# Lagrangian Dual.

Find $x$, subject to

# Lagrangian Dual.

Find $x$, subject to

$f_i(x) \leq 0, i = 1, \ldots m.$

# Lagrangian Dual.

Find $x$, subject to

$f_i(x) \leq 0, i = 1, \ldots m.$

Remember calculus (constrained optimization.)

# Lagrangian Dual.

Find $x$, subject to

$f_i(x) \leq 0, i = 1, \ldots m.$

Remember calculus (constrained optimization.)

Lagrangian:

# Lagrangian Dual.

Find $x$, subject to

$f_i(x) \leq 0, i = 1, \ldots m.$

Remember calculus (constrained optimization.)

Lagrangian: $L(x, \lambda) = \sum_{i=1}^{m} \lambda_i f_i(x)$

# Lagrangian Dual.

Find $x$, subject to

$f_i(x) \leq 0, i = 1, \ldots m.$

Remember calculus (constrained optimization.)

Lagrangian: $L(x,\lambda) = \sum_{i=1}^{m} \lambda_i f_i(x)$

$\lambda_i \geq 0$ - Lagrangian multiplier for inequality $i$.

# Lagrangian Dual.

Find $x$, subject to

$f_i(x) \leq 0, i = 1, \ldots m.$

Remember calculus (constrained optimization.)

Lagrangian: $L(x,\lambda) = \sum_{i=1}^{m} \lambda_i f_i(x)$

$\lambda_i \geq 0$ - Lagrangian multiplier for inequality $i$.

For feasible solution $x$, $L(x,\lambda)$ is

# Lagrangian Dual.

Find $x$, subject to

$f_i(x) \leq 0, i = 1, \ldots m.$

Remember calculus (constrained optimization.)

Lagrangian: $L(x, \lambda) = \sum_{i=1}^{m} \lambda_i f_i(x)$

$\lambda_i \geq 0$ - Lagrangian multiplier for inequality $i$.

For feasible solution $x$, $L(x, \lambda)$ is

# Lagrangian Dual.

Find $x$, subject to

$f_i(x) \leq 0, i = 1, \ldots m.$

Remember calculus (constrained optimization.)

Lagrangian: $L(x,\lambda) = \sum_{i=1}^{m} \lambda_i f_i(x)$

$\lambda_i \geq 0$ - Lagrangian multiplier for inequality $i$.

For feasible solution $x$, $L(x,\lambda)$ is

(A) non-negative in expectation

# Lagrangian Dual.

Find $x$, subject to

$f_i(x) \leq 0, i = 1, \ldots m.$

Remember calculus (constrained optimization.)

Lagrangian: $L(x,\lambda) = \sum_{i=1}^{m} \lambda_i f_i(x)$

$\lambda_i \geq 0$ - Lagrangian multiplier for inequality $i$.

For feasible solution $x$, $L(x,\lambda)$ is

(A) non-negative in expectation

(B) positive for any $\lambda$.

# Lagrangian Dual.

Find $x$, subject to

$f_i(x) \leq 0, i = 1, \ldots m.$

Remember calculus (constrained optimization.)

Lagrangian: $L(x,\lambda) = \sum_{i=1}^{m} \lambda_i f_i(x)$

$\lambda_i \geq 0$ - Lagrangian multiplier for inequality $i$.

For feasible solution $x$, $L(x,\lambda)$ is

(A) non-negative in expectation

(B) positive for any $\lambda$.

(C) non-positive for any valid $\lambda$.

# Lagrangian Dual.

Find $x$, subject to

$f_i(x) \leq 0, i = 1, \ldots m.$

Remember calculus (constrained optimization.)

Lagrangian: $L(x,\lambda) = \sum_{i=1}^{m} \lambda_i f_i(x)$

$\lambda_i \geq 0$ - Lagrangian multiplier for inequality $i$.

For feasible solution $x$, $L(x,\lambda)$ is

(A) non-negative in expectation

(B) positive for any $\lambda$.

(C) non-positive for any valid $\lambda$.

If $\lambda$, where $L(x,\lambda)$ is positive for all $x$

# Lagrangian Dual.

Find $x$, subject to

$f_i(x) \leq 0, i = 1, \ldots m.$

Remember calculus (constrained optimization.)

Lagrangian: $L(x,\lambda) = \sum_{i=1}^{m} \lambda_i f_i(x)$

$\lambda_i \geq 0$ - Lagrangian multiplier for inequality $i$.

For feasible solution $x$, $L(x,\lambda)$ is

(A) non-negative in expectation

(B) positive for any $\lambda$.

(C) non-positive for any valid $\lambda$.

If $\lambda$, where $L(x,\lambda)$ is positive for all $x$

# Lagrangian Dual.

Find $x$, subject to

$f_i(x) \leq 0, i = 1, \ldots m.$

Remember calculus (constrained optimization.)

Lagrangian: $L(x,\lambda) = \sum_{i=1}^{m} \lambda_i f_i(x)$

$\lambda_i \geq 0$ - Lagrangian multiplier for inequality $i$.

For feasible solution $x$, $L(x,\lambda)$ is

(A) non-negative in expectation

(B) positive for any $\lambda$.

(C) non-positive for any valid $\lambda$.

If $\lambda$, where $L(x,\lambda)$ is positive for all $x$

(A) there is no feasible $x$.

# Lagrangian Dual.

Find $x$, subject to

$f_i(x) \leq 0, i = 1, \ldots m.$

Remember calculus (constrained optimization.)

Lagrangian: $L(x,\lambda) = \sum_{i=1}^{m} \lambda_i f_i(x)$

$\lambda_i \geq 0$ - Lagrangian multiplier for inequality $i$.

For feasible solution $x$, $L(x,\lambda)$ is

(A) non-negative in expectation

(B) positive for any $\lambda$.

(C) non-positive for any valid $\lambda$.

If $\lambda$, where $L(x,\lambda)$ is positive for all $x$

(A) there is no feasible $x$.

(B) there is no $x, \lambda$ with $L(x,\lambda) < 0$.

# Lagrangian:constrained optimization.

$$\begin{aligned} \min \quad & f(x) \\ \text{subject to } f_i(x) \leq 0, \quad & i = 1, ..., m \end{aligned}$$

# Lagrangian:constrained optimization.

$$\begin{aligned} \min \quad & f(x) \\ \text{subject to } & f_i(x) \leq 0, \qquad i = 1, ..., m \end{aligned}$$

Lagrangian function:

# Lagrangian:constrained optimization.

$$\begin{array}{rll} \min & f(x) & \\ \text{subject to } f_i(x) \leq 0, & & i = 1, ..., m \end{array}$$

Lagrangian function:

$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$

# Lagrangian:constrained optimization.

$$\begin{aligned} \min \quad & f(x) \\ \text{subject to } f_i(x) \leq 0, & \qquad i = 1, ..., m \end{aligned}$$

Lagrangian function:

$L(x, \lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$

If (primal) $x$ value $v$

# Lagrangian:constrained optimization.

$$\begin{aligned} \min \quad & f(x) \\ \text{subject to } f_i(x) \leq 0, \quad & i = 1, ..., m \end{aligned}$$

Lagrangian function:

$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$

If (primal) $x$ value $v$

For all $\lambda \geq 0$ with $L(x,\lambda) \leq v$

# Lagrangian:constrained optimization.

$$\begin{aligned} \min \quad & f(x) \\ \text{subject to } f_i(x) \leq 0, \quad & i = 1, \ldots, m \end{aligned}$$

Lagrangian function:

$L(x, \lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$

If (primal) $x$ value $v$

For all $\lambda \geq 0$ with $L(x, \lambda) \leq v$

Maximizing $\lambda$ only positive when $f_i(x) = 0$.

# Lagrangian:constrained optimization.

$$\begin{aligned} \min \quad & f(x) \\ \text{subject to } f_i(x) \leq 0, \quad & i = 1,...,m \end{aligned}$$

Lagrangian function:

$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$

If (primal) $x$ value $v$

For all $\lambda \geq 0$ with $L(x,\lambda) \leq v$

Maximizing $\lambda$ only positive when $f_i(x) = 0$.

# Lagrangian:constrained optimization.

$$\begin{aligned} \min \quad & f(x) \\ \text{subject to } f_i(x) \leq 0, \quad & i = 1, ..., m \end{aligned}$$

Lagrangian function:

$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$

If (primal) $x$ value $v$

For all $\lambda \geq 0$ with $L(x,\lambda) \leq v$

Maximizing $\lambda$ only positive when $f_i(x) = 0$.

If there is $\lambda$ with $L(x,\lambda) \geq \alpha$ for all $x$

# Lagrangian:constrained optimization.

$$\begin{aligned} \min \quad & f(x) \\ \text{subject to } & f_i(x) \leq 0, \qquad i = 1, ..., m \end{aligned}$$

Lagrangian function:

$L(x, \lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$

If (primal) $x$ value $v$
For all $\lambda \geq 0$ with $L(x, \lambda) \leq v$
Maximizing $\lambda$ only positive when $f_i(x) = 0$.

If there is $\lambda$ with $L(x, \lambda) \geq \alpha$ for all $x$
Optimum value of program is at least $\alpha$. $OPT \geq \min_x L(x, \lambda)$

# Lagrangian:constrained optimization.

$$\min \quad f(x)$$
$$\text{subject to } f_i(x) \leq 0, \qquad i = 1, ..., m$$

Lagrangian function:

$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$

If (primal) $x$ value $v$

For all $\lambda \geq 0$ with $L(x,\lambda) \leq v$

Maximizing $\lambda$ only positive when $f_i(x) = 0$.

If there is $\lambda$ with $L(x,\lambda) \geq \alpha$ for all $x$

Optimum value of program is at least $\alpha$. $OPT \geq \min_x L(x,\lambda)$

Why?

# Lagrangian:constrained optimization.

$$\begin{aligned} \min \quad & f(x) \\ \text{subject to } f_i(x) \le 0, \quad & i = 1, ..., m \end{aligned}$$

Lagrangian function:

$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$

If (primal) $x$ value $v$

For all $\lambda \ge 0$ with $L(x,\lambda) \le v$

Maximizing $\lambda$ only positive when $f_i(x) = 0$.

If there is $\lambda$ with $L(x,\lambda) \ge \alpha$ for all $x$

Optimum value of program is at least $\alpha$. $OPT \ge \min_x L(x,\lambda)$

Why? A feasible solution has $f_i(x) \le 0$, so $L(x,\lambda) \le f(x)$.

# Lagrangian:constrained optimization.

$$\begin{aligned} \min \quad & f(x) \\ \text{subject to } f_i(x) \le 0, \quad & i = 1, ..., m \end{aligned}$$

Lagrangian function:

$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$

If (primal) $x$ value $v$

For all $\lambda \geq 0$ with $L(x,\lambda) \leq v$

Maximizing $\lambda$ only positive when $f_i(x) = 0$.

If there is $\lambda$ with $L(x,\lambda) \geq \alpha$ for all $x$

Optimum value of program is at least $\alpha$. $OPT \geq \min_x L(x,\lambda)$

Why? A feasible solution has $f_i(x) \leq 0$, so $L(x,\lambda) \leq f(x)$.

# Saddle point.

Lagrangian function:

# Saddle point.

Lagrangian function:

$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$

# Saddle point.

Lagrangian function:

$$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$$

Primal problem.

# Saddle point.

Lagrangian function:

$$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$$

Primal problem.

*x*-player "best defense":

# Saddle point.

Lagrangian function:

$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$

Primal problem.

$x$-player "best defense":

$\min_x \max_\lambda L(x,\lambda)$.

# Saddle point.

Lagrangian function:

$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$

Primal problem.

$x$-player "best defense":

$\min_x \max_\lambda L(x,\lambda)$.

$x$, that minimizes $L(x,\lambda)$ over all $\lambda \geq 0$.

# Saddle point.

Lagrangian function:

$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$

Primal problem.

$x$-player "best defense":

$\min_x \max_\lambda L(x,\lambda)$.

$x$, that minimizes $L(x,\lambda)$ over all $\lambda \geq 0$.

Dual problem:

# Saddle point.

Lagrangian function:

$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$

Primal problem.

$x$-player "best defense":

$\min_x \max_\lambda L(x,\lambda)$.

$x$, that minimizes $L(x,\lambda)$ over all $\lambda \geq 0$.

Dual problem:

$\lambda$-player "best defense":

# Saddle point.

Lagrangian function:

$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$

Primal problem.

$x$-player "best defense":

$\min_x \max_\lambda L(x,\lambda)$.

$x$, that minimizes $L(x,\lambda)$ over all $\lambda \geq 0$.

Dual problem:

$\lambda$-player "best defense":

$\max_\lambda \min_x L(x,\lambda)$.

# Saddle point.

Lagrangian function:

$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$

Primal problem.

$x$-player "best defense":

$\min_x \max_\lambda L(x,\lambda)$.

$x$, that minimizes $L(x,\lambda)$ over all $\lambda \geq 0$.

Dual problem:

$\lambda$-player "best defense":

$\max_\lambda \min_x L(x,\lambda)$.

$\lambda$, that maximizes $L(x,\lambda)$ over all $x$.

# Saddle point.

Lagrangian function:

$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$

Primal problem.

$x$-player "best defense":

$\min_x \max_\lambda L(x,\lambda)$.

$x$, that minimizes $L(x,\lambda)$ over all $\lambda \geq 0$.

Dual problem:

$\lambda$-player "best defense":

$\max_\lambda \min_x L(x,\lambda)$.

$\lambda$, that maximizes $L(x,\lambda)$ over all $x$.

Saddle point: $(x,y)$ with both conditions:

# Saddle point.

Lagrangian function:

$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$

Primal problem.

$x$-player "best defense":

$\min_x \max_\lambda L(x,\lambda)$.

$x$, that minimizes $L(x,\lambda)$ over all $\lambda \geq 0$.

Dual problem:

$\lambda$-player "best defense":

$\max_\lambda \min_x L(x,\lambda)$.

$\lambda$, that maximizes $L(x,\lambda)$ over all $x$.

Saddle point: $(x,y)$ with both conditions:

# Saddle point.

Lagrangian function:

$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$

Primal problem.
$x$-player "best defense":
$\min_x \max_\lambda L(x,\lambda)$.
$x$, that minimizes $L(x,\lambda)$ over all $\lambda \geq 0$.

Dual problem:
$\lambda$-player "best defense":
$\max_\lambda \min_x L(x,\lambda)$.
$\lambda$, that maximizes $L(x,\lambda)$ over all $x$.

Saddle point: $(x,y)$ with both conditions:

For $x$-player: $\nabla_\lambda L'(x,\lambda) \leq 0 \implies f_i'(x) \leq 0$.

# Saddle point.

Lagrangian function:

$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$

Primal problem.

$x$-player "best defense":

$\min_x \max_\lambda L(x,\lambda)$.

$x$, that minimizes $L(x,\lambda)$ over all $\lambda \geq 0$.

Dual problem:

$\lambda$-player "best defense":

$\max_\lambda \min_x L(x,\lambda)$.

$\lambda$, that maximizes $L(x,\lambda)$ over all $x$.

Saddle point: $(x,y)$ with both conditions:

For $x$-player: $\nabla_\lambda L'(x,\lambda) \leq 0 \implies f_i'(x) \leq 0$.

# Saddle point.

Lagrangian function:

$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$

Primal problem.
$x$-player "best defense":
$\min_x \max_\lambda L(x,\lambda)$.
$x$, that minimizes $L(x,\lambda)$ over all $\lambda \geq 0$.

Dual problem:
$\lambda$-player "best defense":
$\max_\lambda \min_x L(x,\lambda)$.
$\lambda$, that maximizes $L(x,\lambda)$ over all $x$.

Saddle point: $(x,y)$ with both conditions:

For $x$-player: $\nabla_\lambda L'(x,\lambda) \leq 0 \implies f_i'(x) \leq 0$.

For $\lambda$ player: $L'(x,\lambda) = \nabla_x f(x) + \sum_{i=1}^{m} \lambda_i \nabla_x f_i(x) = 0$.

