Rules for School...

or..."Rules for taking duals"

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$Ax \leq b, \max cx, x \geq 0 \leftrightarrow y^T A \geq c, \min by, y \geq 0.$

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One more useful trick: Equality constraints.

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One more useful trick: Equality constraints.

"equalities" $\leftrightarrow$ "unrestricted variables."

Maximum Weight Matching.

Bipartite Graph $G = (V, E)$, $w : E \rightarrow Z$.

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$$\forall v : \sum_{e=(u,v)} x_e = 1$$

$$x_e \geq 0$$

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Variable for each constraint. p_v

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Variable for each constraint. p_v unrestricted.

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Constraint for each variable. Edge e, $p_u + p_v \geq w_e$

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Weak duality?

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$\sum_{e \in M} w_e x_e \leq \sum_{e=(u,v) \in M} (p_u + p_v) \leq \sum_v p_u$.

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Strong Duality? Same value solutions.

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Complementary Slackness.

$$\max \sum_e w_e x_e$$

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Dual:

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Complementary slackness:
Only match on tight edges.

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Complementary slackness:

Only match on tight edges.

Nonzero p_u on matched u.

Multicommodity Flow.

Given $G = (V, E)$, and capacity function $c : E \rightarrow Z$, and pairs $(s_1, t_1), \ldots, (s_k, t_k)$ with demands $d_1, \ldots, d_k$.

Multicommodity Flow.

Given $G = (V, E)$, and capacity function $c : E \rightarrow Z$, and pairs $(s_1, t_1), \ldots, (s_k, t_k)$ with demands $d_1, \ldots, d_k$.
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$$\min \mu$$

$$\forall e : \sum_{p \ni e} f_p \leq \mu c_e$$

$$\forall i : \sum_{p \in P_i} f_p = D_i$$

$$f_p \geq 0$$

Take the dual.

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Modify to make it $\geq$, which “goes with” min.

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And only constants on right hand side.

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Introduce variable for each constraint.

Dual.

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Objective: right hand sides.

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d_i - shortest s_i, t_i path length.

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d_i - shortest s_i, t_i path length. Toll problem!

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$$\forall p \in P_i : d_i \leq \sum_{e \in p} d(e)$$

$$\sum_e c_e d_e = 1$$

d_i - shortest s_i, t_i path length. Toll problem!
Weak duality: toll lower bounds routing.
Strong Duality. Tight lower bound.

Dual.

$$\min \mu$$

$$\forall e : \mu c_e - \sum_{p \ni e} f_p \geq 0 \qquad d_e$$

$$\forall i : \sum_{p \in P_i} f_p = D_i \qquad d_i$$

Introduce variable for each constraint.
Introduce constraint for each var:
$\mu \rightarrow \sum_e c_e d_e = 1$. $f_p \rightarrow \forall p \in P_i \; d_i - \sum_{e \in p} d_e \leq 0$.
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Complementary Slackness:

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d_i - shortest s_i, t_i path length. Toll problem!
Weak duality: toll lower bounds routing.
Strong Duality. Tight lower bound. First lecture. Or Experts.
Complementary Slackness: only route on shortest paths
only have toll on congested edges.

Exponential size.

Multicommodity flow.

$$\min \mu$$

$$\forall e : \mu c_e - \sum_{p \ni e} f_p \geq 0$$

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$$f_p \geq 0$$

Exponential size.

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Exponential sized programs?

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Exponential sized programs?

Answer 1:

Exponential size.

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Exponential sized programs?

Answer 1: We solved anyway!

Answer 2:

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Exponential sized programs?

Answer 1: We solved anyway!

Answer 2: Ellipsoid algorithm.

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Exponential sized programs?

Answer 1: We solved anyway!

Answer 2: Ellipsoid algorithm.
 Find violated constraint $\rightarrow$ poly time algorithm.

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Exponential sized programs?

Answer 1: We solved anyway!

Answer 2: Ellipsoid algorithm.
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Answer 3: there is polynomial sized formulation.

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Exponential sized programs?

Answer 1: We solved anyway!

Answer 2: Ellipsoid algorithm.
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Answer 3: there is polynomial sized formulation.
Question: what is it?

Facility location

Set of facilities: F, opening cost f_i for facility i

Facility location

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Set of clients: D.

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d_{ij} - distance between i and j.

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Triangle inequality: $d_{ij} \leq d_{ik} + d_{kj}$.

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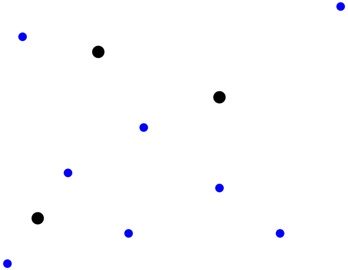

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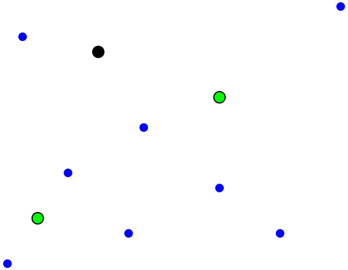

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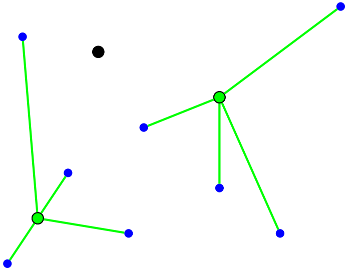

LP and Dual. Interpretation?

$$\begin{aligned} \min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij} \\ \forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1 \\ \forall i \in F, j \in D \quad x_{ij} \leq y_i, \\ x_{ij}, y_i \geq 0 \end{aligned}$$

LP and Dual. Interpretation?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$
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$$\max \sum_j \alpha_j$$
$$\forall i \in F \quad \sum_{j \in D} \beta_{ij} \leq f_i$$
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α_j charge to client.

