Today

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Maximum Weight Matching

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Undergraduate: saw maximum matching!

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Undergraduate: saw maximum matching! (hopefully.) Will review.

Matching.

Given a bipartite graph, $G = (U, V, E)$, with edge weights $w : E \to R$, find a maximum weight matching.

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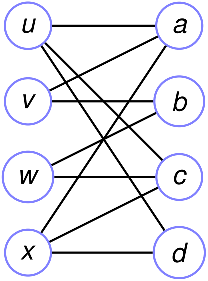

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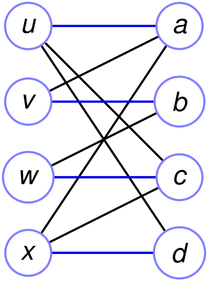

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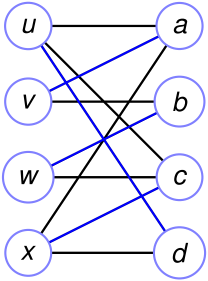

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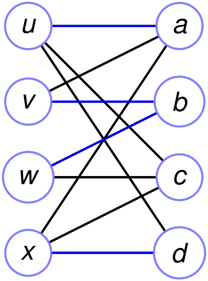

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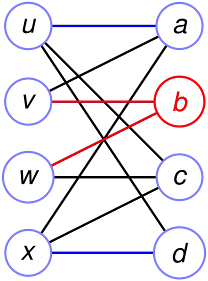

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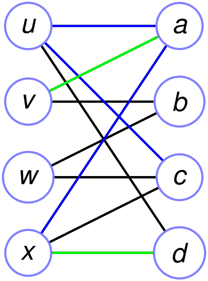

Blue – 3. Green - 2,
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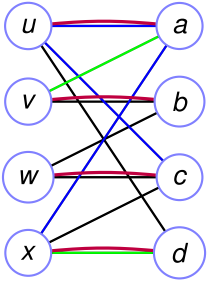

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Solution Value: 7.

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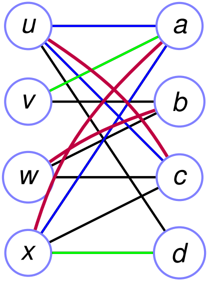

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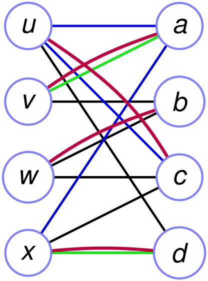

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Solution Value: 7.

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Solution Value: 8.

Applications

Jobs to workers.

Applications

Jobs to workers.

Teachers to classes.

Applications

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Teachers to classes.

Classes to classrooms.

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“The assignment problem”

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"The assignment problem"

Min Weight Matching.

Applications

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Min Weight Matching.
    Negate values and find maximum weight matching.

Vertex Cover

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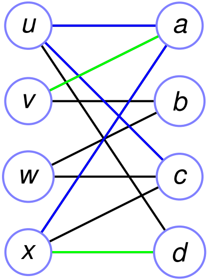
Minimize $\sum_{v \in U \cup V} p(u)$.



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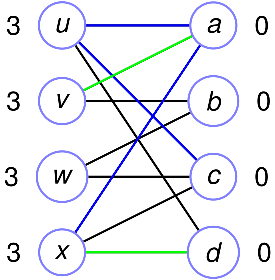

Solution Value: 12.

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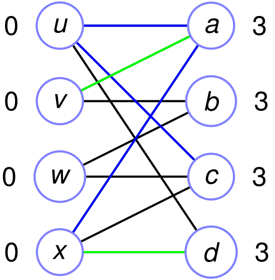

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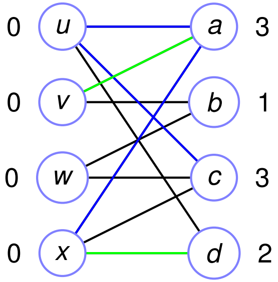

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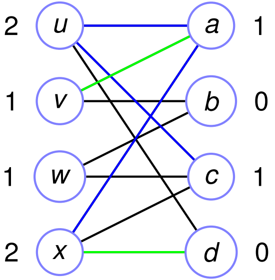

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Cover is upper bound.

