Streaming

Stream: x_1,

Streaming

Stream: $x_1, x_2,$

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Resources: $O(\log^c n)$ storage.

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Today's Goal: find frequent items.

Frequent Items: deterministic.

Additive $\frac{n}{k}$ error.

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Only reasonable for frequent items.

Deteministic Algorithm.

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Example:

State: $k = 3$

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Stream

$[(1,1)]$

1,

Previous State

0

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Example:

State: $k = 3$

Stream

$1, 2$

$[(1,1) - -(2,1)]$

Previous State

$[(1,1)]$

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Example:

State: $k = 3$

Stream

$[(1,2) - -(2,1) - -(3,1)]$

$1, 2, 3, 1$

Previous State

$[(1,1) - -(2,1) - -(3,1)]$

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Example:

State: $k = 3$

Stream

$1,2,3,1,2,4$

$[(1,1) - -(2,1) - -(3,0)]$

Previous State

$[(1,2) - -(2,2) - -(3,1)]$

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Space? $O(k \log n)$

Turnstile Model and Randomization

Stream: $\ldots, (i, c_i), \ldots$

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Space $O(\frac{1}{\varepsilon} \log \frac{1}{\delta} \log n)$.

Count Min Sketch

Sketch

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Sketch – Summary of stream.

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Why t buckets?

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Why t buckets? To get high probability.

Count min sketch:analysis

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Two equations, two unknowns, one solution out of $p(p-1)$ possibilities.

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$h(x) = g(x) \pmod{k}$.

$Pr[h(x) = h(y)]$?

$= Pr[g(x) = g(y) \pmod{k}]$.

If $g(x) = x_1$, the probability of $g(y) = x_2$ is (almost) uniform.

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Sample space: $a \in [1, p-1], b \in [0, p]$.

Consider any $x, y \in [0, p-1]$.

For $x \neq y$.

$Pr[g(x) = x_1, g(y) = x_2]$?

$ax + b = x_1 \pmod{p}$.
$ay + b = x_2 \pmod{p}$.

Two equations, two unknowns, one solution out of $p(p-1)$ possibilities.

$h(x) = g(x) \pmod{k}$.

$Pr[h(x) = h(y)]$?

$= Pr[g(x) = g(y) \pmod{k}]$.

If $g(x) = x_1$, the probability of $g(y) = x_2$ is (almost) uniform.

So $Pr[g(x) = g(y) \pmod{k}] \approx 1/k \leq 2/k$.

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Error in terms of $|f|_2 = \sqrt{\sum_i f_2^2}$.

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Cheybshev: $Pr[|X - \mu| > \Delta] \leq \frac{Var(X)}{\Delta^2}$

Choose $k = \frac{4}{\varepsilon^2}$: $Pr[|X| > \varepsilon |f|_2] \leq \frac{|f|_2^2/k}{\varepsilon^2 |f|_2^2}$

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(1) $\cdots$ $g_i : U \to [-1, +1], h_i : U \to [k]$

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If $>$ half tosses close, median is close!
Exists $t = \Theta(\log \frac{1}{\delta})$ where $\geq \frac{1}{2}$ are close with probability $\geq 1 - \delta$
Total Space: $O(\frac{\log \frac{1}{\delta}}{\varepsilon^2}$

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Sum up

Deterministic:

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stream has items

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Count within additive $\frac{n}{k}$

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Count Min:

stream has $\pm$ counts

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Note: Small Summary of stream.

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Related to projection to few dimensions.

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Count-min sketch gives sparse embeddings: Clarkson-Woodruff 2013

See you on Thurday.