LP and Dual. Interpretation?

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maximize price for client to connect!

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Objective: $\sum_j \alpha_j$ total payment.

LP and Dual. Interpretation?

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Client j travels or pays to open facility i.
Costs client d_{ij} to get to there.

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α_j charge to client.
 maximize price for client to connect!
Objective: $\sum_j \alpha_j$ total payment.
Client j travels or pays to open facility i.
 Costs client d_{ij} to get to there.
 Savings is $\alpha_j - d_{ij}$.

LP and Dual. Interpretation?

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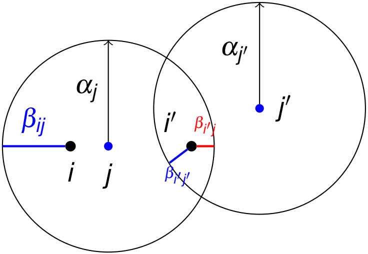

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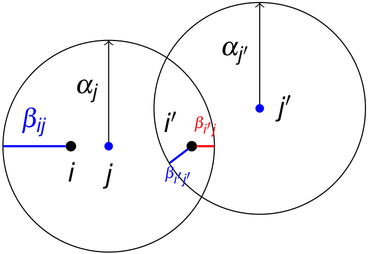

Total payment to facility i at most f_i before opening.

LP and Dual. Interpretation?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$
$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$
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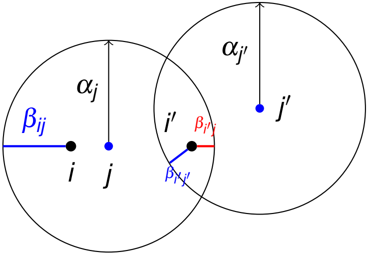

Total payment to facility i at most f_i before opening.
Complementary slackness:

LP and Dual. Interpretation?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$
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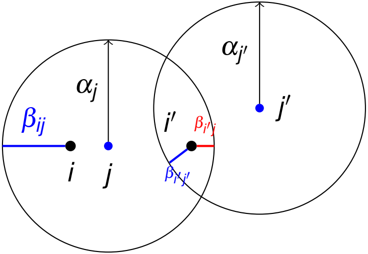

Total payment to facility i at most f_i before opening.
Complementary slackness: $x_{ij} \geq 0$ if and only if $\alpha_j \geq d_{ij}$.

LP and Dual. Interpretation?

$$\min \sum_{i \in F} y_i f_i + \sum_{i \in F, j \in D} x_{ij} d_{ij}$$
$$\forall j \in D \quad \sum_{i \in F} x_{ij} \geq 1$$
$$\forall i \in F, j \in D \quad x_{ij} \leq y_i,$$
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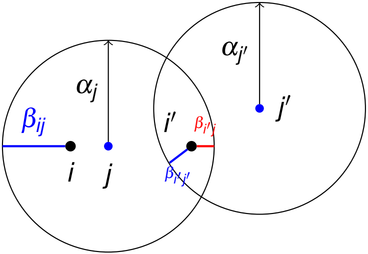
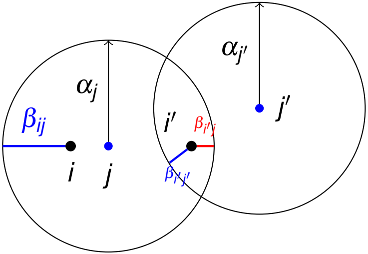
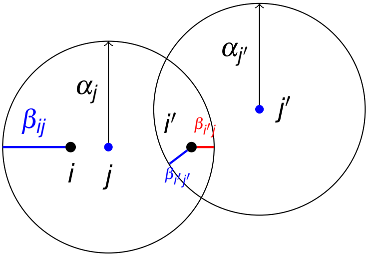


Total payment to facility i at most f_i before opening.
Complementary slackness: $x_{ij} \geq 0$ if and only if $\alpha_j \geq d_{ij}$.
only assign client to "paid to" facilities.

Use Dual.

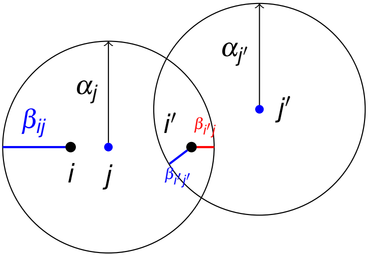


Use Dual.



1. Find solution to primal, (x, y), and dual, (α, β).

Use Dual.



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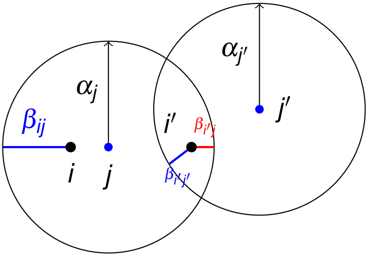

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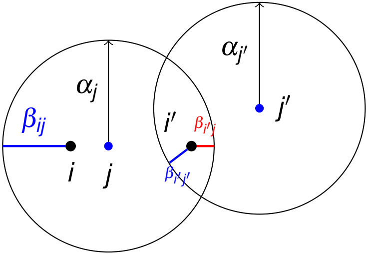

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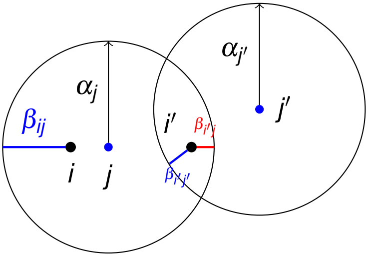

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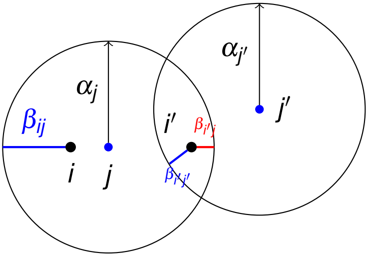