# Saddle point.

Lagrangian function:

$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$

Primal problem.
$x$-player "best defense":
$\min_x \max_\lambda L(x,\lambda)$.
$x$, that minimizes $L(x,\lambda)$ over all $\lambda \geq 0$.

Dual problem:
$\lambda$-player "best defense":
$\max_\lambda \min_x L(x,\lambda)$.
$\lambda$, that maximizes $L(x,\lambda)$ over all $x$.

Saddle point: $(x,y)$ with both conditions:

For $x$-player: $\nabla_\lambda L'(x,\lambda) \leq 0 \implies f_i'(x) \leq 0$.

For $\lambda$ player: $L'(x,\lambda) = \nabla_x f(x) + \sum_{i=1}^{m} \lambda_i \nabla_x f_i(x) = 0$.

At saddle point. Is $\lambda_i \geq 0$ only if $f_i(x) = 0$?

# Saddle point.

Lagrangian function:

$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i f_i(x)$

Primal problem.
$x$-player "best defense":
$\min_x \max_\lambda L(x,\lambda)$.
$x$, that minimizes $L(x,\lambda)$ over all $\lambda \geq 0$.

Dual problem:
$\lambda$-player "best defense":
$\max_\lambda \min_x L(x,\lambda)$.
$\lambda$, that maximizes $L(x,\lambda)$ over all $x$.

Saddle point: $(x,y)$ with both conditions:

For $x$-player: $\nabla_\lambda L'(x,\lambda) \leq 0 \implies f_i'(x) \leq 0$.

For $\lambda$ player: $L'(x,\lambda) = \nabla_x f(x) + \sum_{i=1}^{m} \lambda_i \nabla_x f_i(x) = 0$.

At saddle point. Is $\lambda_i \geq 0$ only if $f_i(x) = 0$? Yes.

# Linear Program.

$\min cx, Ax \geq b$

# Linear Program.

$\min cx, Ax \geq b$

$$
\begin{aligned}
\min \quad & c \cdot x \\
\text{subject to } b_i - a_i \cdot x \leq 0, \quad & i = 1, \ldots, m
\end{aligned}
$$

# Linear Program.

$\min cx, Ax \geq b$

$$\begin{aligned} \text{min} \quad & c \cdot x \\ \text{subject to } b_i - a_i \cdot x \leq 0, \quad & \quad i = 1, ..., m \end{aligned}$$

Lagrangian (Dual):

# Linear Program.

$\min cx, Ax \geq b$

$$\begin{aligned} &\min \quad c \cdot x \\ &\text{subject to } b_i - a_i \cdot x \leq 0, \qquad i = 1, ..., m \end{aligned}$$

Lagrangian (Dual):

$L(\lambda, x) = cx + \sum_i \lambda_i (b_i - a_i x)$.

# Linear Program.

$\min cx, Ax \geq b$

$$\begin{aligned} \min \quad & c \cdot x \\ \text{subject to } & b_i - a_i \cdot x \leq 0, \qquad i = 1, ..., m \end{aligned}$$

Lagrangian (Dual):

$L(\lambda, x) = cx + \sum_i \lambda_i (b_i - a_i x)$.

or

# Linear Program.

$\min cx, Ax \geq b$

$$\begin{aligned} \min \quad & c \cdot x \\ \text{subject to } & b_i - a_i \cdot x \leq 0, \qquad i = 1, ..., m \end{aligned}$$

Lagrangian (Dual):

$L(\lambda, x) = cx + \sum_i \lambda_i (b_i - a_i x)$.

or

$L(\lambda, x) = -(\sum_j x_j (a_j \lambda - c_j)) + b\lambda$.

# Linear Program.

$\min cx, Ax \geq b$

$$\begin{aligned} \min \quad & c \cdot x \\ \text{subject to } b_i - a_i \cdot x \leq 0, \quad & i = 1, ..., m \end{aligned}$$

Lagrangian (Dual):

$L(\lambda, x) = cx + \sum_i \lambda_i (b_i - a_i x)$.

or

$L(\lambda, x) = -(\sum_j x_j (a_j \lambda - c_j)) + b\lambda$.

Best $\lambda$?

# Linear Program.

$\min cx, Ax \geq b$

$$\begin{aligned} \min \quad & c \cdot x \\ \text{subject to } & b_i - a_i \cdot x \leq 0, \qquad i = 1, ..., m \end{aligned}$$

Lagrangian (Dual):

$L(\lambda, x) = cx + \sum_i \lambda_i (b_i - a_i x)$.

or

$L(\lambda, x) = -(\sum_j x_j (a_j \lambda - c_j)) + b\lambda$.

Best $\lambda$?

# Linear Program.

$\min cx, Ax \geq b$

$$\begin{array}{rll} \min & c \cdot x & \\ \text{subject to } & b_i - a_i \cdot x \leq 0, & i = 1, ..., m \end{array}$$

Lagrangian (Dual):

$L(\lambda, x) = cx + \sum_i \lambda_i (b_i - a_i x)$.

or

$L(\lambda, x) = -(\sum_j x_j (a_j \lambda - c_j)) + b\lambda$.

Best $\lambda$?

$\max b \cdot \lambda$

# Linear Program.

$\min cx, Ax \geq b$

$$\begin{array}{rl} \min & c \cdot x \\ \text{subject to } b_i - a_i \cdot x \leq 0, & i = 1, ..., m \end{array}$$

Lagrangian (Dual):

$L(\lambda, x) = cx + \sum_i \lambda_i (b_i - a_i x)$.

or

$L(\lambda, x) = -(\sum_j x_j (a_j \lambda - c_j)) + b\lambda$.

Best $\lambda$?

$\max b \cdot \lambda$ where $a_j \lambda = c_j$.

# Linear Program.

$\min cx, Ax \geq b$

$$\begin{array}{lll} \min & c \cdot x & \\ \text{subject to } & b_i - a_i \cdot x \leq 0, & i = 1, ..., m \end{array}$$

Lagrangian (Dual):

$L(\lambda, x) = cx + \sum_i \lambda_i (b_i - a_i x)$.

or

$L(\lambda, x) = -(\sum_j x_j (a_j \lambda - c_j)) + b\lambda$.

Best $\lambda$?

$\max b \cdot \lambda$ where $a_j \lambda = c_j$.

$\max b\lambda$,

# Linear Program.

$\min cx, Ax \geq b$

$$\begin{aligned} \min \quad & c \cdot x \\ \text{subject to } & b_i - a_i \cdot x \leq 0, \qquad i = 1, ..., m \end{aligned}$$

Lagrangian (Dual):

$L(\lambda, x) = cx + \sum_i \lambda_i (b_i - a_i x)$.

or

$L(\lambda, x) = -(\sum_j x_j (a_j \lambda - c_j)) + b\lambda$.

Best $\lambda$?

$\max b \cdot \lambda$ where $a_j \lambda = c_j$.

$\max b\lambda, \lambda^T A = c$,

# Linear Program.

$\min cx, Ax \geq b$

$$\begin{array}{rl} \min & c \cdot x \\ \text{subject to } b_i - a_i \cdot x \leq 0, & i = 1, ..., m \end{array}$$

Lagrangian (Dual):

$L(\lambda, x) = cx + \sum_i \lambda_i (b_i - a_i x)$.

or

$L(\lambda, x) = -(\sum_j x_j (a_j \lambda - c_j)) + b\lambda$.

Best $\lambda$?

$\max b \cdot \lambda$ where $a_j \lambda = c_j$.

$\max b\lambda, \lambda^T A = c, \lambda \geq 0$

# Linear Program.

$\min cx, Ax \geq b$

$$\begin{aligned} \min \quad & c \cdot x \\ \text{subject to } & b_i - a_i \cdot x \leq 0, \qquad i = 1, ..., m \end{aligned}$$

Lagrangian (Dual):

$L(\lambda, x) = cx + \sum_i \lambda_i (b_i - a_i x)$.

or

$L(\lambda, x) = -(\sum_j x_j (a_j \lambda - c_j)) + b\lambda$.

Best $\lambda$?

$\max b \cdot \lambda$ where $a_j \lambda = c_j$.

$\max b\lambda, \lambda^T A = c, \lambda \geq 0$

Saddle point: complementary slackness.

# Newton's method for root finding.

Find a root of $f(x)$.

# Newton's method for root finding.

Find a root of $f(x)$.

At $x$.

# Newton's method for root finding.

Find a root of $f(x)$.

At $x$.

$$0 = f(x) + f'(x)(t - x).$$

# Newton's method for root finding.

Find a root of $f(x)$.

At $x$.

$0 = f(x) + f'(x)(t - x). \implies t = x - \frac{f(x)}{f'(x)}$

# Newton's method for root finding.

Find a root of $f(x)$.

At $x$.

$0 = f(x) + f'(x)(t - x). \implies t = x - \frac{f(x)}{f'(x)}$

$x_{n+1} = x_n - \frac{f(x)}{f'(x)}$.

# Convergence Analysis.

Choose $\alpha$ where $f(\alpha) = 0$.

# Convergence Analysis.

Choose $\alpha$ where $f(\alpha) = 0$.

$f(\alpha) = f(x) + f'(x_n)(\alpha - x) + R$

# Convergence Analysis.

Choose $\alpha$ where $f(\alpha) = 0$.

$f(\alpha) = f(x) + f'(x_n)(\alpha - x) + R$

$R = \frac{1}{2!} f''(\psi)(\alpha - x)^2$

# Convergence Analysis.

Choose $\alpha$ where $f(\alpha) = 0$.

$f(\alpha) = f(x) + f'(x_n)(\alpha - x) + R$

$R = \frac{1}{2!} f''(\psi)(\alpha - x)^2$

For some $\psi \in [\alpha, x]$. (Assume $\alpha < x$.)

# Convergence Analysis.

Choose $\alpha$ where $f(\alpha) = 0$.

$f(\alpha) = f(x) + f'(x_n)(\alpha - x) + R$

$R = \frac{1}{2!} f''(\psi)(\alpha - x)^2$

For some $\psi \in [\alpha, x]$. (Assume $\alpha < x$.)

Lagrange form of Taylor's series.

# Convergence Analysis.

Choose $\alpha$ where $f(\alpha) = 0$.

$f(\alpha) = f(x) + f'(x_n)(\alpha - x) + R$

$R = \frac{1}{2!} f''(\psi)(\alpha - x)^2$

For some $\psi \in [\alpha, x]$. (Assume $\alpha < x$.)

Lagrange form of Taylor's series.

$0 = f(\alpha) = f(x) + f'(x)(\alpha - x) + \frac{1}{2} f''(\psi)(\alpha - x)^2$

# Convergence Analysis.

Choose $\alpha$ where $f(\alpha) = 0$.

$f(\alpha) = f(x) + f'(x_n)(\alpha - x) + R$

$R = \frac{1}{2!} f''(\psi)(\alpha - x)^2$

For some $\psi \in [\alpha, x]$. (Assume $\alpha < x$.)

Lagrange form of Taylor's series.

$0 = f(\alpha) = f(x) + f'(x)(\alpha - x) + \frac{1}{2} f''(\psi)(\alpha - x)^2$

Rearrange:

# Convergence Analysis.

Choose $\alpha$ where $f(\alpha) = 0$.

$f(\alpha) = f(x) + f'(x_n)(\alpha - x) + R$

$R = \frac{1}{2!} f''(\psi)(\alpha - x)^2$

For some $\psi \in [\alpha, x]$. (Assume $\alpha < x$.)

Lagrange form of Taylor's series.

$0 = f(\alpha) = f(x) + f'(x)(\alpha - x) + \frac{1}{2} f''(\psi)(\alpha - x)^2$

Rearrange:

$\frac{f(x)}{f'(x)} + (\alpha - x) = \frac{-f''(\psi)}{2f'(x)}(\alpha - x)^2$

# Convergence Analysis.

Choose $\alpha$ where $f(\alpha) = 0$.

$f(\alpha) = f(x) + f'(x_n)(\alpha - x) + R$

$R = \frac{1}{2!} f''(\psi)(\alpha - x)^2$

For some $\psi \in [\alpha, x]$. (Assume $\alpha < x$.)

Lagrange form of Taylor's series.

$0 = f(\alpha) = f(x) + f'(x)(\alpha - x) + \frac{1}{2} f''(\psi)(\alpha - x)^2$

Rearrange:
$\frac{f(x)}{f'(x)} + (\alpha - x) = \frac{-f''(\psi)}{2f'(x)}(\alpha - x)^2$

Let $x' = x - \frac{f(x)}{f'(x)}$

# Convergence Analysis.

Choose $\alpha$ where $f(\alpha) = 0$.

$f(\alpha) = f(x) + f'(x_n)(\alpha - x) + R$

$R = \frac{1}{2!} f''(\psi)(\alpha - x)^2$

For some $\psi \in [\alpha, x]$. (Assume $\alpha < x$.)

Lagrange form of Taylor's series.

$0 = f(\alpha) = f(x) + f'(x)(\alpha - x) + \frac{1}{2} f''(\psi)(\alpha - x)^2$

Rearrange:
$\frac{f(x)}{f'(x)} + (\alpha - x) = \frac{-f''(\psi)}{2f'(x)}(\alpha - x)^2$

Let $x' = x - \frac{f(x)}{f'(x)}$

$\alpha - x' = \frac{-f''(\psi)}{2f'(x)}(\alpha - x)^2$

# Convergence Analysis.

Choose $\alpha$ where $f(\alpha) = 0$.

$f(\alpha) = f(x) + f'(x_n)(\alpha - x) + R$

$R = \frac{1}{2!} f''(\psi)(\alpha - x)^2$

For some $\psi \in [\alpha, x]$. (Assume $\alpha < x$.)

Lagrange form of Taylor's series.

$0 = f(\alpha) = f(x) + f'(x)(\alpha - x) + \frac{1}{2} f''(\psi)(\alpha - x)^2$

Rearrange:

$\frac{f(x)}{f'(x)} + (\alpha - x) = \frac{-f''(\psi)}{2f'(x)}(\alpha - x)^2$

Let $x' = x - \frac{f(x)}{f'(x)}$

$\alpha - x' = \frac{-f''(\psi)}{2f'(x)}(\alpha - x)^2$

$|\alpha - x'| = |\frac{f''(\psi)}{2f'(x)}|(\alpha - x)^2$.

# Convergence Analysis.

Choose $\alpha$ where $f(\alpha) = 0$.

$f(\alpha) = f(x) + f'(x_n)(\alpha - x) + R$

$R = \frac{1}{2!} f''(\psi)(\alpha - x)^2$

For some $\psi \in [\alpha, x]$. (Assume $\alpha < x$.)

Lagrange form of Taylor's series.

$0 = f(\alpha) = f(x) + f'(x)(\alpha - x) + \frac{1}{2} f''(\psi)(\alpha - x)^2$

Rearrange:
$\frac{f(x)}{f'(x)} + (\alpha - x) = \frac{-f''(\psi)}{2f'(x)}(\alpha - x)^2$

Let $x' = x - \frac{f(x)}{f'(x)}$

$\alpha - x' = \frac{-f''(\psi)}{2f'(x)}(\alpha - x)^2$

$|\alpha - x'| = |\frac{f''(\psi)}{2f'(x)}|(\alpha - x)^2$.

If $|\alpha - x| < \varepsilon$

# Convergence Analysis.

Choose $\alpha$ where $f(\alpha) = 0$.

$f(\alpha) = f(x) + f'(x_n)(\alpha - x) + R$

$R = \frac{1}{2!} f''(\psi)(\alpha - x)^2$

For some $\psi \in [\alpha, x]$. (Assume $\alpha < x$.)

Lagrange form of Taylor's series.

$0 = f(\alpha) = f(x) + f'(x)(\alpha - x) + \frac{1}{2} f''(\psi)(\alpha - x)^2$

Rearrange:

$\frac{f(x)}{f'(x)} + (\alpha - x) = \frac{-f''(\psi)}{2f'(x)}(\alpha - x)^2$

Let $x' = x - \frac{f(x)}{f'(x)}$

$\alpha - x' = \frac{-f''(\psi)}{2f'(x)}(\alpha - x)^2$

$|\alpha - x'| = |\frac{f''(\psi)}{2f'(x)}|(\alpha - x)^2$.

