_j'$.

Total distance from j to i'.

$d_{ij} + d_{ij'} +$

Analysis.

Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.

Directly connected to (temp open) i'
conflicts with i.
exists j' with $\alpha_{j'} \geq d_{ij'}$ and $\alpha_j \geq d_{i'j'}$.
When i' opens, stops both α_j and α_j'.
α_j' stopped no later (..maybe earlier..)
$\alpha_j \leq \alpha_j'$.
Total distance from j to i'.
$d_{ij} + d_{ij'} + d_{i'j'}$

Analysis.

Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.

Directly connected to (temp open) i'
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When i' opens, stops both α_j and α'_j.
α'_j stopped no later (..maybe earlier..)
$\alpha_j \leq \alpha'_j$.
Total distance from j to i'.
$d_{ij} + d_{ij'} + d_{i'j'} \leq 3\alpha_j$

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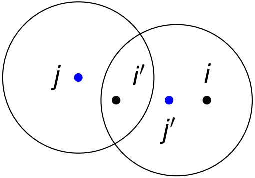

Putting it together!

Claim: Client only pays one facility.
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direct clients dual (α_j) pays for facility and own connections.

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Total Cost:
direct clients dual (α_j) pays for facility and own connections.
plus no more than 3 times indirect client dual.

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Total Cost: 3 times dual.

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3 OPT.

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3 OPT.

Fast!

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Total Cost:
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plus no more than 3 times indirect client dual.
Total Cost: 3 times dual.
feasible dual upper bounds fractional (and integer) primal.

3 OPT.

Fast! Cheap!

Putting it together!

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Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.

Total Cost:
direct clients dual (α_j) pays for facility and own connections.
plus no more than 3 times indirect client dual.
Total Cost: 3 times dual.
feasible dual upper bounds fractional (and integer) primal.

3 OPT.

Fast! Cheap! Safe!

See you on Thursday.