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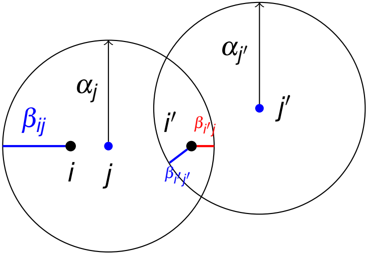

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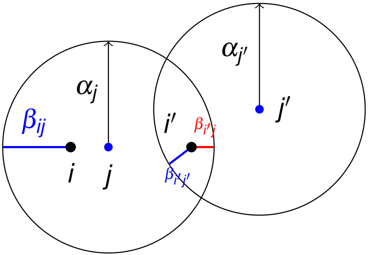

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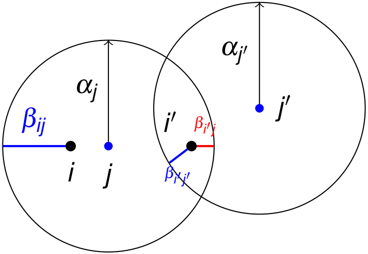

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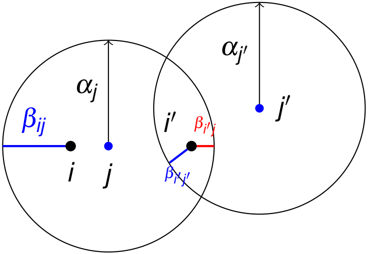

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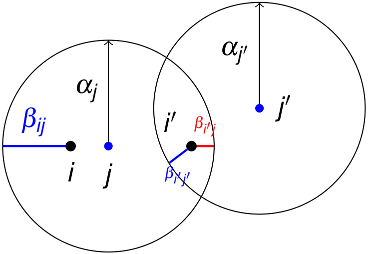

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Claim: Total facility cost is at most $\sum_i f_i y_i$.

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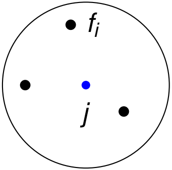

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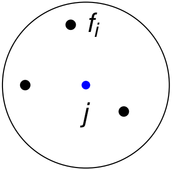

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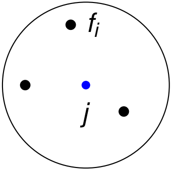

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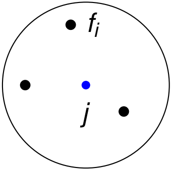

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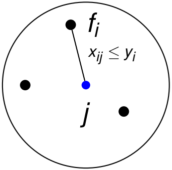

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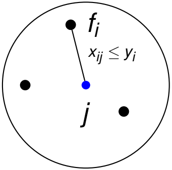

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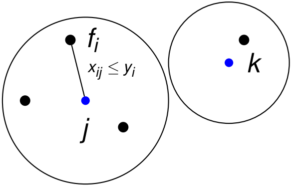

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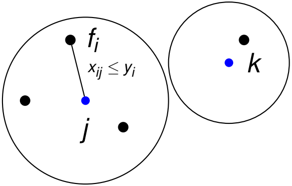

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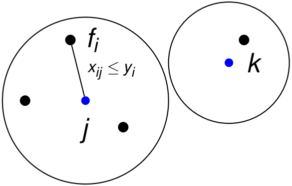

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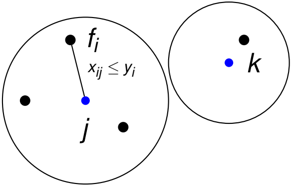

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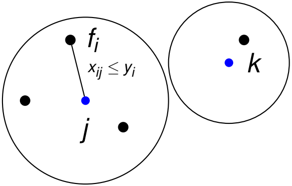

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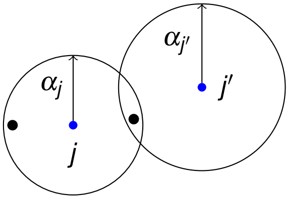

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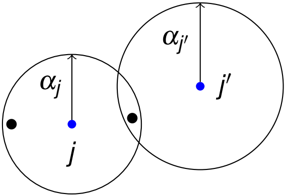

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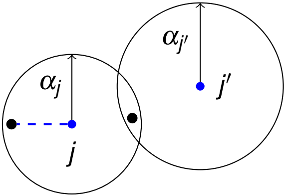

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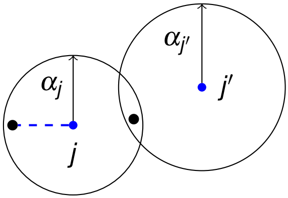

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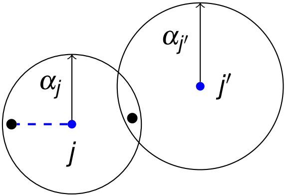

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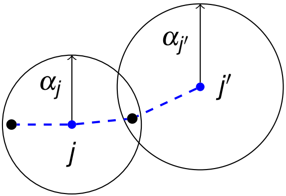

Connection Cost of j: $\leq \alpha_j$.
Connection Cost of j':

Connection Cost.