If $|\alpha - x| < \varepsilon \implies |\alpha - x| \leq |\frac{f''(\psi)}{2f'(x)}|\varepsilon^2$

# Lagrange form for Taylor's Theorem: skipping.

$F(t) = f(t) + f'(t)(x-t) + \frac{f''(t)}{2!}(x-t)^2 \cdots \frac{f^{(k)}}{k!}(x-t)^k$ for $t \in [a,x]$.

# Lagrange form for Taylor's Theorem: skipping.

$F(t) = f(t) + f'(t)(x-t) + \frac{f''(t)}{2!}(x-t)^2 \cdots \frac{f^{(k)}}{k!}(x-t)^k$ for $t \in [a,x]$.

Note $F(x) - F(a) = f(x) - F(a) = R(x)$.

# Lagrange form for Taylor's Theorem: skipping.

$F(t) = f(t) + f'(t)(x-t) + \frac{f''(t)}{2!}(x-t)^2 \cdots \frac{f^{(k)}}{k!}(x-t)^k$ for $t \in [a,x]$.

Note $F(x) - F(a) = f(x) - F(a) = R(x)$. Remainder in Taylor's.

# Lagrange form for Taylor's Theorem: skipping.

$F(t) = f(t) + f'(t)(x-t) + \frac{f''(t)}{2!}(x-t)^2 \cdots \frac{f^{(k)}}{k!}(x-t)^k$ for $t \in [a,x]$.

Note $F(x) - F(a) = f(x) - F(a) = R(x)$. Remainder in Taylor's.

The mean value theorem: There is $\psi \in [a,x]$, where

# Lagrange form for Taylor's Theorem: skipping.

$F(t) = f(t) + f'(t)(x-t) + \frac{f''(t)}{2!}(x-t)^2 \cdots \frac{f^{(k)}}{k!}(x-t)^k$ for $t \in [a,x]$.

Note $F(x) - F(a) = f(x) - F(a) = R(x)$. Remainder in Taylor's.

The mean value theorem: There is $\psi \in [a,x]$, where

$\frac{F'(\psi)}{G'(\psi)} = \frac{F(x)-F(a)}{G(x)-G(a)}$

# Lagrange form for Taylor's Theorem: skipping.

$F(t) = f(t) + f'(t)(x-t) + \frac{f''(t)}{2!}(x-t)^2 \cdots \frac{f^{(k)}}{k!}(x-t)^k$ for $t \in [a,x]$.

Note $F(x) - F(a) = f(x) - F(a) = R(x)$. Remainder in Taylor's.

The mean value theorem: There is $\psi \in [a,x]$, where

$\frac{F'(\psi)}{G'(\psi)} = \frac{F(x)-F(a)}{G(x)-G(a)}$

General version of $G(x) = x$:

# Lagrange form for Taylor's Theorem: skipping.

$F(t) = f(t) + f'(t)(x-t) + \frac{f''(t)}{2!}(x-t)^2 \cdots \frac{f^{(k)}}{k!}(x-t)^k$ for $t \in [a,x]$.

Note $F(x) - F(a) = f(x) - F(a) = R(x)$. Remainder in Taylor's.

The mean value theorem: There is $\psi \in [a,x]$, where

$\frac{F'(\psi)}{G'(\psi)} = \frac{F(x)-F(a)}{G(x)-G(a)}$

General version of $G(x) = x$: $\psi$ has slope equal to average slope.

# Lagrange form for Taylor's Theorem: skipping.

$F(t) = f(t) + f'(t)(x-t) + \frac{f''(t)}{2!}(x-t)^2 \cdots \frac{f^{(k)}}{k!}(x-t)^k$ for $t \in [a,x]$.

Note $F(x) - F(a) = f(x) - F(a) = R(x)$. Remainder in Taylor's.

The mean value theorem: There is $\psi \in [a,x]$, where

$\frac{F'(\psi)}{G'(\psi)} = \frac{F(x)-F(a)}{G(x)-G(a)}$

General version of $G(x) = x$: $\psi$ has slope equal to average slope.

$F'(t)$?

# Lagrange form for Taylor's Theorem: skipping.

$F(t) = f(t) + f'(t)(x-t) + \frac{f''(t)}{2!}(x-t)^2 \cdots \frac{f^{(k)}}{k!}(x-t)^k$ for $t \in [a,x]$.

Note $F(x) - F(a) = f(x) - F(a) = R(x)$. Remainder in Taylor's.

The mean value theorem: There is $\psi \in [a,x]$, where

$\frac{F'(\psi)}{G'(\psi)} = \frac{F(x)-F(a)}{G(x)-G(a)}$

General version of $G(x) = x$: $\psi$ has slope equal to average slope.

$F'(t)$? Use product rule on $k$th term:

# Lagrange form for Taylor's Theorem: skipping.

$F(t) = f(t) + f'(t)(x-t) + \frac{f''(t)}{2!}(x-t)^2 \cdots \frac{f^{(k)}}{k!}(x-t)^k$ for $t \in [a,x]$.

Note $F(x) - F(a) = f(x) - F(a) = R(x)$. Remainder in Taylor's.

The mean value theorem: There is $\psi \in [a,x]$, where

$\frac{F'(\psi)}{G'(\psi)} = \frac{F(x)-F(a)}{G(x)-G(a)}$

General version of $G(x) = x$: $\psi$ has slope equal to average slope.

$F'(t)$? Use product rule on $k$th term:

$\frac{f^{(k+1)}(t)}{k!}(x-t)^k - \frac{f^{(i)}(t)}{(k-1)!}(x-t)^{k-1}$.

# Lagrange form for Taylor's Theorem: skipping.

$F(t) = f(t) + f'(t)(x-t) + \frac{f''(t)}{2!}(x-t)^2 \cdots \frac{f^{(k)}}{k!}(x-t)^k$ for $t \in [a,x]$.

Note $F(x) - F(a) = f(x) - F(a) = R(x)$. Remainder in Taylor's.

The mean value theorem: There is $\psi \in [a,x]$, where

$\frac{F'(\psi)}{G'(\psi)} = \frac{F(x)-F(a)}{G(x)-G(a)}$

General version of $G(x) = x$: $\psi$ has slope equal to average slope.

$F'(t)$? Use product rule on $k$th term:

$\frac{f^{(k+1)}(t)}{k!}(x-t)^k - \frac{f^{(i)}(t)}{(k-1)!}(x-t)^{k-1}$.

Successive terms telescope:

# Lagrange form for Taylor's Theorem: skipping.

$F(t) = f(t) + f'(t)(x-t) + \frac{f''(t)}{2!}(x-t)^2 \cdots \frac{f^{(k)}}{k!}(x-t)^k$ for $t \in [a,x]$.

Note $F(x) - F(a) = f(x) - F(a) = R(x)$. Remainder in Taylor's.

The mean value theorem: There is $\psi \in [a,x]$, where

$\frac{F'(\psi)}{G'(\psi)} = \frac{F(x)-F(a)}{G(x)-G(a)}$

General version of $G(x) = x$: $\psi$ has slope equal to average slope.

$F'(t)$? Use product rule on $k$th term:

$\frac{f^{(k+1)}(t)}{k!}(x-t)^k - \frac{f^{(i)}(t)}{(k-1)!}(x-t)^{k-1}$.

Successive terms telescope: E.g.,

# Lagrange form for Taylor's Theorem: skipping.

$F(t) = f(t) + f'(t)(x-t) + \frac{f''(t)}{2!}(x-t)^2 \cdots \frac{f^{(k)}}{k!}(x-t)^k$ for $t \in [a,x]$.

Note $F(x) - F(a) = f(x) - F(a) = R(x)$. Remainder in Taylor's.

The mean value theorem: There is $\psi \in [a,x]$, where

$\frac{F'(\psi)}{G'(\psi)} = \frac{F(x)-F(a)}{G(x)-G(a)}$

General version of $G(x) = x$: $\psi$ has slope equal to average slope.

$F'(t)$? Use product rule on $k$th term:

$\frac{f^{(k+1)}(t)}{k!}(x-t)^k - \frac{f^{(i)}(t)}{(k-1)!}(x-t)^{k-1}$.

Successive terms telescope: E.g.,

$f'(t) + (f''(t)(x-t) - f'(t)(x-t)^0) = f''(t)(x-t)$.

# Lagrange form for Taylor's Theorem: skipping.

$F(t) = f(t) + f'(t)(x-t) + \frac{f''(t)}{2!}(x-t)^2 \cdots \frac{f^{(k)}}{k!}(x-t)^k$ for $t \in [a,x]$.

Note $F(x) - F(a) = f(x) - F(a) = R(x)$. Remainder in Taylor's.

The mean value theorem: There is $\psi \in [a,x]$, where

$\frac{F'(\psi)}{G'(\psi)} = \frac{F(x)-F(a)}{G(x)-G(a)}$

General version of $G(x) = x$: $\psi$ has slope equal to average slope.

$F'(t)$? Use product rule on $k$th term:
$\frac{f^{(k+1)}(t)}{k!}(x-t)^k - \frac{f^{(i)}(t)}{(k-1)!}(x-t)^{k-1}$.

Successive terms telescope: E.g.,

$f'(t) + (f''(t)(x-t) - f'(t)(x-t)^0) = f''(t)(x-t)$.
So, $F'(t) = \frac{f^{(k+1)}(t)}{k!}(x-t)^k$.

# Lagrange form for Taylor's Theorem: skipping.

$F(t) = f(t) + f'(t)(x-t) + \frac{f''(t)}{2!}(x-t)^2 \cdots \frac{f^{(k)}}{k!}(x-t)^k$ for $t \in [a,x]$.

Note $F(x) - F(a) = f(x) - F(a) = R(x)$. Remainder in Taylor's.

The mean value theorem: There is $\psi \in [a,x]$, where

$\frac{F'(\psi)}{G'(\psi)} = \frac{F(x)-F(a)}{G(x)-G(a)}$

General version of $G(x) = x$: $\psi$ has slope equal to average slope.

$F'(t)$? Use product rule on $k$th term:
$\frac{f^{(k+1)}(t)}{k!}(x-t)^k - \frac{f^{(i)}(t)}{(k-1)!}(x-t)^{k-1}$.

Successive terms telescope: E.g.,

$f'(t) + (f''(t)(x-t) - f'(t)(x-t)^0) = f''(t)(x-t)$.
So, $F'(t) = \frac{f^{(k+1)}(t)}{k!}(x-t)^k$.

$R(x) = F(x) - F(a) = \frac{f^{(k+1)}(\psi)}{k!}(x-\psi)^k \frac{G(x)-G(a)}{G'(\psi)}$

# Lagrange form for Taylor's Theorem: skipping.

$F(t) = f(t) + f'(t)(x-t) + \frac{f''(t)}{2!}(x-t)^2 \cdots \frac{f^{(k)}}{k!}(x-t)^k$ for $t \in [a,x]$.

Note $F(x) - F(a) = f(x) - F(a) = R(x)$. Remainder in Taylor's.

The mean value theorem: There is $\psi \in [a,x]$, where

$\frac{F'(\psi)}{G'(\psi)} = \frac{F(x)-F(a)}{G(x)-G(a)}$

General version of $G(x) = x$: $\psi$ has slope equal to average slope.

$F'(t)$? Use product rule on $k$th term:
$\frac{f^{(k+1)}(t)}{k!}(x-t)^k - \frac{f^{(i)}(t)}{(k-1)!}(x-t)^{k-1}$.

Successive terms telescope: E.g.,

$f'(t) + (f''(t)(x-t) - f'(t)(x-t)^0) = f''(t)(x-t)$.
So, $F'(t) = \frac{f^{(k+1)}(t)}{k!}(x-t)^k$.

$R(x) = F(x) - F(a) = \frac{f^{(k+1)}(\psi)}{k!}(x-\psi)^k \frac{G(x)-G(a)}{G'(\psi)}$

Set $G(t) = (x-t)^{k+1}$, $G'(\psi) = -(k+1)(x-t)^k$, $G(a) = (x-a)^{k+1}$, $G(x) = 0$.

# Lagrange form for Taylor's Theorem: skipping.

$F(t) = f(t) + f'(t)(x-t) + \frac{f''(t)}{2!}(x-t)^2 \cdots \frac{f^{(k)}}{k!}(x-t)^k$ for $t \in [a,x]$.

Note $F(x) - F(a) = f(x) - F(a) = R(x)$. Remainder in Taylor's.

The mean value theorem: There is $\psi \in [a,x]$, where

$\frac{F'(\psi)}{G'(\psi)} = \frac{F(x)-F(a)}{G(x)-G(a)}$

General version of $G(x) = x$: $\psi$ has slope equal to average slope.

$F'(t)$? Use product rule on $k$th term:
$\frac{f^{(k+1)}(t)}{k!}(x-t)^k - \frac{f^{(i)}(t)}{(k-1)!}(x-t)^{k-1}$.

Successive terms telescope: E.g.,

$f'(t) + (f''(t)(x-t) - f'(t)(x-t)^0) = f''(t)(x-t)$.
So, $F'(t) = \frac{f^{(k+1)}(t)}{k!}(x-t)^k$.

$R(x) = F(x) - F(a) = \frac{f^{(k+1)}(\psi)}{k!}(x-\psi)^k \frac{G(x)-G(a)}{G'(\psi)}$

Set $G(t) = (x-t)^{k+1}$, $G'(\psi) = -(k+1)(x-t)^k$, $G(a) = (x-a)^{k+1}$, $G(x) = 0$.

$R(x) = F(x) - F(a) = \frac{f^{(k+1)}(\psi)}{(k+1)!}(x-a)^{k+1}$

# Multivariate version.

Vector $x$, functions: $f_1(x), f_2(x), \ldots f_k(x)$.

# Multivariate version.

Vector $x$, functions: $f_1(x), f_2(x), \ldots f_k(x)$.

$x_{n+1} = x_n - J^{-1}(n)F(x_n)$, where $F(x) = (f_1(x), \ldots, f_n(x))$.

# Multivariate version.

Vector $x$, functions: $f_1(x), f_2(x), \ldots f_k(x)$.

$x_{n+1} = x_n - J^{-1}(n)F(x_n)$, where $F(x) = (f_1(x), \ldots, f_n(x))$.

where

$$J = \begin{bmatrix} \nabla f_1(x) \\ \nabla f_2(x) \\ \vdots \end{bmatrix}$$

# Interior point on the central path.

Find $x$, that minimizes $f_0(x)$ subject to

# Interior point on the central path.

Find $x$, that minimizes $f_0(x)$ subject to

$f_i(x) \leq 0, i = 1, \ldots m.$

# Interior point on the central path.

Find $x$, that minimizes $f_0(x)$ subject to

$f_i(x) \leq 0, i = 1, \ldots m.$

Central path:

# Interior point on the central path.

Find $x$, that minimizes $f_0(x)$ subject to

$f_i(x) \leq 0, i = 1, \ldots m.$

Central path:

$\min t f_0(x) - \sum_{i=1}^{m} \ln(-f_i(x))$

# Interior point on the central path.

Find $x$, that minimizes $f_0(x)$ subject to

$f_i(x) \leq 0, i = 1, \dots m.$

Central path:

$\min t f_0(x) - \sum_{i=1}^{m} \ln(-f_i(x))$

The minimizer, $x(t)$, form the **central path.**

# Interior point on the central path.

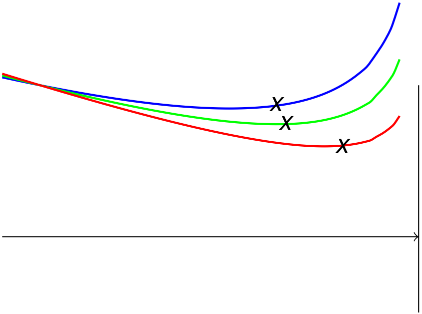
Find $x$, that minimizes $f_0(x)$ subject to

$f_i(x) \le 0, i = 1, \ldots m.$

Central path:

$\min t f_0(x) - \sum_{i=1}^{m} \ln(-f_i(x))$

The minimizer, $x(t)$, form the **central path.**




The sequence of $x$'s are "central path".