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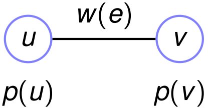

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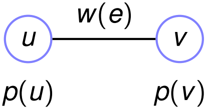
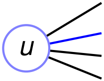


For a matching M, each u is the endpoint of at most one edge in M.



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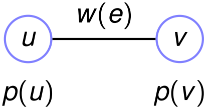

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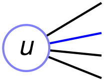

$$\sum_{e=(u,v)\in M} w(e)$$

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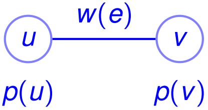

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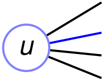

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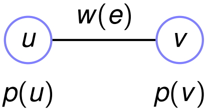

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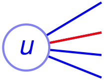

$$\sum_{e=(u,v)\in M} w(e) \leq \sum_{e=(u,v)\in M} (p(u) + p(v)) \leq \sum_{u \in U} p(u) + \sum_{v \in V} p(v)$$

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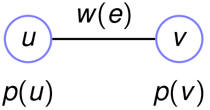

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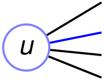

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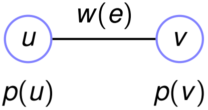

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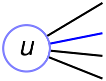

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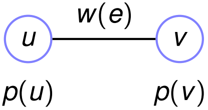

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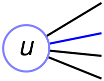

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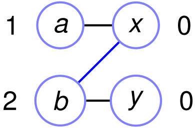
Holds with equality if

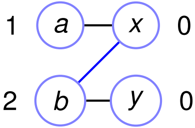
for $e \in M$, $w(e) = p(u) + p(v)$ (Defn: tight edge.) and
perfect matching.

Simple example.



Blue edge – 2, Others – 1.

Simple example.

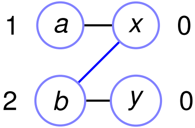


Blue edge – 2, Others – 1.

Using max incident edge.

Value: 3.

Simple example.



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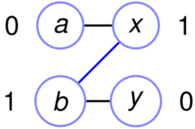
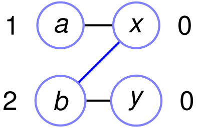


Value: 2.
Same as optimal matching!

Proof of optimality.

Simple example.



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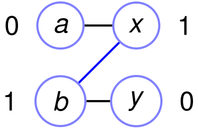

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Matching and cover are optimal,

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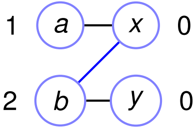

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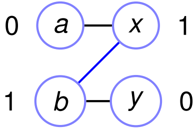

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Matching and cover are optimal,
edges in matching have $w(e) = p(u) + p(v)$. Tight edge.

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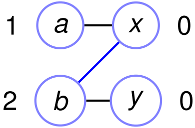

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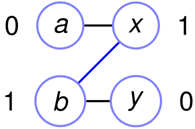

Value: 2.
  Same as optimal matching!

Proof of optimality.

Matching and cover are optimal,
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  all nodes are matched.

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(Rao would have said (A), don't worry.)

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Alg: Start at end, and alternately put in edge or not.

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What if one has a partial matching.

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How do you make it bigger?

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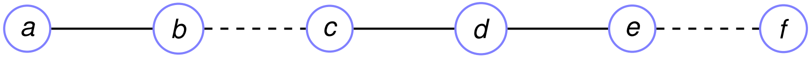

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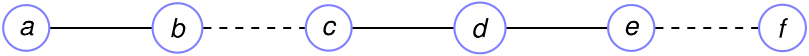

Greedily adding fails.

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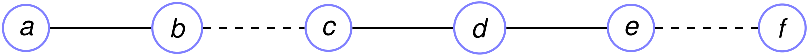
(Rao would have said (A), don't worry.)

Why?
Alg: Start at end, and alternately put in edge or not.

What if one has a partial matching.

How do you make it bigger?