2. For smallest (remaining) α_j,
   (a) Let $N_j = \{i : x_{ij} > 0\}$.
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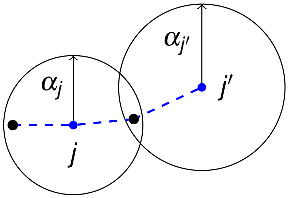

Connection Cost of j: $\leq \alpha_j$.
Connection Cost of j':
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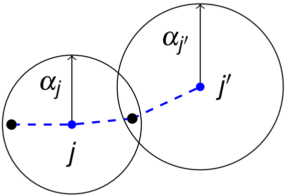

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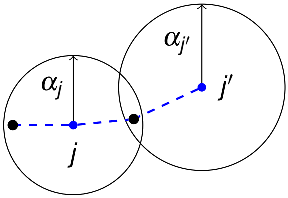

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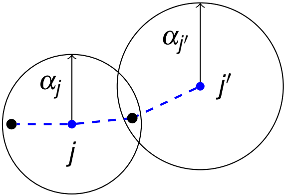

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Total connection cost:
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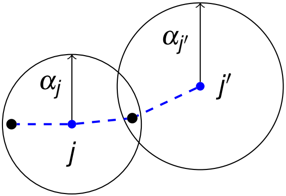

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Previous Slide: Facility cost:

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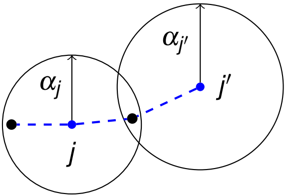

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Previous Slide: Facility cost:
$\leq$ primal "facility" cost $\leq$ Primal OPT.

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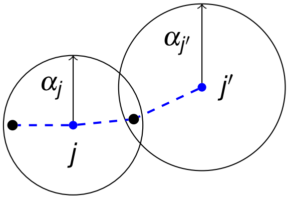

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Total Cost: 4 OPT.

Twist on randomized rounding.

Client j:

Twist on randomized rounding.

Client j: $\sum_i x_{ij} = 1$,

Twist on randomized rounding.

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Expected opening cost:

Twist on randomized rounding.

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Expected opening cost: (note: larger than $f_{\min}$.)

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$D_j = \sum_i x_{ij} d_{ij}$

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$D_j = \sum_i x_{ij} d_{ij}$ Expected connection cost of primal for j.

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Expected connection cost j'

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$D_j = \sum_i x_{ij} d_{ij}$ Expected connection cost of primal for j.

Expected connection cost j' $\alpha_j + \alpha_{j'} + D_j$.

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In step 2: pick in increasing order of $\alpha_j + D_j$.

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$2OPT(D)$ plus connection cost of primal.

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$2OPT(D)$ plus connection cost of primal.

Total expected cost:
Facility cost is at most facility cost of primal.

Twist on randomized rounding.

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Connection cost at most $2OPT$ + connection cost of prmal.
$\rightarrow$ at most $3OPT$.

Primal dual algorithm.

1. Feasible integer solution.

Primal dual algorithm.

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3. Cost of integer solution $\leq \alpha$ times dual value.

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Just did it. Used linear program.

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  Begin with feasible dual.

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Set corresponding primal variable to an integer.

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Recall Dual:

Primal dual algorithm.

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Typically. (If dual is maximization.)
  Begin with feasible dual.
  Raise dual variables until tight constraint.
  Set corresponding primal variable to an integer.

Recall Dual:

$$\max \sum_j \alpha_j$$

$$\forall i \in F \quad \sum_{j \in D} \beta_{ij} \leq f_i$$

$$\forall i \in F, j \in D \quad \alpha_j - \beta_{ij} \leq d_{ij}$$

$$\alpha_j, \beta_{ij} \leq 0$$

Facility location primal dual.

Phase 1:

Facility location primal dual.

Phase 1: 1. Initially $\alpha_j, \beta_{ij} = 0$.

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2. Raise α_j for every (unconnected) client.

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When $\alpha_j = d_{ij}$ for some i
raise β_{ij} at same rate

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Intuition:Paying β_{ij} to open i.

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Stop when $\sum_i \beta_{ij} = f_i$.

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Why? Dual: $\sum_i \beta_{ij} \leq f_i$

Intuition: facility paid for.

Temporarily open i.

Connect all tight ji clients j to i.

Facility location primal dual.

Phase 1: 1. Initially $\alpha_j, \beta_{ij} = 0$.
2. Raise α_j for every (unconnected) client.
When $\alpha_j = d_{ij}$ for some i
raise β_{ij} at same rate Why? Dual: $\alpha_j - \beta_{ij} \leq d_{ij}$.
Intuition:Paying β_{ij} to open i.
Stop when $\sum_i \beta_{ij} = f_i$.
Why? Dual: $\sum_i \beta_{ij} \leq f_i$
Intuition: facility paid for.
Temporarily open i.
Connect all tight ji clients j to i.

Facility location primal dual.

Phase 1: 1. Initially $\alpha_j, \beta_{ij} = 0$.
2. Raise α_j for every (unconnected) client.
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Temporarily open i.
Connect all tight *ji* clients *j* to *i*.

3. Continue until all clients connected.

Facility location primal dual.

Phase 1: 1. Initially $\alpha_j, \beta_{ij} = 0$.

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Intuition: facility paid for.

Temporarily open i.

Connect all tight ji clients j to i.

3. Continue until all clients connected.

Phase 2:

Facility location primal dual.

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2. Raise α_j for every (unconnected) client.
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Intuition: facility paid for.
Temporarily open i.
Connect all tight *ji* clients *j* to *i*.

3. Continue until all clients connected.

Phase 2:
Make “edge” between two facilities if paid by a common client.

Facility location primal dual.

Phase 1: 1. Initially $\alpha_j, \beta_{ij} = 0$.
2. Raise α_j for every (unconnected) client.
When $\alpha_j = d_{ij}$ for some i
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Why? Dual: $\sum_i \beta_{ij} \leq f_i$
Intuition: facility paid for.
Temporarily open i.
Connect all tight *ji* clients *j* to *i*.

3. Continue until all clients connected.

Phase 2:
Make "edge" between two facilities if paid by a common client.
Permanently open an independent set of facilities in graph.

Facility location primal dual.

Phase 1: 1. Initially $\alpha_j, \beta_{ij} = 0$.
2. Raise α_j for every (unconnected) client.
When $\alpha_j = d_{ij}$ for some i
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Make "edge" between two facilities if paid by a common client.
Permanently open an independent set of facilities in graph.

