Problem Set 1

1. \([\epsilon\text{-optimal strategies}]

A strategy pair \((x, y)\) for a zero sum game is \(\epsilon\)-optimal if a player can gain at most \(\epsilon\) by deviating from his strategy assuming that the other player does not deviate,

(a) Given a strategy pair \((x, y)\) suggest an efficient procedure to verify that \((x, y)\) is \(\epsilon\)-optimal.

(b) Show that the value \(x^TAy\) of an \(\epsilon\) optimal strategy is within \(\epsilon\) of the value of the game.

2. \([\text{Irrelevant Attributes}]

Given a function of \(n\) variables, which is, in fact, the OR of \(r\) of them, where \(r \ll n\). Consider an algorithm to predict the function which maintains weights \(w_1, \cdots, w_n\) and predicts 1 when \(w_1x_1 + \cdots + w_nx_n \geq n\) and 0 otherwise. Each time the prediction is incorrect halve or double the appropriate weights. Complete the description of the algorithm and show it makes \(O(r \log n)\) mistakes.

3. \([\text{The matching game}]

The matching game is played over the complete weighted bipartite graph \(G(V, E)\). The edge player plays an edge \(e \in E\) while the vertex player player plays a vertex \(v \in V\) and if \(v \in e\) the edge player pays \(1/w_e\) to the vertex player.

(a) If the vertex player plays a uniformly random vertex what is the best response for the edge player?

(b) If the edge player plays a uniformly random edge from the maximum weight matching what is the best response for the vertex player?

(c) Are these two strategies optimal for this game? If not, what are a pair of strategies that are optimal for this game.

(d) Show how to use the multiplicative weights algorithm to find a fractional matching with cost within \((1 + \epsilon)\) factor of optimal. (A fractional matching is an assignment \(x_e\) to the edges such that for each vertex \(\sum_{e \ni u} x_e \leq 1\).)