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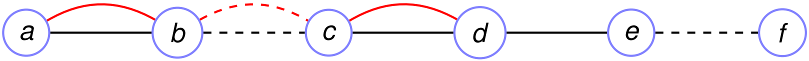

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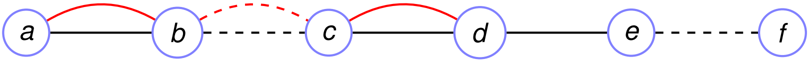

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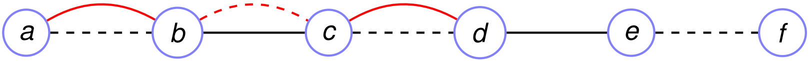

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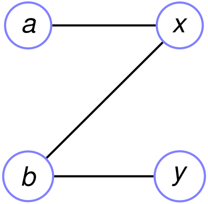

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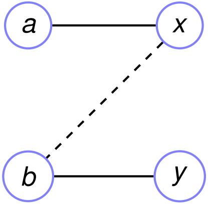

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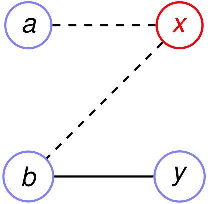

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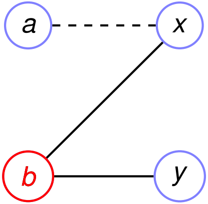

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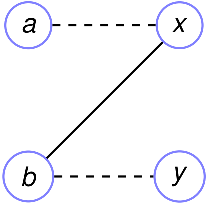

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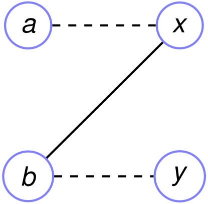

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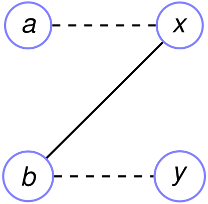

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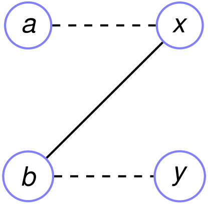

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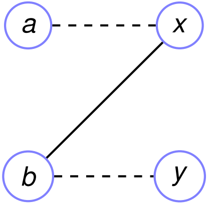

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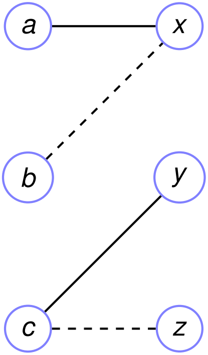

Can't increase matching size.

No alternating path from (a) to (y).

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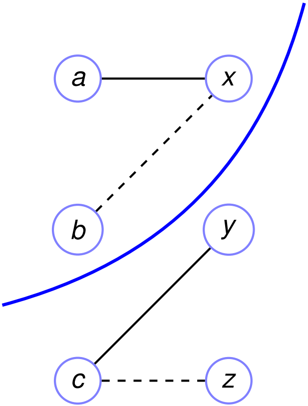

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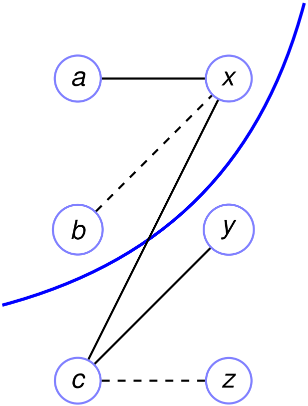

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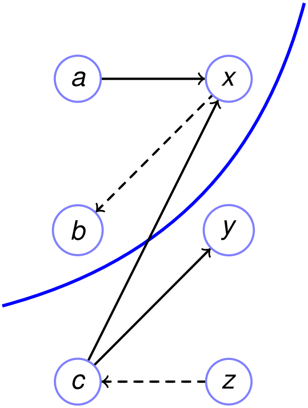

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Use directed graph!
Cut in this graph.

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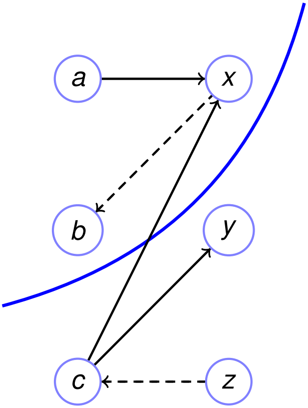

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Given matching.