# Lagrangian Dual and Central Path.

$\min tf_0(x) - \sum_{i=1} \ln(-f_i(x))$

# Lagrangian Dual and Central Path.

$\min tf_0(x) - \sum_{i=1} \ln(-f_i(x))$

Optimality condition?

# Lagrangian Dual and Central Path.

$\min tf_0(x) - \sum_{i=1} \ln(-f_i(x))$

Optimality condition?

# Lagrangian Dual and Central Path.

$\min t f_0(x) - \sum_{i=1} \ln(-f_i(x))$

Optimality condition?

Derivative:

# Lagrangian Dual and Central Path.

$\min tf_0(x) - \sum_{i=1} \ln(-f_i(x))$

Optimality condition?

Derivative: $t\nabla f_0(x) - \sum_{i=1} \frac{\nabla f_i(x)}{f_i(x)} = 0$

# Lagrangian Dual and Central Path.

$\min tf_0(x) - \sum_{i=1} \ln(-f_i(x))$

Optimality condition?

Derivative: $t\nabla f_0(x) - \sum_{i=1} \frac{\nabla f_i(x)}{f_i(x)} = 0$

Or, $\nabla f_0(x) = \sum_{i=1} \frac{\nabla f_i(x)}{tf_i(x)}$

# Lagrangian Dual and Central Path.

$\min tf_0(x) - \sum_{i=1} \ln(-f_i(x))$

Optimality condition?

Derivative: $t\nabla f_0(x) - \sum_{i=1} \frac{\nabla f_i(x)}{f_i(x)} = 0$

Or, $\nabla f_0(x) = \sum_{i=1} \frac{\nabla f_i(x)}{tf_i(x)}$ (Opposing force fields.)

# Lagrangian Dual and Central Path.

$\min tf_0(x) - \sum_{i=1} \ln(-f_i(x))$

Optimality condition?

Derivative: $t\nabla f_0(x) - \sum_{i=1} \frac{\nabla f_i(x)}{f_i(x)} = 0$

Or, $\nabla f_0(x) = \sum_{i=1} \frac{\nabla f_i(x)}{tf_i(x)}$ (Opposing force fields.)

Recall, Lagrangian: $L(\lambda, x) = f_0(x) + \sum_i \lambda_i f_i(x)$.

# Lagrangian Dual and Central Path.

$\min tf_0(x) - \sum_{i=1} \ln(-f_i(x))$

Optimality condition?

Derivative: $t\nabla f_0(x) - \sum_{i=1} \frac{\nabla f_i(x)}{f_i(x)} = 0$

Or, $\nabla f_0(x) = \sum_{i=1} \frac{\nabla f_i(x)}{tf_i(x)}$ (Opposing force fields.)

Recall, Lagrangian: $L(\lambda, x) = f_0(x) + \sum_i \lambda_i f_i(x)$.

Fix $\lambda$, optimize for $x^*$ give valid lower bound on solution.

# Lagrangian Dual and Central Path.

$\min tf_0(x) - \sum_{i=1} \ln(-f_i(x))$

Optimality condition?

Derivative: $t\nabla f_0(x) - \sum_{i=1} \frac{\nabla f_i(x)}{f_i(x)} = 0$

Or, $\nabla f_0(x) = \sum_{i=1} \frac{\nabla f_i(x)}{tf_i(x)}$ (Opposing force fields.)

Recall, Lagrangian: $L(\lambda, x) = f_0(x) + \sum_i \lambda_i f_i(x)$.

Fix $\lambda$, optimize for $x^*$ give valid lower bound on solution.
Optimality Condition.

# Lagrangian Dual and Central Path.

$\min tf_0(x) - \sum_{i=1} \ln(-f_i(x))$

Optimality condition?

Derivative: $t\nabla f_0(x) - \sum_{i=1} \frac{\nabla f_i(x)}{f_i(x)} = 0$

Or, $\nabla f_0(x) = \sum_{i=1} \frac{\nabla f_i(x)}{tf_i(x)}$ (Opposing force fields.)

Recall, Lagrangian: $L(\lambda, x) = f_0(x) + \sum_i \lambda_i f_i(x)$.

Fix $\lambda$, optimize for $x^*$ give valid lower bound on solution.
Optimality Condition.
Derivative:

# Lagrangian Dual and Central Path.

$\min tf_0(x) - \sum_{i=1} \ln(-f_i(x))$

Optimality condition?

Derivative: $t\nabla f_0(x) - \sum_{i=1} \frac{\nabla f_i(x)}{f_i(x)} = 0$

Or, $\nabla f_0(x) = \sum_{i=1} \frac{\nabla f_i(x)}{tf_i(x)}$ (Opposing force fields.)

Recall, Lagrangian: $L(\lambda, x) = f_0(x) + \sum_i \lambda_i f_i(x)$.

Fix $\lambda$, optimize for $x^*$ give valid lower bound on solution.
Optimality Condition.
Derivative: $\nabla f_0(x) + \sum_{i=1} \lambda_i \nabla f_i(x) = 0$.

# Lagrangian Dual and Central Path.

$\min tf_0(x) - \sum_{i=1} \ln(-f_i(x))$

Optimality condition?

Derivative: $t\nabla f_0(x) - \sum_{i=1} \frac{\nabla f_i(x)}{f_i(x)} = 0$

Or, $\nabla f_0(x) = \sum_{i=1} \frac{\nabla f_i(x)}{tf_i(x)}$ (Opposing force fields.)

Recall, Lagrangian: $L(\lambda, x) = f_0(x) + \sum_i \lambda_i f_i(x)$.

Fix $\lambda$, optimize for $x^*$ give valid lower bound on solution.
Optimality Condition.
Derivative: $\nabla f_0(x) + \sum_{i=1} \lambda_i \nabla f_i(x) = 0$.

Take $\lambda_i^{(t)} = -\frac{1}{tf_i(x)}$.

# Lagrangian Dual and Central Path.

$\min tf_0(x) - \sum_{i=1} \ln(-f_i(x))$

Optimality condition?

Derivative: $t\nabla f_0(x) - \sum_{i=1} \frac{\nabla f_i(x)}{f_i(x)} = 0$

Or, $\nabla f_0(x) = \sum_{i=1} \frac{\nabla f_i(x)}{tf_i(x)}$ (Opposing force fields.)

Recall, Lagrangian: $L(\lambda, x) = f_0(x) + \sum_i \lambda_i f_i(x)$.

Fix $\lambda$, optimize for $x^*$ give valid lower bound on solution.
Optimality Condition.
Derivative: $\nabla f_0(x) + \sum_{i=1} \lambda_i \nabla f_i(x) = 0$.

Take $\lambda_i^{(t)} = -\frac{1}{tf_i(x)}$. $x(t) = x^*(\lambda^{(t)})$!

# Lagrangian Dual and Central Path.

$\min tf_0(x) - \sum_{i=1} \ln(-f_i(x))$

Optimality condition?

Derivative: $t\nabla f_0(x) - \sum_{i=1} \frac{\nabla f_i(x)}{f_i(x)} = 0$ $\nabla f_0(x) - \sum_{i=1} \frac{1}{tf_i(x)} \nabla f_i(x) = 0$

Or, $\nabla f_0(x) = \sum_{i=1} \frac{\nabla f_i(x)}{tf_i(x)}$ (Opposing force fields.)

Recall, Lagrangian: $L(\lambda, x) = f_0(x) + \sum_i \lambda_i f_i(x)$.

Fix $\lambda$, optimize for $x^*$ give valid lower bound on solution.

Optimality Condition.

Derivative: $\nabla f_0(x) + \sum_{i=1} \lambda_i \nabla f_i(x) = 0$.

Take $\lambda_i^{(t)} = -\frac{1}{tf_i(x)}$. $x(t) = x^*(\lambda^{(t)})$! Same optimal point!

# Lagrangian Dual and Central Path.

$\min t f_0(x) - \sum_{i=1} \ln(-f_i(x))$

Optimality condition?

Derivative: $t\nabla f_0(x) - \sum_{i=1} \frac{\nabla f_i(x)}{f_i(x)} = 0$

Or, $\nabla f_0(x) = \sum_{i=1} \frac{\nabla f_i(x)}{t f_i(x)}$ (Opposing force fields.)

Recall, Lagrangian: $L(\lambda, x) = f_0(x) + \sum_i \lambda_i f_i(x)$.

Fix $\lambda$, optimize for $x^*$ give valid lower bound on solution.
Optimality Condition.
Derivative: $\nabla f_0(x) + \sum_{i=1} \lambda_i \nabla f_i(x) = 0$.

Take $\lambda_i^{(t)} = -\frac{1}{t f_i(x)}$. $x(t) = x^*(\lambda^{(t)})$! Same optimal point!

Value? Found $\lambda$ where:

# Lagrangian Dual and Central Path.

$\min tf_0(x) - \sum_{i=1} \ln(-f_i(x))$

Optimality condition?

Derivative: $t\nabla f_0(x) - \sum_{i=1} \frac{\nabla f_i(x)}{f_i(x)} = 0$

Or, $\nabla f_0(x) = \sum_{i=1} \frac{\nabla f_i(x)}{tf_i(x)}$ (Opposing force fields.)

Recall, Lagrangian: $L(\lambda, x) = f_0(x) + \sum_i \lambda_i f_i(x)$.

Fix $\lambda$, optimize for $x^*$ give valid lower bound on solution.

Optimality Condition.

Derivative: $\nabla f_0(x) + \sum_{i=1} \lambda_i \nabla f_i(x) = 0$.

Take $\lambda_i^{(t)} = -\frac{1}{tf_i(x)}$. $x(t) = x^*(\lambda^{(t)})$! Same optimal point!

Value? Found $\lambda$ where:

$\min_x L(\lambda, x) = f_0(x) + \sum_{i=1} \lambda_i f_i(x) = f_0(x) - \frac{m}{t} \leq \min_x \max_\lambda L(\lambda, x)$.

# Lagrangian Dual and Central Path.

$\min t f_0(x) - \sum_{i=1} \ln(-f_i(x))$

Optimality condition?

Derivative: $t\nabla f_0(x) - \sum_{i=1} \frac{\nabla f_i(x)}{f_i(x)} = 0$

Or, $\nabla f_0(x) = \sum_{i=1} \frac{\nabla f_i(x)}{t f_i(x)}$ (Opposing force fields.)

Recall, Lagrangian: $L(\lambda, x) = f_0(x) + \sum_i \lambda_i f_i(x)$.

Fix $\lambda$, optimize for $x^*$ give valid lower bound on solution.

Optimality Condition.

Derivative: $\nabla f_0(x) + \sum_{i=1} \lambda_i \nabla f_i(x) = 0$.

Take $\lambda_i^{(t)} = -\frac{1}{t f_i(x)}$. $x(t) = x^*(\lambda^{(t)})$! Same optimal point!

Value? Found $\lambda$ where:

$\min_x L(\lambda, x) = f_0(x) + \sum_{i=1} \lambda_i f_i(x) = f_0(x) - \frac{m}{t} \leq \min_x \max_\lambda L(\lambda, x)$.

Central point $x(t)$ within $\frac{m}{t}$ of optimal primal!!!!

# Lagrangian Dual and Central Path.

$\min tf_0(x) - \sum_{i=1} \ln(-f_i(x))$

Optimality condition?

Derivative: $t\nabla f_0(x) - \sum_{i=1} \frac{\nabla f_i(x)}{f_i(x)} = 0$

Or, $\nabla f_0(x) = \sum_{i=1} \frac{\nabla f_i(x)}{tf_i(x)}$ (Opposing force fields.)

Recall, Lagrangian: $L(\lambda, x) = f_0(x) + \sum_i \lambda_i f_i(x)$.

Fix $\lambda$, optimize for $x^*$ give valid lower bound on solution.

Optimality Condition.

Derivative: $\nabla f_0(x) + \sum_{i=1} \lambda_i \nabla f_i(x) = 0$.

Take $\lambda_i^{(t)} = -\frac{1}{tf_i(x)}$. $x(t) = x^*(\lambda^{(t)})$! Same optimal point!

Value? Found $\lambda$ where:

$\min_x L(\lambda, x) = f_0(x) + \sum_{i=1} \lambda_i f_i(x) = f_0(x) - \frac{m}{t} \leq \min_x \max_\lambda L(\lambda, x)$.

Central point $x(t)$ within $\frac{m}{t}$ of optimal primal!!!!

$L(\lambda, x(t)) \geq f_0(x) - \frac{m}{t}$

# Lagrangian Dual and Central Path.

$\min tf_0(x) - \sum_{i=1} \ln(-f_i(x))$

Optimality condition?

Derivative: $t\nabla f_0(x) - \sum_{i=1} \frac{\nabla f_i(x)}{f_i(x)} = 0$

Or, $\nabla f_0(x) = \sum_{i=1} \frac{\nabla f_i(x)}{tf_i(x)}$ (Opposing force fields.)

Recall, Lagrangian: $L(\lambda, x) = f_0(x) + \sum_i \lambda_i f_i(x)$.

Fix $\lambda$, optimize for $x^*$ give valid lower bound on solution.
Optimality Condition.
Derivative: $\nabla f_0(x) + \sum_{i=1} \lambda_i \nabla f_i(x) = 0$.

Take $\lambda_i^{(t)} = -\frac{1}{tf_i(x)}$. $x(t) = x^*(\lambda^{(t)})$! Same optimal point!

Value? Found $\lambda$ where:
$\min_x L(\lambda, x) = f_0(x) + \sum_{i=1} \lambda_i f_i(x) = f_0(x) - \frac{m}{t} \leq \min_x \max_\lambda L(\lambda, x)$.

Central point $x(t)$ within $\frac{m}{t}$ of optimal primal!!!!
$L(\lambda, x(t)) \geq f_0(x) - \frac{m}{t} \implies \min_x L(\lambda, x) + \frac{m}{t} \geq f_0(x)$

# Lagrangian Dual and Central Path.

$\min tf_0(x) - \sum_{i=1} \ln(-f_i(x))$

Optimality condition?

Derivative: $t\nabla f_0(x) - \sum_{i=1} \frac{\nabla f_i(x)}{f_i(x)} = 0$

Or, $\nabla f_0(x) = \sum_{i=1} \frac{\nabla f_i(x)}{tf_i(x)}$ (Opposing force fields.)

Recall, Lagrangian: $L(\lambda, x) = f_0(x) + \sum_i \lambda_i f_i(x)$.

Fix $\lambda$, optimize for $x^*$ give valid lower bound on solution.
Optimality Condition.
Derivative: $\nabla f_0(x) + \sum_{i=1} \lambda_i \nabla f_i(x) = 0$.

Take $\lambda_i^{(t)} = -\frac{1}{tf_i(x)}$. $x(t) = x^*(\lambda^{(t)})$! Same optimal point!

Value? Found $\lambda$ where:
$\min_x L(\lambda, x) = f_0(x) + \sum_{i=1} \lambda_i f_i(x) = f_0(x) - \frac{m}{t} \leq \min_x \max_\lambda L(\lambda, x)$.

Central point $x(t)$ within $\frac{m}{t}$ of optimal primal!!!!
$L(\lambda, x(t)) \geq f_0(x) - \frac{m}{t} \implies \min_x L(\lambda, x) + \frac{m}{t} \geq f_0(x)$
$\implies OPT + \frac{m}{t} \geq f_0(x)$

# Central path.

$\min_x f_0(x), f_i(x) \leq 0$.

# Central path.

$\min_x f_0(x), f_i(x) \leq 0.$

$\min_x tf_0(x) - \sum_{i>0} \ln(-f_i(x))$

# Central path.

$\min_x f_0(x), f_i(x) \leq 0$.

$\min_x tf_0(x) - \sum_{i>0} \ln(-f_i(x))$

Optimal: $x(t)$ is feasible.