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Temporarily open i.
Connect all tight ji clients j to i.

3. Continue until all clients connected.

Phase 2:
Make "edge" between two facilities if paid by a common client.
Permanently open an independent set of facilities in graph.

For client j, connected facility i is opened.

Facility location primal dual.

Phase 1: 1. Initially $\alpha_j, \beta_{ij} = 0$.
2. Raise α_j for every (unconnected) client.
When $\alpha_j = d_{ij}$ for some i
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Why? Dual: $\sum_i \beta_{ij} \le f_i$
Intuition: facility paid for.
Temporarily open i.
Connect all tight *ji* clients *j* to *i*.

3. Continue until all clients connected.

Phase 2:
Make "edge" between two facilities if paid by a common client.
Permanently open an independent set of facilities in graph.

For client *j*, connected facility *i* is opened. Good.

Facility location primal dual.

Phase 1: 1. Initially $\alpha_j, \beta_{ij} = 0$.
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Intuition: facility paid for.
Temporarily open i.
Connect all tight ji clients j to i.

3. Continue until all clients connected.

Phase 2:
Make "edge" between two facilities if paid by a common client.
Permanently open an independent set of facilities in graph.

For client j, connected facility i is opened. Good.
Connected facility not open

Facility location primal dual.

Phase 1: 1. Initially $\alpha_j, \beta_{ij} = 0$.
2. Raise α_j for every (unconnected) client.
When $\alpha_j = d_{ij}$ for some i
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Why? Dual: $\sum_i \beta_{ij} \leq f_i$
Intuition: facility paid for.
Temporarily open i.
Connect all tight ji clients j to i.

3. Continue until all clients connected.

Phase 2:
Make “edge” between two facilities if paid by a common client.
Permanently open an independent set of facilities in graph.

For client j, connected facility i is opened. Good.
Connected facility not open
$\rightarrow$ exists client j' paid i and connected to open facility.

Facility location primal dual.

Phase 1: 1. Initially $\alpha_j, \beta_{ij} = 0$.
2. Raise α_j for every (unconnected) client.
When $\alpha_j = d_{ij}$ for some i
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Why? Dual: $\sum_i \beta_{ij} \leq f_i$
Intuition: facility paid for.
Temporarily open i.
Connect all tight ji clients j to i.

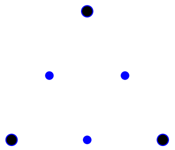
3. Continue until all clients connected.

Phase 2:
Make "edge" between two facilities if paid by a common client.
Permanently open an independent set of facilities in graph.

For client j, connected facility i is opened. Good.
Connected facility not open
$\rightarrow$ exists client j' paid i and connected to open facility.
Connect j to j''s open facility.

Example.

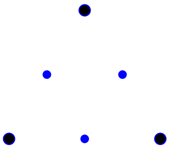
Constraints for dual.



Example.

Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

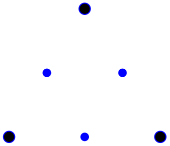


Example.

Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

$\alpha_i - \beta_{ij} \leq d_{ij}$.



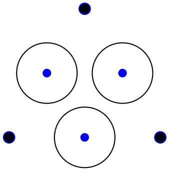
Example.

Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

$\alpha_i - \beta_{ij} \leq d_{ij}$.

Grow α_j.



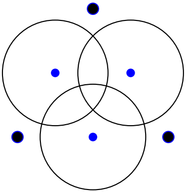
Example.

Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

$\alpha_i - \beta_{ij} \leq d_{ij}$.

Grow α_j.



Example.

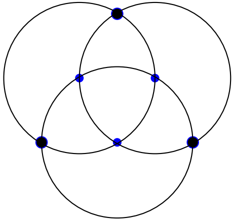
Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

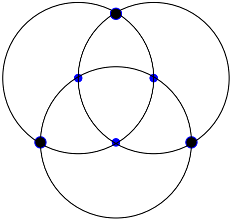
$\alpha_i - \beta_{ij} \leq d_{ij}$.

Grow α_j.

$\alpha_j = d_{ij}$!



Example.



Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

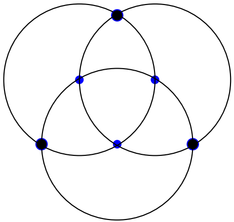
$\alpha_i - \beta_{ij} \leq d_{ij}$.

Grow α_j.

$\alpha_j = d_{ij}$!

Tight constraint:

Example.



Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

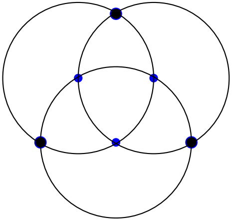
$\alpha_i - \beta_{ij} \leq d_{ij}$.

Grow α_j.

$\alpha_j = d_{ij}$!

Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

Example.



Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

$\alpha_i - \beta_{ij} \leq d_{ij}$.

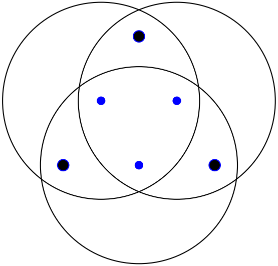
Grow α_j.

$\alpha_j = d_{ij}$!

Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

Grow β_{ij} (and α_j).

Example.



Constraints for dual.

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Grow α_j.

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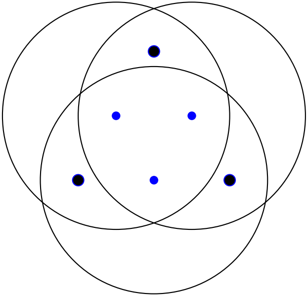
Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

Grow β_{ij} (and α_j).

Example.