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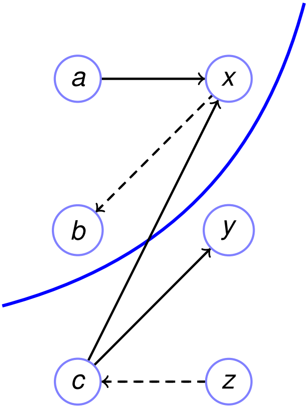

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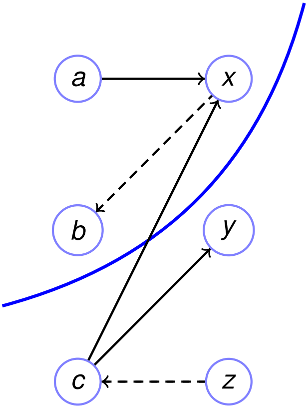

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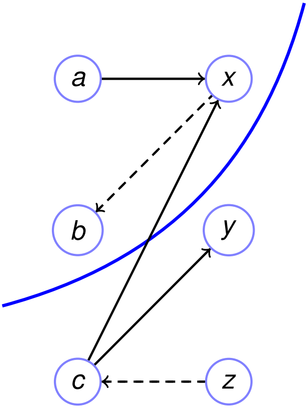

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Until everything matched

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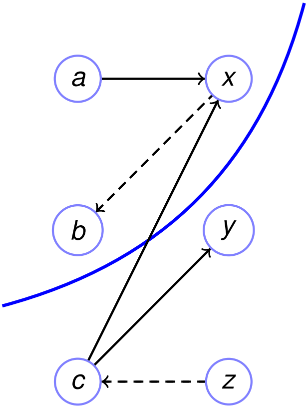

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Algorithm:
Given matching.
Direct unmatched edges U to V, matched V to U.
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Until everything matched ... or output a cut.

Back to Maximum Weight Matching.

Want vertex cover (price function) $p(\cdot)$ and matching where.

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Goal: perfect matching on tight edges.

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Use alt./aug. paths of tight edges.

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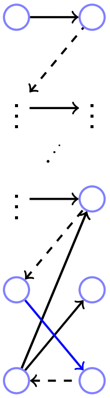

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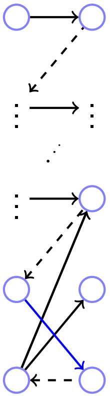

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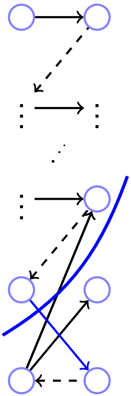

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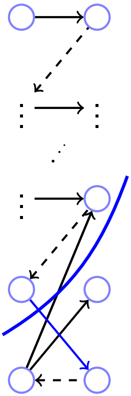

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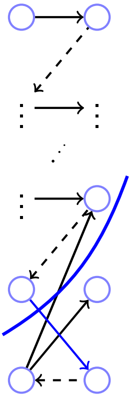

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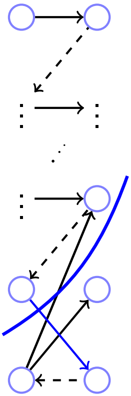

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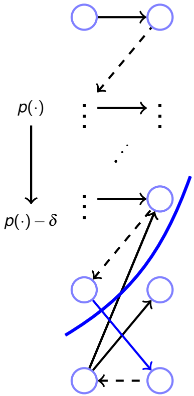

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Lower prices in S_U,

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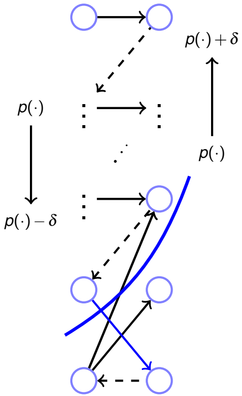

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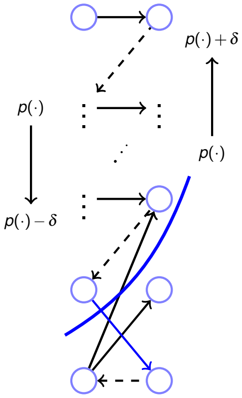

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Nontight edges leaving cut, go from S_U, T_V.