# Central path.

$\min_x f_0(x), f_i(x) \leq 0.$

$\min_x tf_0(x) - \sum_{i>0} \ln(-f_i(x))$

Optimal: $x(t)$ is feasible.

$f_0(x(t)) \leq OPT + \frac{m}{t}$

# Central path.

$\min_x f_0(x), f_i(x) \leq 0$.

$\min_x tf_0(x) - \sum_{i>0} \ln(-f_i(x))$

Optimal: $x(t)$ is feasible.

$f_0(x(t)) \leq OPT + \frac{m}{t}$

Algorithm: take $t \rightarrow \infty$.

# Central path.

$\min_x f_0(x), f_i(x) \leq 0$.

$\min_x t f_0(x) - \sum_{i>0} \ln(-f_i(x))$

Optimal: $x(t)$ is feasible.

$f_0(x(t)) \leq OPT + \frac{m}{t}$

Algorithm: take $t \to \infty$.

Finding $x(t)$?

# Central path.

$\min_x f_0(x), f_i(x) \leq 0$.

$\min_x tf_0(x) - \sum_{i>0} \ln(-f_i(x))$

Optimal: $x(t)$ is feasible.

$f_0(x(t)) \leq OPT + \frac{m}{t}$

Algorithm: take $t \to \infty$.

Finding $x(t)$?

Assume you have $x(t)$, change $t = \mu t$, for $\mu > 1$.

# Central path.

$\min_x f_0(x), f_i(x) \leq 0.$

$\min_x tf_0(x) - \sum_{i>0} \ln(-f_i(x))$

Optimal: $x(t)$ is feasible.

$f_0(x(t)) \leq OPT + \frac{m}{t}$

Algorithm: take $t \to \infty$.

Finding $x(t)$?

Assume you have $x(t)$, change $t = \mu t$, for $\mu > 1$.
Find $x(\mu t)$.

# Central path.

$\min_x f_0(x), f_i(x) \leq 0.$

$\min_x tf_0(x) - \sum_{i>0} \ln(-f_i(x))$

Optimal: $x(t)$ is feasible.

$f_0(x(t)) \leq OPT + \frac{m}{t}$

Algorithm: take $t \to \infty$.

Finding $x(t)$?

Assume you have $x(t)$, change $t = \mu t$, for $\mu > 1$.
Find $x(\mu t)$.
Idea: newton's method.

# Central path.

$\min_x f_0(x), f_i(x) \leq 0.$

$\min_x tf_0(x) - \sum_{i>0} \ln(-f_i(x))$

Optimal: $x(t)$ is feasible.

$f_0(x(t)) \leq OPT + \frac{m}{t}$

Algorithm: take $t \to \infty$.

Finding $x(t)$?

Assume you have $x(t)$, change $t = \mu t$, for $\mu > 1$.
Find $x(\mu t)$.
Idea: newton's method.

# Central path.

$\min_x f_0(x), f_i(x) \leq 0.$

$\min_x tf_0(x) - \sum_{i>0} \ln(-f_i(x))$

Optimal: $x(t)$ is feasible.

$f_0(x(t)) \leq OPT + \frac{m}{t}$

Algorithm: take $t \to \infty$.

Finding $x(t)$?

Assume you have $x(t)$, change $t = \mu t$, for $\mu > 1$.
Find $x(\mu t)$.
Idea: newton's method.
Should show new optimal point not too different from old.

# Central path.

$\min_x f_0(x), f_i(x) \leq 0$.

$\min_x tf_0(x) - \sum_{i>0} \ln(-f_i(x))$

Optimal: $x(t)$ is feasible.

$f_0(x(t)) \leq OPT + \frac{m}{t}$

Algorithm: take $t \to \infty$.

Finding $x(t)$?

Assume you have $x(t)$, change $t = \mu t$, for $\mu > 1$.
Find $x(\mu t)$.
Idea: newton's method.
Should show new optimal point not too different from old.

Next.

# Central Path evolution.

Old point $x = x(t)$ versus $x^+ = x(\mu t)$? Minimizing: $\mu t f_0(x) - \sum_{i=1}^n \ln(-f_i(t))$.

# Central Path evolution.

Old point $x = x(t)$ versus $x^+ = x(\mu t)$? Minimizing: $\mu t f_0(x) - \sum_{i=1}^{n} \ln(-f_i(t))$.

Difference in new objective from old optimal point to new:

# Central Path evolution.

Old point $x = x(t)$ versus $x^+ = x(\mu t)$? Minimizing: $\mu t f_0(x) - \sum_{i=1}^{n} \ln(-f_i(t))$.

Difference in new objective from old optimal point to new:

$\mu t f_0(x) - \sum_{i=1}^{m} \ln(-f_i(x)) - \mu t f_0(x^+) + \sum_{i=1}^{m} \ln(-f_i(x^+))$

# Central Path evolution.

Old point $x = x(t)$ versus $x^+ = x(\mu t)$? Minimizing: $\mu t f_0(x) - \sum_{i=1}^{n} \ln(-f_i(t))$.

Difference in new objective from old optimal point to new:

$$\mu t f_0(x) - \sum_{i=1}^{m} \ln(-f_i(x)) - \mu t f_0(x^+) + \sum_{i=1}^{m} \ln(-f_i(x^+))$$

Simplify: $\mu t f_0(x) - \mu t f_0(x^+) + \sum_i \ln(-\frac{f_i(x^+)}{f_i(x)})$

# Central Path evolution.

Old point $x = x(t)$ versus $x^+ = x(\mu t)$? Minimizing: $\mu t f_0(x) - \sum_{i=1}^{n} \ln(-f_i(t))$.

Difference in new objective from old optimal point to new:

$$\mu t f_0(x) - \sum_{i=1}^{m} \ln(-f_i(x)) - \mu t f_0(x^+) + \sum_{i=1}^{m} \ln(-f_i(x^+))$$

Simplify: $\mu t f_0(x) - \mu t f_0(x^+) + \sum_i \ln(-\frac{f_i(x^+)}{f_i(x)})$

Let $\lambda_i = -\frac{1}{t f_i(x)}$. Remember: $L(\lambda, x) = f_0(x) + \sum_i \lambda_i f_i(x)$.

# Central Path evolution.

Old point $x = x(t)$ versus $x^+ = x(\mu t)$? Minimizing: $\mu t f_0(x) - \sum_{i=1}^{n} \ln(-f_i(t))$.

Difference in new objective from old optimal point to new:

$$\mu t f_0(x) - \sum_{i=1}^{m} \ln(-f_i(x)) - \mu t f_0(x^+) + \sum_{i=1}^{m} \ln(-f_i(x^+))$$

Simplify: $\mu t f_0(x) - \mu t f_0(x^+) + \sum_i \ln(-\frac{f_i(x^+)}{f_i(x)})$

Let $\lambda_i = -\frac{1}{t f_i(x)}$. Remember: $L(\lambda, x) = f_0(x) + \sum_i \lambda_i f_i(x)$.

$$f_0(x) - L(\lambda, x') \leq \frac{m}{t}$$

# Central Path evolution.

Old point $x = x(t)$ versus $x^+ = x(\mu t)$? Minimizing: $\mu t f_0(x) - \sum_{i=1}^{n} \ln(-f_i(t))$.

Difference in new objective from old optimal point to new:

$$\mu t f_0(x) - \sum_{i=1}^{m} \ln(-f_i(x)) - \mu t f_0(x^+) + \sum_{i=1}^{m} \ln(-f_i(x^+))$$

Simplify: $\mu t f_0(x) - \mu t f_0(x^+) + \sum_i \ln(-\frac{f_i(x^+)}{f_i(x)})$

Let $\lambda_i = -\frac{1}{tf_i(x)}$. Remember: $L(\lambda, x) = f_0(x) + \sum_i \lambda_i f_i(x)$.

$$f_0(x) - L(\lambda, x') \leq \frac{m}{t} \text{ since } \sum_i \lambda_i f_i(x) = -\frac{m}{t}.$$

# Central Path evolution.

Old point $x = x(t)$ versus $x^+ = x(\mu t)$? Minimizing: $\mu t f_0(x) - \sum_{i=1}^{n} \ln(-f_i(t))$.

Difference in new objective from old optimal point to new:

$$\mu t f_0(x) - \sum_{i=1}^{m} \ln(-f_i(x)) - \mu t f_0(x^+) + \sum_{i=1}^{m} \ln(-f_i(x^+))$$

Simplify: $\mu t f_0(x) - \mu t f_0(x^+) + \sum_i \ln(-\frac{f_i(x^+)}{f_i(x)})$

Let $\lambda_i = -\frac{1}{t f_i(x)}$. Remember: $L(\lambda, x) = f_0(x) + \sum_i \lambda_i f_i(x)$.

$f_0(x) - L(\lambda, x') \leq \frac{m}{t}$ since $\sum_i \lambda_i f_i(x) = -\frac{m}{t}$.

$\mu t f_0(x) - \mu t f_0(x^+) + \sum_i \ln(-\mu t \lambda_i f_i(x^+)) - m \ln \mu$

# Central Path evolution.

Old point $x = x(t)$ versus $x^+ = x(\mu t)$? Minimizing: $\mu t f_0(x) - \sum_{i=1}^{n} \ln(-f_i(t))$.

Difference in new objective from old optimal point to new:

$$\mu t f_0(x) - \sum_{i=1}^{m} \ln(-f_i(x)) - \mu t f_0(x^+) + \sum_{i=1}^{m} \ln(-f_i(x^+))$$

Simplify: $\mu t f_0(x) - \mu t f_0(x^+) + \sum_i \ln(-\frac{f_i(x^+)}{f_i(x)})$

Let $\lambda_i = -\frac{1}{t f_i(x)}$. Remember: $L(\lambda, x) = f_0(x) + \sum_i \lambda_i f_i(x)$.

$$f_0(x) - L(\lambda, x') \leq \frac{m}{t} \text{ since } \sum_i \lambda_i f_i(x) = -\frac{m}{t}.$$

$\mu t f_0(x) - \mu t f_0(x^+) + \sum_i \ln(-\mu t \lambda_i f_i(x^+)) - m \ln \mu$

$\ln(-x) = ln(1 - (1 + x)) \leq -(1 + x)$

$$\leq \mu t f_0(x) - \mu t f_0(x^+) - \sum_i (1 + \lambda_i \mu t f_i(x^+)) - m \ln \mu$$

# Central Path evolution.

Old point $x = x(t)$ versus $x^+ = x(\mu t)$? Minimizing: $\mu t f_0(x) - \sum_{i=1}^{n} \ln(-f_i(t))$.

Difference in new objective from old optimal point to new:

$$\mu t f_0(x) - \sum_{i=1}^{m} \ln(-f_i(x)) - \mu t f_0(x^+) + \sum_{i=1}^{m} \ln(-f_i(x^+))$$

Simplify: $\mu t f_0(x) - \mu t f_0(x^+) + \sum_i \ln(-\frac{f_i(x^+)}{f_i(x)})$

Let $\lambda_i = -\frac{1}{tf_i(x)}$. Remember: $L(\lambda, x) = f_0(x) + \sum_i \lambda_i f_i(x)$.

$$f_0(x) - L(\lambda, x') \leq \frac{m}{t} \text{ since } \sum_i \lambda_i f_i(x) = -\frac{m}{t}.$$

$\mu t f_0(x) - \mu t f_0(x^+) + \sum_i \ln(-\mu t \lambda_i f_i(x^+)) - m \ln \mu$

$$\ln(-x) = ln(1 - (1 + x)) \leq -(1 + x)$$

$$\leq \mu t f_0(x) - \mu t f_0(x^+) - \sum_i (1 + \lambda_i \mu t f_i(x^+)) - m \ln \mu$$

$$= \mu t f_0(x) - \mu t f_0(x^+) - \mu t \sum_i \lambda_i f_i(x^+) - m - m \ln \mu$$

# Central Path evolution.

Old point $x = x(t)$ versus $x^+ = x(\mu t)$? Minimizing: $\mu t f_0(x) - \sum_{i=1}^n \ln(-f_i(t))$.

Difference in new objective from old optimal point to new:

$$\mu t f_0(x) - \sum_{i=1}^m \ln(-f_i(x)) - \mu t f_0(x^+) + \sum_{i=1}^m \ln(-f_i(x^+))$$

Simplify: $\mu t f_0(x) - \mu t f_0(x^+) + \sum_i \ln(-\frac{f_i(x^+)}{f_i(x)})$

Let $\lambda_i = -\frac{1}{t f_i(x)}$. Remember: $L(\lambda, x) = f_0(x) + \sum_i \lambda_i f_i(x)$.

$$f_0(x) - L(\lambda, x') \leq \frac{m}{t} \text{ since } \sum_i \lambda_i f_i(x) = -\frac{m}{t}.$$

$\mu t f_0(x) - \mu t f_0(x^+) + \sum_i \ln(-\mu t \lambda_i f_i(x^+)) - m \ln \mu$

$$\ln(-x) = ln(1 - (1 + x)) \leq -(1 + x)$$

$$\leq \mu t f_0(x) - \mu t f_0(x^+) - \sum_i (1 + \lambda_i \mu t f_i(x^+)) - m \ln \mu$$

$$= \mu t f_0(x) - \mu t f_0(x^+) - \mu t \sum_i \lambda_i f_i(x^+) - m - m \ln \mu$$

$$= \mu t (f_0(x) - (f_0(x^+) + \sum_i \lambda_i f_i(x^+)) - m - m \ln \mu$$

# Central Path evolution.

Old point $x = x(t)$ versus $x^+ = x(\mu t)$? Minimizing: $\mu t f_0(x) - \sum_{i=1}^{n} \ln(-f_i(t))$.

Difference in new objective from old optimal point to new:

$$\mu t f_0(x) - \sum_{i=1}^{m} \ln(-f_i(x)) - \mu t f_0(x^+) + \sum_{i=1}^{m} \ln(-f_i(x^+))$$

Simplify: $\mu t f_0(x) - \mu t f_0(x^+) + \sum_i \ln(-\frac{f_i(x^+)}{f_i(x)})$

Let $\lambda_i = -\frac{1}{t f_i(x)}$. Remember: $L(\lambda, x) = f_0(x) + \sum_i \lambda_i f_i(x)$.

$f_0(x) - L(\lambda, x') \leq \frac{m}{t}$ since $\sum_i \lambda_i f_i(x) = -\frac{m}{t}$.

$\mu t f_0(x) - \mu t f_0(x^+) + \sum_i \ln(-\mu t \lambda_i f_i(x^+)) - m \ln \mu$

$\ln(-x) = ln(1 - (1 + x)) \leq -(1 + x)$

$$\begin{aligned}
&\leq \mu t f_0(x) - \mu t f_0(x^+) - \textstyle\sum_i (1 + \lambda_i \mu t f_i(x^+)) - m \ln \mu \\
&= \mu t f_0(x) - \mu t f_0(x^+) - \mu t \textstyle\sum_i \lambda_i f_i(x^+) - m - m \ln \mu \\
&= \mu t (f_0(x) - (f_0(x^+) + \textstyle\sum_i \lambda_i f_i(x^+)) - m - m \ln \mu \\
&= \mu t (f_0(x) - L(\lambda, x^+)) - m - m \ln \mu
\end{aligned}$$

# Central Path evolution.

Old point $x = x(t)$ versus $x^+ = x(\mu t)$? Minimizing: $\mu t f_0(x) - \sum_{i=1}^{n} \ln(-f_i(t))$.