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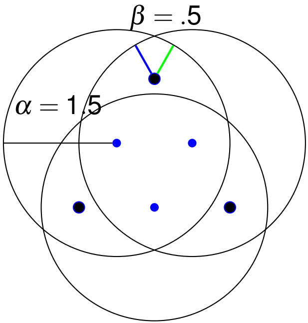
Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

Grow β_{ij} (and α_j).

Example.



Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

$\alpha_i - \beta_{ij} \leq d_{ij}$.

Grow α_j.

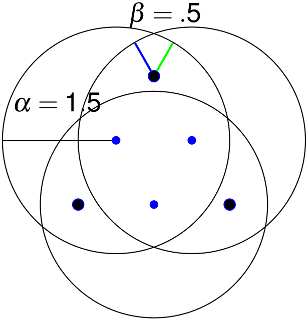
$\alpha_j = d_{ij}$!

Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

Grow β_{ij} (and α_j).

$\sum_j \beta_{ij} = f_i$ for all facilities.

Example.



Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

$\alpha_i - \beta_{ij} \leq d_{ij}$.

Grow α_j.

$\alpha_j = d_{ij}$!

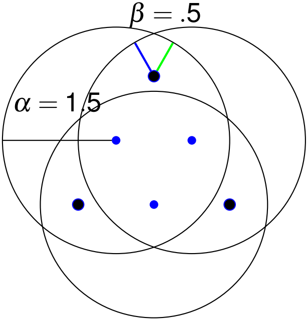
Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

Grow β_{ij} (and α_j).

$\sum_j \beta_{ij} = f_i$ for all facilities.

Tight: $\sum_j \beta_{ij} \leq f_i$

Example.



Constraints for dual.

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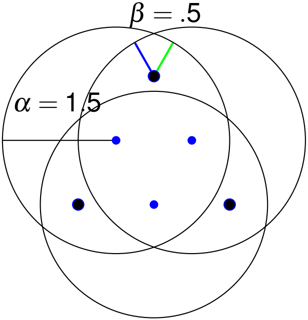
$\sum_j \beta_{ij} = f_i$ for all facilities.

Tight: $\sum_j \beta_{ij} \leq f_i$

Example.



Constraints for dual.

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$\alpha_i - \beta_{ij} \leq d_{ij}$.

Grow α_j.

$\alpha_j = d_{ij}$!

Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

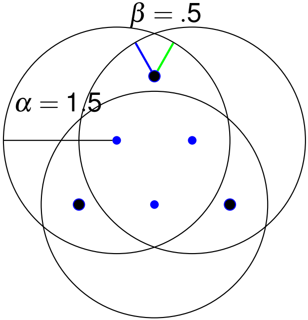
Grow β_{ij} (and α_j).

$\sum_j \beta_{ij} = f_i$ for all facilities.

Tight: $\sum_j \beta_{ij} \leq f_i$

LP Cost: $\sum_j \alpha_j$

Example.



Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

$\alpha_i - \beta_{ij} \leq d_{ij}$.

Grow α_j.

$\alpha_j = d_{ij}$!

Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

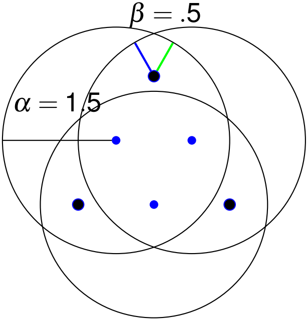
Grow β_{ij} (and α_j).

$\sum_j \beta_{ij} = f_i$ for all facilities.

Tight: $\sum_j \beta_{ij} \leq f_i$

LP Cost: $\sum_j \alpha_j = 4.5$

Example.



Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

$\alpha_i - \beta_{ij} \leq d_{ij}$.

Grow α_j.

$\alpha_j = d_{ij}$!

Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

Grow β_{ij} (and α_j).

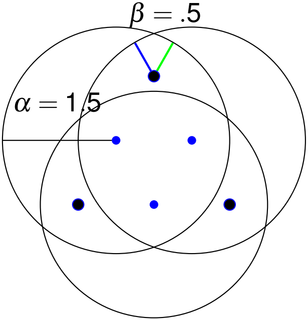
$\sum_j \beta_{ij} = f_i$ for all facilities.

Tight: $\sum_j \beta_{ij} \leq f_i$

LP Cost: $\sum_j \alpha_j = 4.5$

Temporarily open all facilities.

Example.



Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

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Grow α_j.

$\alpha_j = d_{ij}$!

Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

Grow β_{ij} (and α_j).

$\sum_j \beta_{ij} = f_i$ for all facilities.

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LP Cost: $\sum_j \alpha_j = 4.5$

Temporarily open all facilities.

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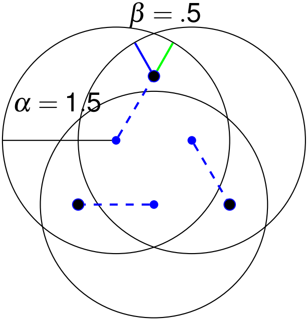

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Grow β_{ij} (and α_j).

$\sum_j \beta_{ij} = f_i$ for all facilities.

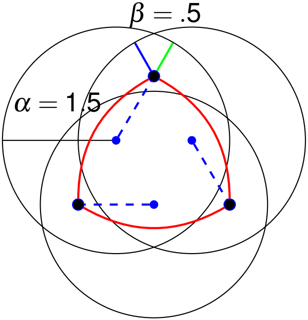
Tight: $\sum_j \beta_{ij} \leq f_i$

LP Cost: $\sum_j \alpha_j = 4.5$

Temporarily open all facilities.

Assign Clients to “paid to” open facility.

Example.



Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

$\alpha_i - \beta_{ij} \leq d_{ij}$.

Grow α_j.

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Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

Grow β_{ij} (and α_j).

$\sum_j \beta_{ij} = f_i$ for all facilities.