Lower prices in S_U, raise prices in S_V,
all explored edges still tight,
matched edges still tight

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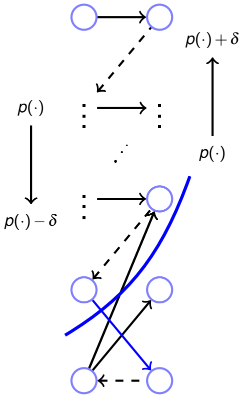

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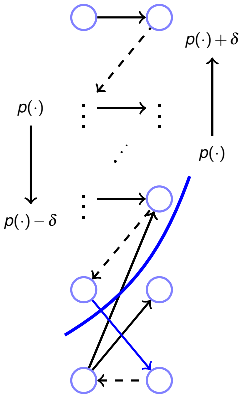

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What's delta?

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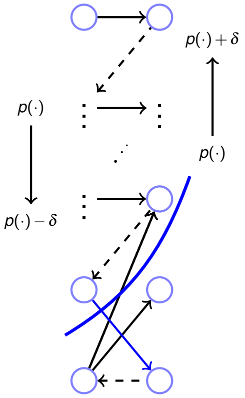

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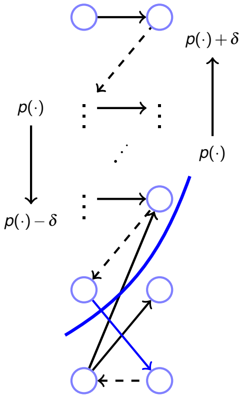

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What's delta? $w(e) < p(u) + p(v) \rightarrow$
$\delta = min_{e \in (S_U \times T_V)} p(u) + p(v) - w(e)$.

Some details

Add 0 value edges, so that optimal solution contains perfect matching.

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Update prices (find minimum delta.)

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$p(u) =$ maximum incident edge for $u \in U$, 0 otherwise.

Main Work:
breadth first search from unmatched nodes finds cut.
Update prices (find minimum delta.)

Some details

Add 0 value edges, so that optimal solution contains perfect matching.

Beginning "Matcher" Solution: $M = \{\}$.

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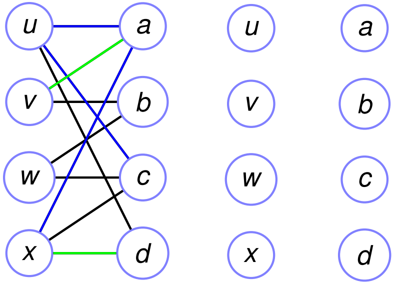
$O(n)$ iterations per augmentation.
$O(n)$ augmentations.

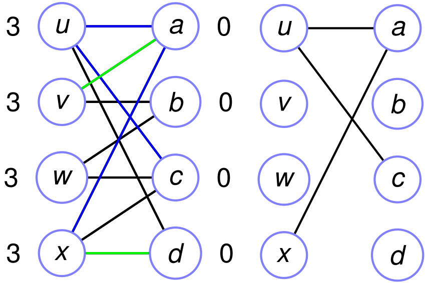
$O(n^2 m)$ time.

Example

Weight legend:
black 1, green 2, blue 3



Example



Weight legend:
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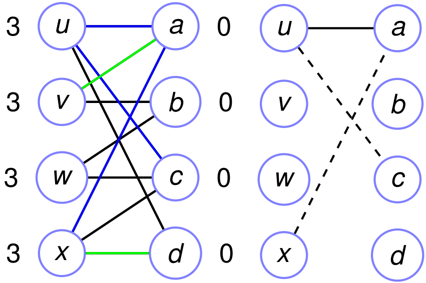
Tight edges for inital prices.

Example



Weight legend:
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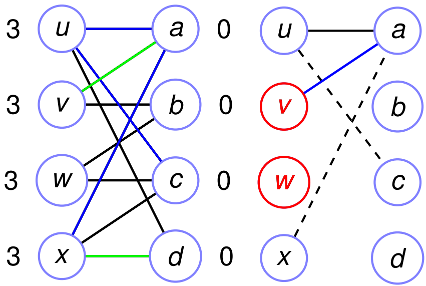
Tight edges for inital prices.
Max matching in tight edges.
dashed means matched.

Example



Weight legend:
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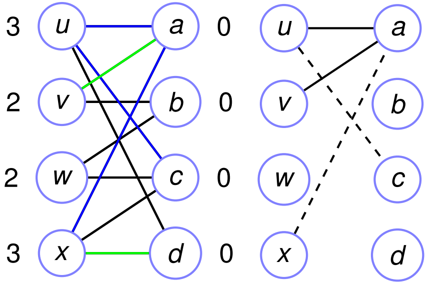
Tight edges for inital prices.
Max matching in tight edges.
dashed means matched.
No augmenting path $\rightarrow$
reachable: $S = \{u, w\}$
Blue edge on right soon
to be tight!