Difference in new objective from old optimal point to new:

$$\mu t f_0(x) - \sum_{i=1}^{m} \ln(-f_i(x)) - \mu t f_0(x^+) + \sum_{i=1}^{m} \ln(-f_i(x^+))$$

Simplify: $\mu t f_0(x) - \mu t f_0(x^+) + \sum_i \ln(-\frac{f_i(x^+)}{f_i(x)})$

Let $\lambda_i = -\frac{1}{t f_i(x)}$. Remember: $L(\lambda, x) = f_0(x) + \sum_i \lambda_i f_i(x)$.

$$f_0(x) - L(\lambda, x') \leq \frac{m}{t} \text{ since } \sum_i \lambda_i f_i(x) = -\frac{m}{t}.$$

$\mu t f_0(x) - \mu t f_0(x^+) + \sum_i \ln(-\mu t \lambda_i f_i(x^+)) - m \ln \mu$

$$\ln(-x) = ln(1 - (1 + x)) \leq -(1 + x)$$

$$\begin{aligned}
&\leq \mu t f_0(x) - \mu t f_0(x^+) - \sum_i (1 + \lambda_i \mu t f_i(x^+)) - m \ln \mu \\
&= \mu t f_0(x) - \mu t f_0(x^+) - \mu t \sum_i \lambda_i f_i(x^+) - m - m \ln \mu \\
&= \mu t (f_0(x) - (f_0(x^+) + \sum_i \lambda_i f_i(x^+)) - m - m \ln \mu \\
&= \mu t (f_0(x) - L(\lambda, x^+)) - m - m \ln \mu \\
&\leq \mu t (\frac{m}{t}) - m - m \ln \mu
\end{aligned}$$

# Central Path evolution.

Old point $x = x(t)$ versus $x^+ = x(\mu t)$? Minimizing: $\mu t f_0(x) - \sum_{i=1}^{n} \ln(-f_i(t))$.

Difference in new objective from old optimal point to new:

$\mu t f_0(x) - \sum_{i=1}^{m} \ln(-f_i(x)) - \mu t f_0(x^+) + \sum_{i=1}^{m} \ln(-f_i(x^+))$

Simplify: $\mu t f_0(x) - \mu t f_0(x^+) + \sum_i \ln(-\frac{f_i(x^+)}{f_i(x)})$

Let $\lambda_i = -\frac{1}{t f_i(x)}$. Remember: $L(\lambda, x) = f_0(x) + \sum_i \lambda_i f_i(x)$.

$f_0(x) - L(\lambda, x') \leq \frac{m}{t}$ since $\sum_i \lambda_i f_i(x) = -\frac{m}{t}$.

$\mu t f_0(x) - \mu t f_0(x^+) + \sum_i \ln(-\mu t \lambda_i f_i(x^+)) - m \ln \mu$

$\ln(-x) = ln(1-(1+x)) \leq -(1+x)$

$$\leq \mu t f_0(x) - \mu t f_0(x^+) - \sum_i (1 + \lambda_i \mu t f_i(x^+)) - m \ln \mu$$
$$= \mu t f_0(x) - \mu t f_0(x^+) - \mu t \sum_i \lambda_i f_i(x^+) - m - m \ln \mu$$
$$= \mu t (f_0(x) - (f_0(x^+) + \sum_i \lambda_i f_i(x^+)) - m - m \ln \mu$$
$$= \mu t (f_0(x) - L(\lambda, x^+)) - m - m \ln \mu$$
$$\leq \mu t (\frac{m}{t}) - m - m \ln \mu$$
$$= m(\mu - 1 - \ln \mu)$$

# ...Central Path Evolution

Old point $x = x(t)$ versus $x^+ = x(\mu t)$?

# ...Central Path Evolution

Old point $x = x(t)$ versus $x^+ = x(\mu t)$?

Minimizing: $F(x) = \mu t f_0(x) + \sum_{i=0}^{n} \ln(-f_i(t))$.

# ...Central Path Evolution

Old point $x = x(t)$ versus $x^+ = x(\mu t)$?
  Minimizing: $F(x) = \mu t f_0(x) + \sum_{i=0}^{n} \ln(-f_i(t))$.

We proved: $F(x) - F(x^+) \leq m(\mu - 1 - \ln \mu)$.

# ...Central Path Evolution

Old point $x = x(t)$ versus $x^+ = x(\mu t)$?
Minimizing: $F(x) = \mu t f_0(x) + \sum_{i=0}^{n} \ln(-f_i(t))$.

We proved: $F(x) - F(x^+) \leq m(\mu - 1 - \ln \mu)$.

Choose $\mu = 1 + \delta$.

# ...Central Path Evolution

Old point $x = x(t)$ versus $x^+ = x(\mu t)$?
Minimizing: $F(x) = \mu t f_0(x) + \sum_{i=0}^{n} \ln(-f_i(t))$.

We proved: $F(x) - F(x^+) \leq m(\mu - 1 - \ln \mu)$.

Choose $\mu = 1 + \delta$.
$\ln(1+x) \approx x - x^2/2. \implies \ln(1+\delta) = \delta - \delta^2/2$

# ...Central Path Evolution

Old point $x = x(t)$ versus $x^+ = x(\mu t)$?
Minimizing: $F(x) = \mu t f_0(x) + \sum_{i=0}^{n} \ln(-f_i(t))$.

We proved: $F(x) - F(x^+) \leq m(\mu - 1 - \ln \mu)$.

Choose $\mu = 1 + \delta$.
$\ln(1+x) \approx x - x^2/2. \implies \ln(1+\delta) = \delta - \delta^2/2$

$F(x) - F(x^+) = m(1 + \delta - 1 - (\delta - \delta^2/2))$

# ...Central Path Evolution

Old point $x = x(t)$ versus $x^+ = x(\mu t)$?

Minimizing: $F(x) = \mu t f_0(x) + \sum_{i=0}^{n} \ln(-f_i(t))$.

We proved: $F(x) - F(x^+) \leq m(\mu - 1 - \ln \mu)$.

Choose $\mu = 1 + \delta$.

$\ln(1+x) \approx x - x^2/2. \implies \ln(1+\delta) = \delta - \delta^2/2$

$F(x) - F(x^+) = m(1 + \delta - 1 - (\delta - \delta^2/2)) = m(\delta^2/2)$.

# ...Central Path Evolution

Old point $x = x(t)$ versus $x^+ = x(\mu t)$?
  Minimizing: $F(x) = \mu t f_0(x) + \sum_{i=0}^{n} \ln(-f_i(t))$.

We proved: $F(x) - F(x^+) \leq m(\mu - 1 - \ln \mu)$.

Choose $\mu = 1 + \delta$.
  $\ln(1+x) \approx x - x^2/2. \implies \ln(1+\delta) = \delta - \delta^2/2$

$F(x) - F(x^+) = m(1 + \delta - 1 - (\delta - \delta^2/2)) = m(\delta^2/2)$.

Choose $\delta = \frac{1}{\sqrt{m}}$ or $\mu = (1+\delta)$.

# ...Central Path Evolution

Old point $x = x(t)$ versus $x^+ = x(\mu t)$?
Minimizing: $F(x) = \mu t f_0(x) + \sum_{i=0}^{n} \ln(-f_i(t))$.

We proved: $F(x) - F(x^+) \leq m(\mu - 1 - \ln \mu)$.

Choose $\mu = 1 + \delta$.
$\ln(1+x) \approx x - x^2/2. \implies \ln(1+\delta) = \delta - \delta^2/2$

$F(x) - F(x^+) = m(1 + \delta - 1 - (\delta - \delta^2/2)) = m(\delta^2/2)$.

Choose $\delta = \frac{1}{\sqrt{m}}$ or $\mu = (1+\delta)$.
$F(x) - F(x^+) = (\delta^2/2)) = \frac{1}{2}$.

# ...Central Path Evolution

Old point $x = x(t)$ versus $x^+ = x(\mu t)$?
Minimizing: $F(x) = \mu t f_0(x) + \sum_{i=0}^{n} \ln(-f_i(t))$.

We proved: $F(x) - F(x^+) \leq m(\mu - 1 - \ln \mu)$.

Choose $\mu = 1 + \delta$.
$\ln(1+x) \approx x - x^2/2. \implies \ln(1+\delta) = \delta - \delta^2/2$

$F(x) - F(x^+) = m(1 + \delta - 1 - (\delta - \delta^2/2)) = m(\delta^2/2)$.

Choose $\delta = \frac{1}{\sqrt{m}}$ or $\mu = (1+\delta)$.
$F(x) - F(x^+) = (\delta^2/2)) = \frac{1}{2}$.

Modifying $t$ by factor of $(1 + \frac{1}{\sqrt{m}})$, optimal still close!

# ...Central Path Evolution

Old point $x = x(t)$ versus $x^+ = x(\mu t)$?
Minimizing: $F(x) = \mu t f_0(x) + \sum_{i=0}^{n} \ln(-f_i(t))$.

We proved: $F(x) - F(x^+) \leq m(\mu - 1 - \ln \mu)$.

Choose $\mu = 1 + \delta$.
$\ln(1+x) \approx x - x^2/2. \implies \ln(1+\delta) = \delta - \delta^2/2$

$F(x) - F(x^+) = m(1 + \delta - 1 - (\delta - \delta^2/2)) = m(\delta^2/2)$.

Choose $\delta = \frac{1}{\sqrt{m}}$ or $\mu = (1 + \delta)$.
$F(x) - F(x^+) = (\delta^2/2)) = \frac{1}{2}$.

Modifying $t$ by factor of $(1 + \frac{1}{\sqrt{m}})$, optimal still close!

$t$ can be arbitrarily large!

# ...Central Path Evolution

Old point $x = x(t)$ versus $x^+ = x(\mu t)$?
Minimizing: $F(x) = \mu t f_0(x) + \sum_{i=0}^{n} \ln(-f_i(t))$.

We proved: $F(x) - F(x^+) \leq m(\mu - 1 - \ln \mu)$.

Choose $\mu = 1 + \delta$.
$\ln(1+x) \approx x - x^2/2. \implies \ln(1+\delta) = \delta - \delta^2/2$

$F(x) - F(x^+) = m(1 + \delta - 1 - (\delta - \delta^2/2)) = m(\delta^2/2)$.

Choose $\delta = \frac{1}{\sqrt{m}}$ or $\mu = (1+\delta)$.
$F(x) - F(x^+) = (\delta^2/2)) = \frac{1}{2}$.

Modifying $t$ by factor of $(1 + \frac{1}{\sqrt{m}})$, optimal still close!

$t$ can be arbitrarily large!

# ...Central Path Evolution

Old point $x = x(t)$ versus $x^+ = x(\mu t)$?
Minimizing: $F(x) = \mu t f_0(x) + \sum_{i=0}^{n} \ln(-f_i(t))$.

We proved: $F(x) - F(x^+) \leq m(\mu - 1 - \ln \mu)$.

Choose $\mu = 1 + \delta$.
$\ln(1+x) \approx x - x^2/2. \implies \ln(1+\delta) = \delta - \delta^2/2$

$F(x) - F(x^+) = m(1 + \delta - 1 - (\delta - \delta^2/2)) = m(\delta^2/2)$.

Choose $\delta = \frac{1}{\sqrt{m}}$ or $\mu = (1+\delta)$.
$F(x) - F(x^+) = (\delta^2/2)) = \frac{1}{2}$.

Modifying $t$ by factor of $(1 + \frac{1}{\sqrt{m}})$, optimal still close!

$t$ can be arbitrarily large!

The value of the objective function can be gigantic

# ...Central Path Evolution

Old point $x = x(t)$ versus $x^+ = x(\mu t)$?
Minimizing: $F(x) = \mu t f_0(x) + \sum_{i=0}^{n} \ln(-f_i(t))$.

We proved: $F(x) - F(x^+) \leq m(\mu - 1 - \ln \mu)$.

Choose $\mu = 1 + \delta$.
$\ln(1+x) \approx x - x^2/2. \implies \ln(1+\delta) = \delta - \delta^2/2$

$F(x) - F(x^+) = m(1 + \delta - 1 - (\delta - \delta^2/2)) = m(\delta^2/2)$.

Choose $\delta = \frac{1}{\sqrt{m}}$ or $\mu = (1+\delta)$.
$F(x) - F(x^+) = (\delta^2/2)) = \frac{1}{2}$.

Modifying $t$ by factor of $(1 + \frac{1}{\sqrt{m}})$, optimal still close!

$t$ can be arbitrarily large!

The value of the objective function can be gigantic and change by an enormous amount.

# ...Central Path Evolution

Old point $x = x(t)$ versus $x^+ = x(\mu t)$?
  Minimizing: $F(x) = \mu t f_0(x) + \sum_{i=0}^{n} \ln(-f_i(t))$.

We proved: $F(x) - F(x^+) \leq m(\mu - 1 - \ln \mu)$.

Choose $\mu = 1 + \delta$.
  $\ln(1+x) \approx x - x^2/2. \implies \ln(1+\delta) = \delta - \delta^2/2$

$F(x) - F(x^+) = m(1 + \delta - 1 - (\delta - \delta^2/2)) = m(\delta^2/2)$.

Choose $\delta = \frac{1}{\sqrt{m}}$ or $\mu = (1+\delta)$.
  $F(x) - F(x^+) = (\delta^2/2)) = \frac{1}{2}$.

Modifying $t$ by factor of $(1 + \frac{1}{\sqrt{m}})$, optimal still close!

$t$ can be arbitrarily large!

The value of the objective function can be gigantic
  and change by an enormous amount.
    but current point is very close to optimal.

# ...Central Path Evolution

Old point $x = x(t)$ versus $x^+ = x(\mu t)$?
  Minimizing: $F(x) = \mu t f_0(x) + \sum_{i=0}^{n} \ln(-f_i(t))$.

We proved: $F(x) - F(x^+) \leq m(\mu - 1 - \ln \mu)$.

Choose $\mu = 1 + \delta$.
  $\ln(1+x) \approx x - x^2/2. \implies \ln(1+\delta) = \delta - \delta^2/2$

$F(x) - F(x^+) = m(1 + \delta - 1 - (\delta - \delta^2/2)) = m(\delta^2/2)$.

Choose $\delta = \frac{1}{\sqrt{m}}$ or $\mu = (1 + \delta)$.
  $F(x) - F(x^+) = (\delta^2/2)) = \frac{1}{2}$.

Modifying $t$ by factor of $(1 + \frac{1}{\sqrt{m}})$, optimal still close!

$t$ can be arbitrarily large!

The value of the objective function can be gigantic
  and change by an enormous amount.
    but current point is very close to optimal.

Intuition here?

# An attempt at intuition.

$\min \sum_i x_i, x \geq 0.$

# An attempt at intuition.

$\min \sum_i x_i, x \geq 0.$

optimum is 0.

# An attempt at intuition.

$\min \sum_i x_i, x \geq 0$.

optimum is 0.

Central path: $F_t(x) = t \sum_i x_i + \sum_i \ln x_i$

# An attempt at intuition.

$\min \sum_i x_i, x \geq 0$.

optimum is 0.

Central path: $F_t(x) = t\sum_i x_i + \sum_i \ln x_i$

Optimum: $x_i = \frac{1}{t}$.

# An attempt at intuition.

$\min \sum_i x_i, x \geq 0.$

optimum is 0.

Central path: $F_t(x) = t\sum_i x_i + \sum_i \ln x_i$

Optimum: $x_i = \frac{1}{t}$.

$t \to \mu t$

# An attempt at intuition.

$\min \sum_i x_i, x \geq 0$.

optimum is 0.

Central path: $F_t(x) = t\sum_i x_i + \sum_i \ln x_i$

Optimum: $x_i = \frac{1}{t}$.

$t \to \mu t$

New optimum: $x_i^+ = \frac{1}{\mu t}$.

# An attempt at intuition.

$\min \sum_i x_i, x \geq 0$.

optimum is 0.

Central path: $F_t(x) = t\sum_i x_i + \sum_i \ln x_i$

Optimum: $x_i = \frac{1}{t}$.

$t \rightarrow \mu t$

New optimum: $x_i^+ = \frac{1}{\mu t}$.

Notice: the change in $x$ is quite small.

# An attempt at intuition.

$\min \sum_i x_i, x \geq 0$.

optimum is 0.

Central path: $F_t(x) = t\sum_i x_i + \sum_i \ln x_i$

Optimum: $x_i = \frac{1}{t}$.

$t \rightarrow \mu t$

New optimum: $x_i^+ = \frac{1}{\mu t}$.

Notice: the change in $x$ is quite small.
Roughly $(\mu - 1)\frac{1}{t} = \frac{1}{\sqrt{m}t}$ where $t$ is large.