Tight: $\sum_j \beta_{ij} \leq f_i$

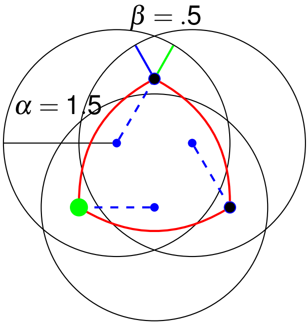
LP Cost: $\sum_j \alpha_j = 4.5$

Temporarily open all facilities.

Assign Clients to “paid to” open facility.

Connect facilities with common client.

Example.



Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

$\alpha_i - \beta_{ij} \leq d_{ij}$.

Grow α_j.

$\alpha_j = d_{ij}$!

Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

Grow β_{ij} (and α_j).

$\sum_j \beta_{ij} = f_i$ for all facilities.

Tight: $\sum_j \beta_{ij} \leq f_i$

LP Cost: $\sum_j \alpha_j = 4.5$

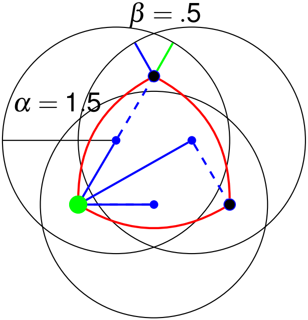
Temporarily open all facilities.

Assign Clients to “paid to” open facility.

Connect facilities with common client.

Open independent set.

Example.



Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

$\alpha_i - \beta_{ij} \leq d_{ij}$.

Grow α_j.

$\alpha_j = d_{ij}$!

Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

Grow β_{ij} (and α_j).

$\sum_j \beta_{ij} = f_i$ for all facilities.

Tight: $\sum_j \beta_{ij} \leq f_i$

LP Cost: $\sum_j \alpha_j = 4.5$

Temporarily open all facilities.

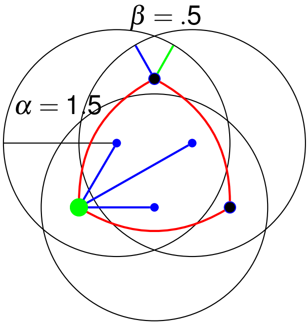
Assign Clients to “paid to” open facility.

Connect facilities with common client.

Open independent set.

Connect to “killer” client’s facility.

Example.



Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

$\alpha_i - \beta_{ij} \leq d_{ij}$.

Grow α_j.

$\alpha_j = d_{ij}$!

Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

Grow β_{ij} (and α_j).

$\sum_j \beta_{ij} = f_i$ for all facilities.

Tight: $\sum_j \beta_{ij} \leq f_i$

LP Cost: $\sum_j \alpha_j = 4.5$

Temporarily open all facilities.

Assign Clients to “paid to” open facility.

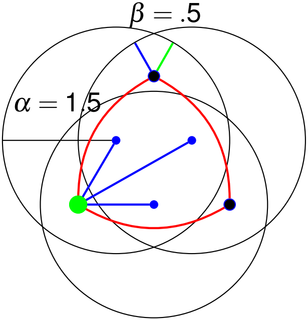
Connect facilities with common client.

Open independent set.

Connect to “killer” client’s facility.

Cost: 1 + 3.7

Example.



Constraints for dual.

$\sum_j \beta_{ij} \leq f_i$

$\alpha_i - \beta_{ij} \leq d_{ij}$.

Grow α_j.

$\alpha_j = d_{ij}$!

Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.

Grow β_{ij} (and α_j).

$\sum_j \beta_{ij} = f_i$ for all facilities.

Tight: $\sum_j \beta_{ij} \leq f_i$

LP Cost: $\sum_j \alpha_j = 4.5$

Temporarily open all facilities.

Assign Clients to "paid to" open facility.

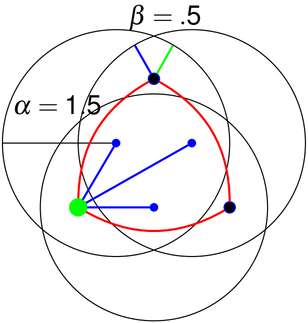
Connect facilities with common client.

Open independent set.

Connect to "killer" client's facility.

Cost: 1 + 3.7 = 4.7.

Example.



Constraints for dual.
$\sum_j \beta_{ij} \leq f_i$
$\alpha_i - \beta_{ij} \leq d_{ij}$.
Grow α_j.
$\alpha_j = d_{ij}$!
Tight constraint: $\alpha_j - \beta_{ij} \leq d_{ij}$.
Grow β_{ij} (and α_j).
$\sum_j \beta_{ij} = f_i$ for all facilities.
Tight: $\sum_j \beta_{ij} \leq f_i$
LP Cost: $\sum_j \alpha_j = 4.5$

Temporarily open all facilities.
Assign Clients to “paid to” open facility.
Connect facilities with common client.
Open independent set.
Connect to “killer” client’s facility.
Cost: 1 + 3.7 = 4.7.
A bit more than the LP cost.

Analysis

Claim: Client only pays one facility.

Analysis

Claim: Client only pays one facility.

Independent set of facilities.

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Claim: S_i - directly connected clients to open facility i.

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Claim: S_i - directly connected clients to open facility i.
$f_i + \sum_{j \in S_i} d_{ij} \leq \sum_j \alpha_j$.

Analysis

Claim: Client only pays one facility.

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Claim: S_i - directly connected clients to open facility i.
$f_i + \sum_{j \in S_i} d_{ij} \leq \sum_j \alpha_j$.

Proof:

Analysis

Claim: Client only pays one facility.

Independent set of facilities.

Claim: S_i - directly connected clients to open facility i.
$f_i + \sum_{j \in S_i} d_{ij} \leq \sum_j \alpha_j$.

Proof:
$f_i = \sum_{j \in S_i} \beta_{ij}$

Analysis

Claim: Client only pays one facility.

Independent set of facilities.