Example



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Adjust prices...$\delta = 1$
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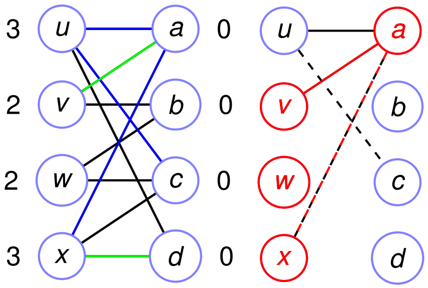

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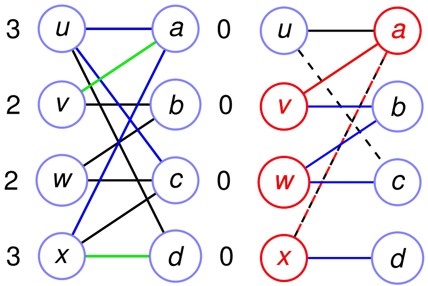

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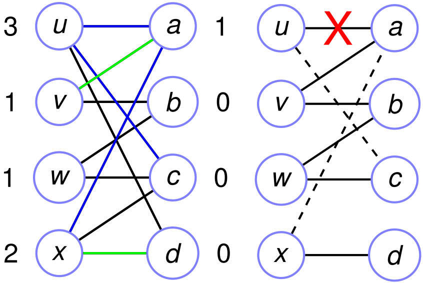
Blue edges minimally non-tight.

Example



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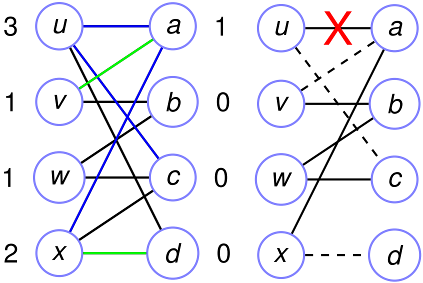
Blue edges minimally non-tight.
Adjust prices.
Some more tight edges.
And X shows
a “new” nontight edge.

Example



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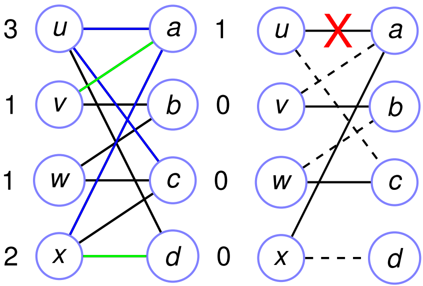

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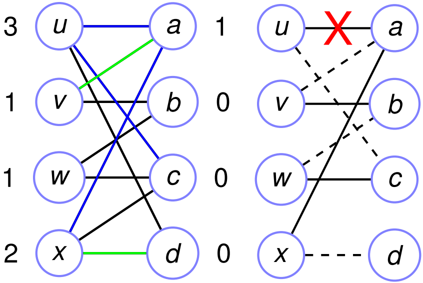

All matched edges tight.

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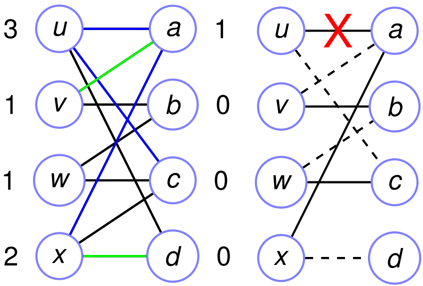

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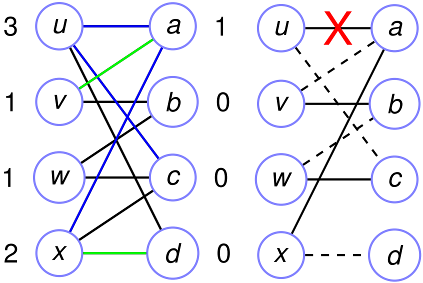

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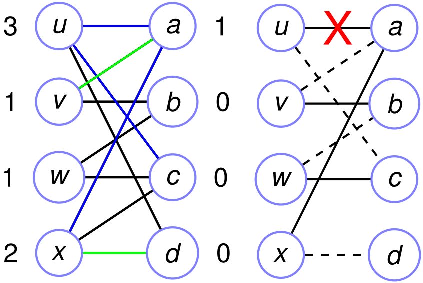
All matched edges tight.
Perfect matching. Feasible price function.