# An attempt at intuition.

$\min \sum_i x_i, x \geq 0.$

optimum is 0.

Central path: $F_t(x) = t\sum_i x_i + \sum_i \ln x_i$

Optimum: $x_i = \frac{1}{t}$.

$t \to \mu t$

New optimum: $x_i^+ = \frac{1}{\mu t}$.

Notice: the change in $x$ is quite small.
Roughly $(\mu - 1)\frac{1}{t} = \frac{1}{\sqrt{m}t}$ where $t$ is large.

Intuitively: new point is very close to old point.

# Slightly more generally.

Only one vertex on polytope.

# Slightly more generally.

Only one vertex on polytope.

$n$ inequalities, $n$ unkonwns:

# Slightly more generally.

Only one vertex on polytope.

$n$ inequalities, $n$ unkonwns: $\min cx, Ax \geq b$.

# Slightly more generally.

Only one vertex on polytope.

$n$ inequalities, $n$ unkonwns: $\min cx, Ax \geq b$.

Is solution bounded or unbounded?

# Slightly more generally.

Only one vertex on polytope.

$n$ inequalities, $n$ unkonwns: $\min cx, Ax \geq b$.

Is solution bounded or unbounded?

Alg: Linear equation solve for intersection of $n$ inequalities, check if there is some direction of improvement.

# Slightly more generally.

Only one vertex on polytope.

$n$ inequalities, $n$ unkonwns: $\min cx, Ax \geq b$.

Is solution bounded or unbounded?

Alg: Linear equation solve for intersection of $n$ inequalities, check if there is some direction of improvement.

Evolution of central path.

# Slightly more generally.

Only one vertex on polytope.

$n$ inequalities, $n$ unkonwns: $\min cx, Ax \geq b$.

Is solution bounded or unbounded?

Alg: Linear equation solve for intersection of $n$ inequalities, check if there is some direction of improvement.

Evolution of central path.

Optimal $x(t)$:

# Slightly more generally.

Only one vertex on polytope.

$n$ inequalities, $n$ unkonwns: $\min cx, Ax \geq b$.

Is solution bounded or unbounded?

Alg: Linear equation solve for intersection of $n$ inequalities, check if there is some direction of improvement.

Evolution of central path.

Optimal $x(t)$: $\nabla f_0(x) + \sum_{i=1} \frac{1}{tf_i(x)} \nabla f_i(x) = 0$

# Slightly more generally.

Only one vertex on polytope.

$n$ inequalities, $n$ unkonwns: $\min cx, Ax \geq b$.

Is solution bounded or unbounded?

Alg: Linear equation solve for intersection of $n$ inequalities, check if there is some direction of improvement.

Evolution of central path.

Optimal $x(t)$: $\nabla f_0(x) + \sum_{i=1} \frac{1}{tf_i(x)} \nabla f_i(x) = 0$

$tc = -\sum_i \frac{a_i}{a_i x - b_i}$

# Slightly more generally.

Only one vertex on polytope.

$n$ inequalities, $n$ unkonwns: $\min cx, Ax \geq b$.

Is solution bounded or unbounded?

Alg: Linear equation solve for intersection of $n$ inequalities, check if there is some direction of improvement.

Evolution of central path.

Optimal $x(t)$: $\nabla f_0(x) + \sum_{i=1} \frac{1}{tf_i(x)} \nabla f_i(x) = 0$

$tc = -\sum_i \frac{a_i}{a_i x - b_i}$

# Slightly more generally.

Only one vertex on polytope.

$n$ inequalities, $n$ unkonwns: $\min cx, Ax \geq b$.

Is solution bounded or unbounded?

Alg: Linear equation solve for intersection of $n$ inequalities, check if there is some direction of improvement.

Evolution of central path.

Optimal $x(t)$: $\nabla f_0(x) + \sum_{i=1} \frac{1}{tf_i(x)} \nabla f_i(x) = 0$

$tc = -\sum_i \frac{a_i}{a_i x - b_i}$

$s_i = a_i x - b_i$.

# Slightly more generally.

Only one vertex on polytope.

$n$ inequalities, $n$ unkonwns: $\min cx, Ax \geq b$.

Is solution bounded or unbounded?

Alg: Linear equation solve for intersection of $n$ inequalities, check if there is some direction of improvement.

Evolution of central path.

Optimal $x(t)$: $\nabla f_0(x) + \sum_{i=1} \frac{1}{tf_i(x)} \nabla f_i(x) = 0$

$tc = -\sum_i \frac{a_i}{a_i x - b_i}$

$s_i = a_i x - b_i$. “Distance’ to constraint.

# Slightly more generally.

Only one vertex on polytope.

$n$ inequalities, $n$ unkonwns: $\min cx, Ax \geq b$.

Is solution bounded or unbounded?

Alg: Linear equation solve for intersection of $n$ inequalities, check if there is some direction of improvement.

Evolution of central path.

Optimal $x(t)$: $\nabla f_0(x) + \sum_{i=1} \frac{1}{tf_i(x)} \nabla f_i(x) = 0$

$tc = -\sum_i \frac{a_i}{a_i x - b_i}$

$s_i = a_i x - b_i$. “Distance’ to constraint.

Recall previous example: $x \geq 0$, the $x_i$ are slack variables.

# Slightly more generally.

Only one vertex on polytope.

$n$ inequalities, $n$ unkonwns: $\min cx, Ax \geq b$.

Is solution bounded or unbounded?

Alg: Linear equation solve for intersection of $n$ inequalities, check if there is some direction of improvement.

Evolution of central path.

Optimal $x(t)$: $\nabla f_0(x) + \sum_{i=1} \frac{1}{tf_i(x)} \nabla f_i(x) = 0$
$tc = -\sum_i \frac{a_i}{a_i x - b_i}$

$s_i = a_i x - b_i$. "Distance' to constraint.
Recall previous example: $x \geq 0$, the $x_i$ are slack variables.
$s = Ax - b$.

# Slightly more generally.

Only one vertex on polytope.

$n$ inequalities, $n$ unkonwns: $\min cx, Ax \geq b$.

Is solution bounded or unbounded?

Alg: Linear equation solve for intersection of $n$ inequalities, check if there is some direction of improvement.

Evolution of central path.

Optimal $x(t)$: $\nabla f_0(x) + \sum_{i=1} \frac{1}{tf_i(x)} \nabla f_i(x) = 0$
$tc = -\sum_i \frac{a_i}{a_i x - b_i}$

$s_i = a_i x - b_i$. "Distance' to constraint.
Recall previous example: $x \geq 0$, the $x_i$ are slack variables.
$s = Ax - b$.

Given solution to $x(t)$ with $b - Ax(t) = s(t)$.

# Slightly more generally.

Only one vertex on polytope.

$n$ inequalities, $n$ unkonwns: $\min cx, Ax \geq b$.

Is solution bounded or unbounded?

Alg: Linear equation solve for intersection of $n$ inequalities, check if there is some direction of improvement.

Evolution of central path.

Optimal $x(t)$: $\nabla f_0(x) + \sum_{i=1} \frac{1}{tf_i(x)} \nabla f_i(x) = 0$
$tc = -\sum_i \frac{a_i}{a_i x - b_i}$

$s_i = a_i x - b_i$. "Distance' to constraint.
Recall previous example: $x \geq 0$, the $x_i$ are slack variables.
$s = Ax - b$.

Given solution to $x(t)$ with $b - Ax(t) = s(t)$.

# Slightly more generally.

Only one vertex on polytope.

$n$ inequalities, $n$ unkonwns: $\min cx, Ax \geq b$.

Is solution bounded or unbounded?

Alg: Linear equation solve for intersection of $n$ inequalities, check if there is some direction of improvement.

Evolution of central path.

Optimal $x(t)$: $\nabla f_0(x) + \sum_{i=1} \frac{1}{tf_i(x)} \nabla f_i(x) = 0$
$tc = -\sum_i \frac{a_i}{a_i x - b_i}$

$s_i = a_i x - b_i$. "Distance' to constraint.
Recall previous example: $x \geq 0$, the $x_i$ are slack variables.
$s = Ax - b$.

Given solution to $x(t)$ with $b - Ax(t) = s(t)$.
Then $Ax(\mu t) - b = s(t)/\mu$ works.

# Slightly more generally.

Only one vertex on polytope.

$n$ inequalities, $n$ unkonwns: $\min cx, Ax \geq b$.

Is solution bounded or unbounded?

Alg: Linear equation solve for intersection of $n$ inequalities, check if there is some direction of improvement.

Evolution of central path.

Optimal $x(t)$: $\nabla f_0(x) + \sum_{i=1} \frac{1}{tf_i(x)} \nabla f_i(x) = 0$
$tc = -\sum_i \frac{a_i}{a_i x - b_i}$

$s_i = a_i x - b_i$. “Distance’ to constraint.
Recall previous example: $x \geq 0$, the $x_i$ are slack variables.
$s = Ax - b$.

Given solution to $x(t)$ with $b - Ax(t) = s(t)$.
Then $Ax(\mu t) - b = s(t)/\mu$ works.

Since only $n$ inequalities, can just solve to get next point.

# Slightly more generally.

Only one vertex on polytope.

$n$ inequalities, $n$ unkonwns: $\min cx, Ax \geq b$.

Is solution bounded or unbounded?

Alg: Linear equation solve for intersection of $n$ inequalities, check if there is some direction of improvement.

Evolution of central path.

Optimal $x(t)$: $\nabla f_0(x) + \sum_{i=1} \frac{1}{tf_i(x)} \nabla f_i(x) = 0$
$tc = -\sum_i \frac{a_i}{a_i x - b_i}$

$s_i = a_i x - b_i$. "Distance' to constraint.
Recall previous example: $x \geq 0$, the $x_i$ are slack variables.
$s = Ax - b$.

Given solution to $x(t)$ with $b - Ax(t) = s(t)$.
Then $Ax(\mu t) - b = s(t)/\mu$ works.

Since only $n$ inequalities, can just solve to get next point.

Answer is easy too.

# More generally.

General $Ax \geq b, \min cx$.

# More generally.

General $Ax \geq b, \min cx$.

Given solution to $x(t)$ with $b - Ax(t) = s(t)$.

# More generally.

General $Ax \geq b, \min cx$.

Given solution to $x(t)$ with $b - Ax(t) = s(t)$.

# More generally.

General $Ax \geq b, \min cx$.

Given solution to $x(t)$ with $b - Ax(t) = s(t)$.
Then $b - Ax(\mu t) = s(t)/\mu$ works.

# More generally.

General $Ax \geq b, \min cx$.

Given solution to $x(t)$ with $b - Ax(t) = s(t)$.
Then $b - Ax(\mu t) = s(t)/\mu$ works.

Overdetermined if more than $n$ inequalities, so maybe not possible.

# More generally.

General $Ax \geq b, \min cx$.

Given solution to $x(t)$ with $b - Ax(t) = s(t)$.
Then $b - Ax(\mu t) = s(t)/\mu$ works.

Overdetermined if more than $n$ inequalities, so maybe not possible.

So, need to find solution to: $\mu tc = -\sum_i \frac{a_i}{a_i x - b_i}$

# More generally.

General $Ax \geq b, \min cx$.

Given solution to $x(t)$ with $b - Ax(t) = s(t)$.
Then $b - Ax(\mu t) = s(t)/\mu$ works.

Overdetermined if more than $n$ inequalities, so maybe not possible.

So, need to find solution to: $\mu tc = -\sum_i \frac{a_i}{a_i x - b_i}$

# More generally.

General $Ax \geq b, \min cx$.

Given solution to $x(t)$ with $b - Ax(t) = s(t)$.
Then $b - Ax(\mu t) = s(t)/\mu$ works.

Overdetermined if more than $n$ inequalities, so maybe not possible.

So, need to find solution to: $\mu tc = -\sum_i \frac{a_i}{a_i x - b_i}$

Showed solution is at least close in value to old solution on $F(x)$.

# More generally.

General $Ax \geq b, \min cx$.

Given solution to $x(t)$ with $b - Ax(t) = s(t)$.
Then $b - Ax(\mu t) = s(t)/\mu$ works.

Overdetermined if more than $n$ inequalities, so maybe not possible.

So, need to find solution to: $\mu tc = -\sum_i \frac{a_i}{a_i x - b_i}$

Showed solution is at least close in value to old solution on $F(x)$.

One thing to note:

# More generally.

General $Ax \geq b, \min cx$.

Given solution to $x(t)$ with $b - Ax(t) = s(t)$.
Then $b - Ax(\mu t) = s(t)/\mu$ works.

Overdetermined if more than $n$ inequalities, so maybe not possible.

So, need to find solution to: $\mu tc = -\sum_i \frac{a_i}{a_i x - b_i}$

Showed solution is at least close in value to old solution on $F(x)$.

One thing to note:
if you know the optimal vertex (tight constraints).

# More generally.

General $Ax \geq b, \min cx$.

Given solution to $x(t)$ with $b - Ax(t) = s(t)$.
Then $b - Ax(\mu t) = s(t)/\mu$ works.

Overdetermined if more than $n$ inequalities, so maybe not possible.

So, need to find solution to: $\mu tc = -\sum_i \frac{a_i}{a_i x - b_i}$

Showed solution is at least close in value to old solution on $F(x)$.

One thing to note:
if you know the optimal vertex (tight constraints).
then you are done.

# More generally.

General $Ax \geq b, \min cx$.

Given solution to $x(t)$ with $b - Ax(t) = s(t)$.
Then $b - Ax(\mu t) = s(t)/\mu$ works.

Overdetermined if more than $n$ inequalities, so maybe not possible.

So, need to find solution to: $\mu tc = -\sum_i \frac{a_i}{a_i x - b_i}$

Showed solution is at least close in value to old solution on $F(x)$.

One thing to note:
if you know the optimal vertex (tight constraints).
then you are done.

# More generally.

General $Ax \geq b, \min cx$.

Given solution to $x(t)$ with $b - Ax(t) = s(t)$.
Then $b - Ax(\mu t) = s(t)/\mu$ works.

Overdetermined if more than $n$ inequalities, so maybe not possible.

So, need to find solution to: $\mu tc = -\sum_i \frac{a_i}{a_i x - b_i}$

Showed solution is at least close in value to old solution on $F(x)$.

One thing to note:
if you know the optimal vertex (tight constraints).
then you are done.

Idea: close enough to tight constraints.

# More generally.

General $Ax \geq b, \min cx$.

Given solution to $x(t)$ with $b - Ax(t) = s(t)$.
Then $b - Ax(\mu t) = s(t)/\mu$ works.

Overdetermined if more than $n$ inequalities, so maybe not possible.

So, need to find solution to: $\mu tc = -\sum_i \frac{a_i}{a_i x - b_i}$

Showed solution is at least close in value to old solution on $F(x)$.

One thing to note:
if you know the optimal vertex (tight constraints).
then you are done.

Idea: close enough to tight constraints. Done.

# More generally.

General $Ax \geq b, \min cx$.

Given solution to $x(t)$ with $b - Ax(t) = s(t)$.
Then $b - Ax(\mu t) = s(t)/\mu$ works.

Overdetermined if more than $n$ inequalities, so maybe not possible.

So, need to find solution to: $\mu tc = -\sum_i \frac{a_i}{a_i x - b_i}$

Showed solution is at least close in value to old solution on $F(x)$.

One thing to note:
if you know the optimal vertex (tight constraints).
then you are done.

Idea: close enough to tight constraints. Done.

Close enough to a vertex, can jump to vertex.

# More generally.

General $Ax \geq b, \min cx$.

Given solution to $x(t)$ with $b - Ax(t) = s(t)$.
Then $b - Ax(\mu t) = s(t)/\mu$ works.

Overdetermined if more than $n$ inequalities, so maybe not possible.

So, need to find solution to: $\mu tc = -\sum_i \frac{a_i}{a_i x - b_i}$

Showed solution is at least close in value to old solution on $F(x)$.

One thing to note:
if you know the optimal vertex (tight constraints).
then you are done.