Claim: S_i - directly connected clients to open facility i.
$f_i + \sum_{j \in S_i} d_{ij} \leq \sum_j \alpha_j$.

Proof:
$f_i = \sum_{j \in S_i} \beta_{ij} = \sum_{j \in S_i} \alpha_j - d_{ij}$.

Analysis

Claim: Client only pays one facility.

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Since directly connected: $\beta_{ij} = \alpha_j - d_{ij}$.

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Analysis.

Claim: Client j is indirectly connected to i

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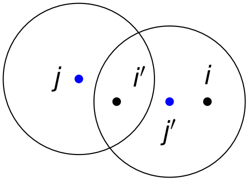
Directly connected to (temp open) i'



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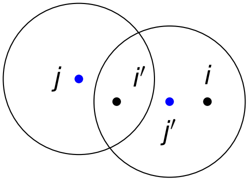
Directly connected to (temp open) i'
has common client j' with some facility i.



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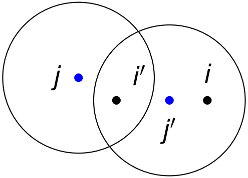

Directly connected to (temp open) i'
has common client j' with some facility i.
client j' has $\alpha_{j'} \geq d_{ij'}$ and $\alpha_j \geq d_{i'j'}$.

Analysis.

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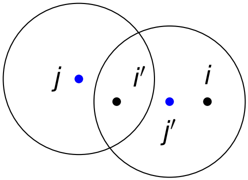

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When i' opens, stops both α_j and α_j'.

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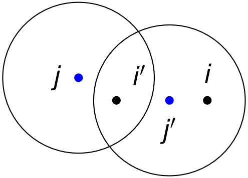

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When i' opens, stops both α_j and α'_j.
$\alpha_{j'}$ stopped no later

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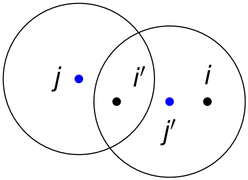

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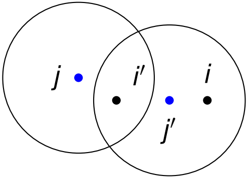

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$\alpha_{j'} \leq \alpha_j$.

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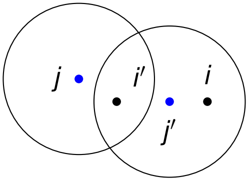

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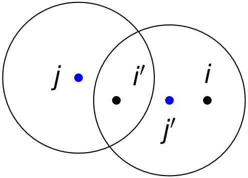
When i' opens, stops both α_j and α_j'.
$\alpha_{j'}$ stopped no later (..maybe earlier..)
$\alpha_{j'} \leq \alpha_j$.
Total distance from j to j'.

Analysis.

Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.



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$d_{ji'} +$

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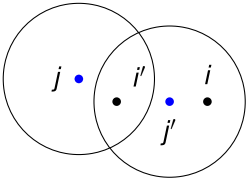

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Total distance from j to j'.
$d_{ji'} + d_{i'j'} +$

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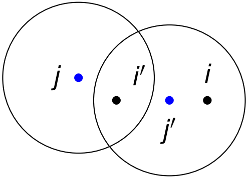

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$d_{ji'} + d_{i'j'} + d_{j'i}$

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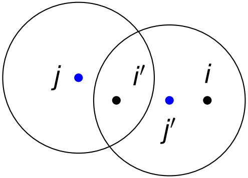

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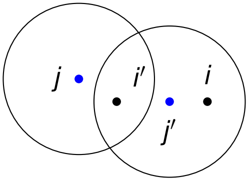

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$d_{ji'} + d_{i'j'} + d_{j'i} \leq 3\alpha_j$ □

Putting it together!

Claim: Client only pays one facility.

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Total Cost:

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Total Cost:
direct clients dual (α_j) pays for facility and own connections.

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Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.

Total Cost:
direct clients dual (α_j) pays for facility and own connections.
plus no more than 3 times indirect client dual.

Putting it together!

Claim: Client only pays one facility.
Claim: S_i - directly connected clients to open facility i.
$f_i + \sum_{j \in S_i} d_{ij} \leq \sum_j \alpha_j$.
Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.

Total Cost:
direct clients dual (α_j) pays for facility and own connections.
plus no more than 3 times indirect client dual.
Total Cost: 3 times dual.

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Claim: S_i - directly connected clients to open facility i.
$f_i + \sum_{j \in S_i} d_{ij} \leq \sum_j \alpha_j$.
Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.

Total Cost:
direct clients dual (α_j) pays for facility and own connections.
plus no more than 3 times indirect client dual.
Total Cost: 3 times dual.
feasible dual upper bounds fractional (and integer) primal.

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3 OPT.

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Total Cost: 3 times dual.
feasible dual upper bounds fractional (and integer) primal.

3 OPT.

Fast!

Putting it together!

Claim: Client only pays one facility.
Claim: S_i - directly connected clients to open facility i.
$f_i + \sum_{j \in S_i} d_{ij} \le \sum_j \alpha_j$.
Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \le 3\alpha_j$.

Total Cost:
direct clients dual (α_j) pays for facility and own connections.
plus no more than 3 times indirect client dual.
Total Cost: 3 times dual.
feasible dual upper bounds fractional (and integer) primal.

3 OPT.

Fast! Cheap!

Putting it together!

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Claim: S_i - directly connected clients to open facility i.
$f_i + \sum_{j \in S_i} d_{ij} \leq \sum_j \alpha_j$.
Claim: Client j is indirectly connected to i
$\rightarrow d_{ij} \leq 3\alpha_j$.

Total Cost:
direct clients dual (α_j) pays for facility and own connections.
plus no more than 3 times indirect client dual.
Total Cost: 3 times dual.
feasible dual upper bounds fractional (and integer) primal.

3 OPT.

Fast! Cheap! Safe!