Example



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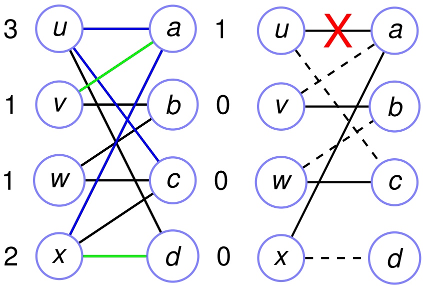
All matched edges tight.
Perfect matching. Feasible price function. Values the same.

Example



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Perfect matching. Feasible price function. Values the same. Optimal!

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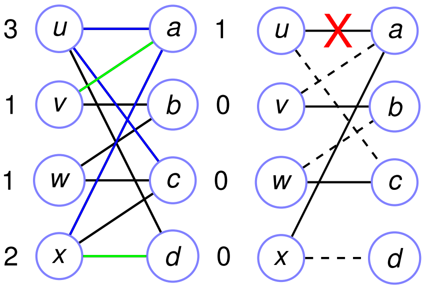

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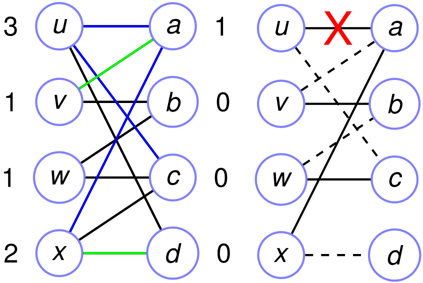
Example



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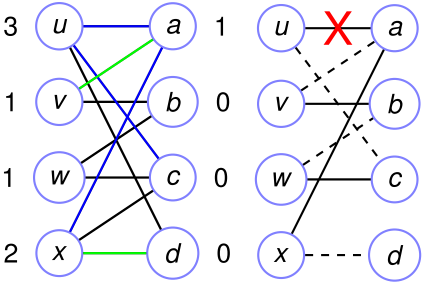

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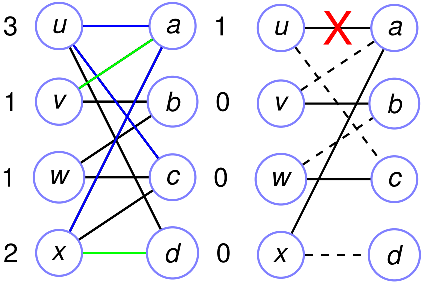
Notice:
no weights on the right problem.
retain previous matching through price changes.

Example



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Notice:
no weights on the right problem.
retain previous matching through price changes.
retains edges in failed search through price changes.

Linear Program.

How?

Linear Program.

How? From lecture warmup.

Linear Program.

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Linear program: $\max cx, Ax \leq b, x \geq 0$
Dual: $\min y^T b, y^T A \geq c, y \geq 0$

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Note: Dual variables correspond to primal equations and vice versa.

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First inequality from $b \geq Ax$

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Complementary slackness:
(1) $x_j > 0 \implies a^{(j)} y = c_j$
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What does multiplying by 0 do?

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What does multiplying by 0 do?
Zero and one. My love is won. Nothing and nothing done.

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(2) $\implies y^T b = \sum_i y_i b_i$

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Similarly: (1) $\implies y^T Ax = cx$.

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Similarly: (1) $\implies y^T Ax = cx$.

Complementary slackness conditions imply optimality.

nnnn

Perfect Matching

Linear program: $\max \sum_e w_e x_e$, $\forall v : \sum_{e=(u,v)} x_e \leq 1$, $x_e \geq 0$

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Perfect Matching

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In this case:

Perfect Matching

Linear program: $\max \sum_e w_e x_e$, $\forall v : \sum_{e=(u,v)} x_e \leq 1$, $x_e \geq 0$

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Are things as easy or as hard as $0, 1, 2, \ldots$...?

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...see you on Tuesday