Idea: close enough to tight constraints. Done.

Close enough to a vertex, can jump to vertex.

Cramer's rule, gives estimate of how close the closest two vertices can be.

# Interior Point Method.

Find central point.

# Interior Point Method.

Find central point.

Recall: $F(x) = tf_0(x) - \sum_i \log(-f_i(x))$.

# Interior Point Method.

Find central point.

Recall: $F(x) = tf_0(x) - \sum_i \log(-f_i(x))$.

Find point: $G(x) = \nabla F(x) = 0$.

# Interior Point Method.

Find central point.

Recall: $F(x) = tf_0(x) - \sum_i \log(-f_i(x))$.

Find point: $G(x) = \nabla F(x) = 0$.

Newton: find all zeros

# Interior Point Method.

Find central point.

Recall: $F(x) = tf_0(x) - \sum_i \log(-f_i(x))$.

Find point: $G(x) = \nabla F(x) = 0$.

Newton: find all zeros of vector valued $G(x)$!

# Interior Point Method.

Find central point.

Recall: $F(x) = tf_0(x) - \sum_i \log(-f_i(x))$.

Find point: $G(x) = \nabla F(x) = 0$.

Newton: find all zeros of vector valued $G(x)$!

$g_1(x) = \frac{t\partial f_0(x)}{\partial x_1} - \sum_i \frac{\partial f_i(x)}{\partial x_1} \frac{1}{f_i(x)}$

# Interior Point Method.

Find central point.

Recall: $F(x) = tf_0(x) - \sum_i \log(-f_i(x))$.

Find point: $G(x) = \nabla F(x) = 0$.

Newton: find all zeros of vector valued $G(x)$!

$g_1(x) = \frac{t\partial f_0(x)}{\partial x_1} - \sum_i \frac{\partial f_i(x)}{\partial x_1} \frac{1}{f_i(x)}$

Newton:

# Interior Point Method.

Find central point.

Recall: $F(x) = tf_0(x) - \sum_i \log(-f_i(x))$.

Find point: $G(x) = \nabla F(x) = 0$.

Newton: find all zeros of vector valued $G(x)$!

$g_1(x) = \frac{t\partial f_0(x)}{\partial x_1} - \sum_i \frac{\partial f_i(x)}{\partial x_1} \frac{1}{f_i(x)}$

Newton:

$\implies |(x_{n+1} - \alpha)| \leq |\frac{g'(\psi)}{2g''(x_n)}|(x_n - \alpha)^2$.

# Interior Point Method.

Find central point.

Recall: $F(x) = tf_0(x) - \sum_i \log(-f_i(x))$.

Find point: $G(x) = \nabla F(x) = 0$.

Newton: find all zeros of vector valued $G(x)$!

$g_1(x) = \frac{t\partial f_0(x)}{\partial x_1} - \sum_i \frac{\partial f_i(x)}{\partial x_1} \frac{1}{f_i(x)}$

Newton:
$\implies |(x_{n+1} - \alpha)| \leq |\frac{g'(\psi)}{2g''(x_n)}|(x_n - \alpha)^2$.

Recall, distance for $x$ to $x^+$ is pretty small.

# Interior Point Method.

Find central point.

Recall: $F(x) = tf_0(x) - \sum_i \log(-f_i(x))$.

Find point: $G(x) = \nabla F(x) = 0$.

Newton: find all zeros of vector valued $G(x)$!

$g_1(x) = \frac{t\partial f_0(x)}{\partial x_1} - \sum_i \frac{\partial f_i(x)}{\partial x_1} \frac{1}{f_i(x)}$

Newton:
$\implies |(x_{n+1} - \alpha)| \leq |\frac{g'(\psi)}{2g''(x_n)}|(x_n - \alpha)^2$.

Recall, distance for $x$ to $x^+$ is pretty small.
On the order of $1/t$.

# Interior Point Method.

Find central point.

Recall: $F(x) = tf_0(x) - \sum_i \log(-f_i(x))$.

Find point: $G(x) = \nabla F(x) = 0$.

Newton: find all zeros of vector valued $G(x)$!

$g_1(x) = \frac{t\partial f_0(x)}{\partial x_1} - \sum_i \frac{\partial f_i(x)}{\partial x_1} \frac{1}{f_i(x)}$

Newton:
$\implies |(x_{n+1} - \alpha)| \leq |\frac{g'(\psi)}{2g''(x_n)}|(x_n - \alpha)^2$.

Recall, distance for $x$ to $x^+$ is pretty small.
On the order of $1/t$.

# Interior Point Method.

Find central point.

Recall: $F(x) = tf_0(x) - \sum_i \log(-f_i(x))$.

Find point: $G(x) = \nabla F(x) = 0$.

Newton: find all zeros of vector valued $G(x)$!

$g_1(x) = \frac{t\partial f_0(x)}{\partial x_1} - \sum_i \frac{\partial f_i(x)}{\partial x_1}\frac{1}{f_i(x)}$

Newton:
$\implies |(x_{n+1} - \alpha)| \leq |\frac{g'(\psi)}{2g''(x_n)}|(x_n - \alpha)^2$.

Recall, distance for $x$ to $x^+$ is pretty small.
On the order of $1/t$.

What about the ratio? $|\frac{g''(\psi)}{2g'(x_n)}|$

# Behavior of log barrier.

What about the ratio? $\left|\frac{g''(\psi)}{2g'(x_n)}\right|$

# Behavior of log barrier.

What about the ratio? $|\frac{g''(\psi)}{2g'(x_n)}|$

What if $f(x) = \log x$ and recall $g(x) = f'(x)$?

# Behavior of log barrier.

What about the ratio? $|\frac{g''(\psi)}{2g'(x_n)}|$

What if $f(x) = \log x$ and recall $g(x) = f'(x)$?

$(\log x)' = 1/x$, $(\log x)'' = -1/x^2$, $(\log x)''' = 2/x^3$.

# Behavior of log barrier.

What about the ratio? $|\frac{g''(\psi)}{2g'(x_n)}|$

What if $f(x) = \log x$ and recall $g(x) = f'(x)$?

$(\log x)' = 1/x$, $(\log x)'' = -1/x^2$, $(\log x)''' = 2/x^3$.

$|(\log x)'''| = |\frac{x}{2}(\log x)''|$.

# Behavior of log barrier.

What about the ratio? $|\frac{g''(\psi)}{2g'(x_n)}|$

What if $f(x) = \log x$ and recall $g(x) = f'(x)$?

$(\log x)' = 1/x$, $(\log x)'' = -1/x^2$, $(\log x)''' = 2/x^3$.

$|(\log x)'''| = |\frac{x}{2}(\log x)''|$.

Thus, this ratio is around $1/x$.

# Behavior of log barrier.

What about the ratio? $|\frac{g''(\psi)}{2g'(x_n)}|$

What if $f(x) = \log x$ and recall $g(x) = f'(x)$?

$(\log x)' = 1/x$, $(\log x)'' = -1/x^2$, $(\log x)''' = 2/x^3$.

$|(\log x)'''| = |\frac{x}{2}(\log x)''|$.

Thus, this ratio is around $1/x$.

Thus, $|\frac{g''(\psi)}{2g'(x_n)}|(x - x^+) \leq 1/2$.

# Behavior of log barrier.

What about the ratio? $|\frac{g''(\psi)}{2g'(x_n)}|$

What if $f(x) = \log x$ and recall $g(x) = f'(x)$?

$(\log x)' = 1/x$, $(\log x)'' = -1/x^2$, $(\log x)''' = 2/x^3$.

$|(\log x)'''| = |\frac{x}{2}(\log x)''|$.

Thus, this ratio is around $1/x$.

Thus, $|\frac{g''(\psi)}{2g'(x_n)}|(x - x^+) \leq 1/2$.

Quadratic convergence: ratio is small.

# Newton's Method.

$$f(x) = \log(a_i x - b_i)$$

# Newton's Method.

$$f(x) = \log(a_i x - b_i)$$

$$s_i = a_i x - b_i$$

# Newton's Method.

$$f(x) = \log(a_i x - b_i)$$

$$s_i = a_i x - b_i$$

$$f'(x) = \frac{a_i}{s_i}$$

# Newton's Method.

$$f(x) = \log(a_i x - b_i)$$

$$s_i = a_i x - b_i$$

$$f'(x) = \frac{a_i}{s_i} \qquad f''(x) = \frac{1}{s_i^2} a_i a_i^T$$

# Newton's Method.

$$f(x) = \log(a_i x - b_i)$$

$$s_i = a_i x - b_i$$

$$f'(x) = \frac{a_i}{s_i} \qquad f''(x) = \frac{1}{s_i^2} a_i a_i^T \qquad f'''(a) = a_i^{\otimes 3} \frac{1}{s_i^3}$$

# Newton's Method.

$$f(x) = \log(a_i x - b_i)$$

$$s_i = a_i x - b_i$$

$$f'(x) = \frac{a_i}{s_i} \qquad f''(x) = \frac{1}{s_i^2} a_i a_i^T \qquad f'''(a) = a_i^{\otimes 3} \frac{1}{s_i^3}$$

$$f(x+u) = f(x) + f'(x) \cdot u + u^{\otimes 2} \cdot f''(x)$$

# Newton's Method.

$$f(x) = \log(a_i x - b_i)$$

$$s_i = a_i x - b_i$$

$$f'(x) = \frac{a_i}{s_i} \qquad f''(x) = \frac{1}{s_i^2} a_i a_i^T \qquad f'''(a) = a_i^{\otimes 3} \frac{1}{s_i^3}$$

$$f(x+u) = f(x) + f'(x) \cdot u + u^{\otimes 2} \cdot f''(x)$$

The minimizer?

# Newton's Method.

$f(x) = \log(a_i x - b_i)$

$s_i = a_i x - b_i$

$f'(x) = \frac{a_i}{s_i} \quad f''(x) = \frac{1}{s_i^2} a_i a_i^T \quad f'''(a) = a_i^{\otimes 3} \frac{1}{s_i^3}$

$f(x+u) = f(x) + f'(x) \cdot u + u^{\otimes 2} \cdot f''(x)$

The minimizer? $-\frac{1}{2}(f''(x))^{-1} f'(x)$.

# Newton's Method.

$f(x) = \log(a_i x - b_i)$

$s_i = a_i x - b_i$

$f'(x) = \frac{a_i}{s_i} \quad f''(x) = \frac{1}{s_i^2} a_i a_i^T \quad f'''(a) = a_i^{\otimes 3} \frac{1}{s_i^3}$

$f(x+u) = f(x) + f'(x) \cdot u + u^{\otimes 2} \cdot f''(x)$

The minimizer? $-\frac{1}{2}(f''(x))^{-1} f'(x)$.

# Newton's Method.

$f(x) = \log(a_i x - b_i)$

$s_i = a_i x - b_i$

$f'(x) = \frac{a_i}{s_i} \quad f''(x) = \frac{1}{s_i^2} a_i a_i^T \quad f'''(a) = a_i^{\otimes 3} \frac{1}{s_i^3}$

$f(x+u) = f(x) + f'(x) \cdot u + u^{\otimes 2} \cdot f''(x)$

The minimizer? $-\frac{1}{2}(f''(x))^{-1} f'(x)$.

Self-Concordance: $|f'''(x)| \leq 2f''(x)^3/2$.

# Newton's Method.

$f(x) = \log(a_i x - b_i)$

$s_i = a_i x - b_i$

$f'(x) = \frac{a_i}{s_i} \quad f''(x) = \frac{1}{s_i^2} a_i a_i^T \quad f'''(a) = a_i^{\otimes 3} \frac{1}{s_i^3}$

$f(x+u) = f(x) + f'(x) \cdot u + u^{\otimes 2} \cdot f''(x)$

The minimizer? $-\frac{1}{2}(f''(x))^{-1} f'(x)$.

Self-Concordance: $|f'''(x)| \leq 2f''(x)^3/2$.

Newton Method: $f(x) - f(x*) \leq 1/2$, it converges quadratically.

# Another Type of IPM strategy.

$\min xc, Ax \geq b$.

# Another Type of IPM strategy.

$\min xc, Ax \geq b$.

$F(x) = tcx - \sum_i \log(a_i x - b_i)$.

# Another Type of IPM strategy.

$\min xc, Ax \geq b$.

$F(x) = tcx - \sum_i \log(a_i x - b_i)$.

$\nabla F(x) = tc - \sum_i \frac{a_i}{s_i}$

# Another Type of IPM strategy.

$\min xc, Ax \geq b$.

$F(x) = tcx - \sum_i \log(a_i x - b_i)$.

$\nabla F(x) = tc - \sum_i \frac{a_i}{s_i}$

Introduce dual variables: $\lambda_i$.

# Another Type of IPM strategy.

$\min xc, Ax \geq b$.

$F(x) = tcx - \sum_i \log(a_i x - b_i)$.

$\nabla F(x) = tc - \sum_i \frac{a_i}{s_i}$

Introduce dual variables: $\lambda_i$.
 Approximate Complementary slackness.

# Another Type of IPM strategy.

$\min xc, Ax \geq b$.

$F(x) = tcx - \sum_i \log(a_i x - b_i)$.

$\nabla F(x) = tc - \sum_i \frac{a_i}{s_i}$

Introduce dual variables: $\lambda_i$.

Approximate Complementary slackness.

$\lambda_i s_i = \frac{1}{t}$

# Another Type of IPM strategy.

$\min xc, Ax \geq b$.

$F(x) = tcx - \sum_i \log(a_i x - b_i)$.

$\nabla F(x) = tc - \sum_i \frac{a_i}{s_i}$

Introduce dual variables: $\lambda_i$.

Approximate Complementary slackness.

$\lambda_i s_i = \frac{1}{t}$ verse $\lambda_i s_i = 0$

# Another Type of IPM strategy.

$\min xc, Ax \geq b$.

$F(x) = tcx - \sum_i \log(a_i x - b_i)$.

$\nabla F(x) = tc - \sum_i \frac{a_i}{s_i}$

Introduce dual variables: $\lambda_i$.

Approximate Complementary slackness.

$\lambda_i s_i = \frac{1}{t}$ verse $\lambda_i s_i = 0$

$s = b - Ax$

# Another Type of IPM strategy.

$\min xc, Ax \geq b$.

$F(x) = tcx - \sum_i \log(a_i x - b_i)$.

$\nabla F(x) = tc - \sum_i \frac{a_i}{s_i}$

Introduce dual variables: $\lambda_i$.

Approximate Complementary slackness.

$\lambda_i s_i = \frac{1}{t}$ verse $\lambda_i s_i = 0$

$s = b - Ax$

# Another Type of IPM strategy.

$\min xc, Ax \geq b.$

$F(x) = tcx - \sum_i \log(a_i x - b_i).$

$\nabla F(x) = tc - \sum_i \frac{a_i}{s_i}$

Introduce dual variables: $\lambda_i$.
Approximate Complementary slackness.
$\lambda_i s_i = \frac{1}{t}$ verse $\lambda_i s_i = 0$
$s = b - Ax$

Gives another possibility:

# Another Type of IPM strategy.

$\min xc, Ax \geq b$.

$F(x) = tcx - \sum_i \log(a_i x - b_i)$.

$\nabla F(x) = tc - \sum_i \frac{a_i}{s_i}$

Introduce dual variables: $\lambda_i$.

Approximate Complementary slackness.

$\lambda_i s_i = \frac{1}{t}$ verse $\lambda_i s_i = 0$

$s = b - Ax$

Gives another possibility:

Explicitly maintain primal-dual soluion: $(x, s)$

# Another Type of IPM strategy.

$\min xc, Ax \geq b$.

$F(x) = tcx - \sum_i \log(a_i x - b_i)$.

$\nabla F(x) = tc - \sum_i \frac{a_i}{s_i}$

Introduce dual variables: $\lambda_i$.
Approximate Complementary slackness.
$\lambda_i s_i = \frac{1}{t}$ verse $\lambda_i s_i = 0$
$s = b - Ax$

Gives another possibility:
Explicitly maintain primal-dual soluion: $(x, s)$