Metric spaces.

A metric space X, $d(i,j)$ where
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(A) X from R^d and $d(\cdot,\cdot)$ is Euclidean distance.

(B) X from R^d and $d(\cdot,\cdot)$ is squared Euclidean distance.

(C) X- vertices in graph, $d(i,j)$ is shortest path distances in graph.

(D) X is a set of vectors and $d(u,v)$ is $u \cdot v$.

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(A) Obeys triangle inequality.

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Input to TSP, facility location, some layout problems, ..., metric labelling.

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Hard problems.

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Approximate metric on trees?

Approximate metric using a tree.

Tree metric:

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X is nodes of tree with edge weights

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Hierarchically well separated tree metric:

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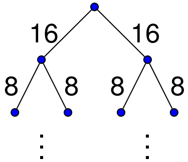
Hierarchically well separated tree metric:
  Tree weights are geometrically decreasing.



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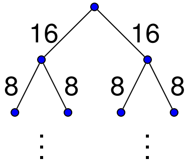

Map X into tree.

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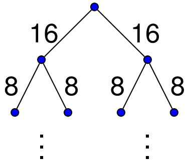

Map X into tree.
(i) No distance shrinks. (dominating)

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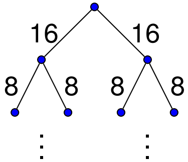

Map X into tree.
 (i) No distance shrinks. (dominating)
 (ii) Every distance stretches $\leq \alpha$

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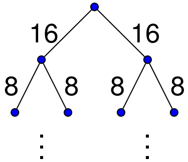

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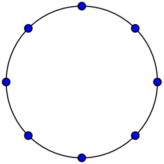

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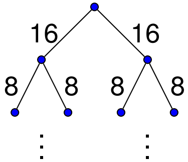

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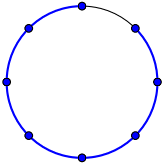

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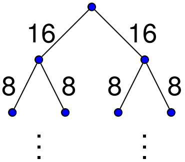

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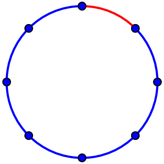

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Distance 1 goes to $n-1$!

Map metric onto tree?

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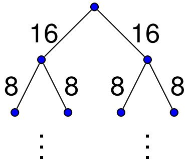

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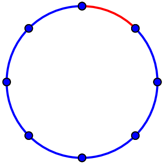

Map metric onto tree?

Distance 1 goes to $n-1$!
Bummer.

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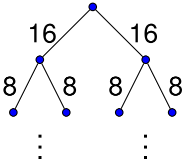
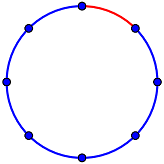


Probabilistic Tree embedding.
  Map X into tree.
  (i) No distance shrinks. (dominating)
  (ii) Every distance stretches $\leq \alpha$
    in expectation.

Map metric onto tree?



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Fix it up chappie!

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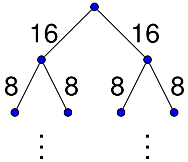

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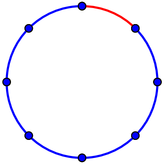

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Fix it up chappie!

For cycle, remove a random edge

Approximate metric using a tree.

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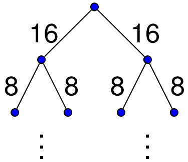

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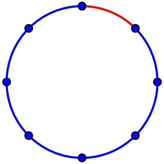

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Map metric onto tree?

Fix it up chappie!

For cycle, remove a random edge get a tree.

Approximate metric using a tree.

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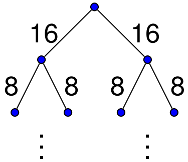

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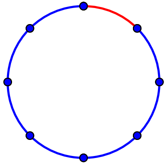

Distance 1 goes to $n-1$!
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Map metric onto tree?

Fix it up chappie!

For cycle, remove a random edge get a tree.

Stretch of edge: $\frac{n-1}{n} \times 1$

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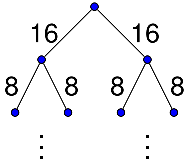

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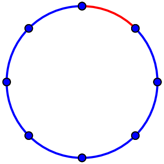

Distance 1 goes to $n-1$!
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Map metric onto tree?

Fix it up chappie!

For cycle, remove a random edge get a tree.

Stretch of edge: $\frac{n-1}{n} \times 1 + \frac{1}{n} \times (n-1)$

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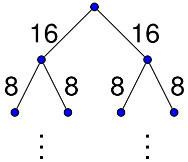

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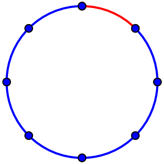

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For cycle, remove a random edge get a tree.

Stretch of edge: $\frac{n-1}{n} \times 1 + \frac{1}{n} \times (n-1) \approx$

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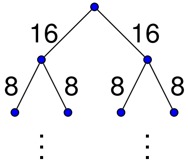

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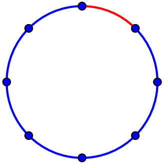

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For cycle, remove a random edge get a tree.

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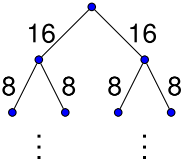

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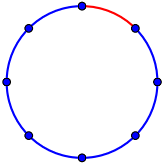

Distance 1 goes to $n-1$!
  Bummer.

Map metric onto tree?

Fix it up chappie!

For cycle, remove a random edge get a tree.

Stretch of edge: $\frac{n-1}{n} \times 1 + \frac{1}{n} \times (n-1) \approx 2$

General metrics?

Probabilistic Tree embedding.

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Map X into tree.

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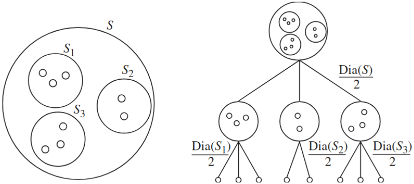
Idea of decomposition.



Algorithm

Algorithm: (X, d), $\mathrm{diam}(X) \le D$, $|X| = n$, $d(i, j) \ge 1$

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Tree has internal node for each level of call.

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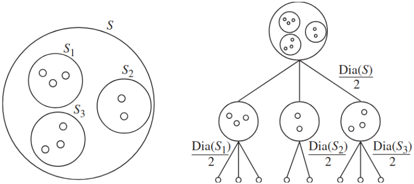

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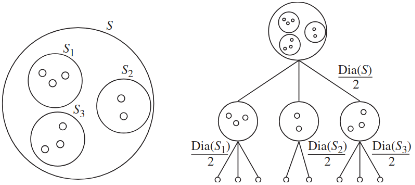

Claim 1: $d_T(x,y) \geq d(x,y)$.

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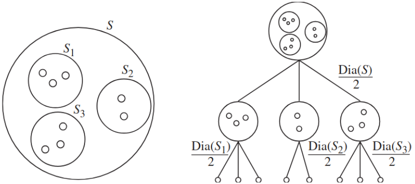

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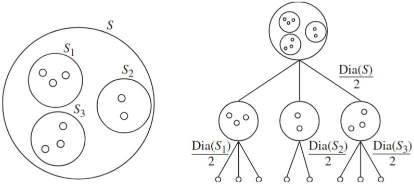

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Analysis: idea

Claim: $E[d_T(x, y)] = O(\log n) d(x, y)$.

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random permutation to deal with this

Analysis: (x, y)

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At some point x is in some Δ level ball.

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Call this set X_Δ.

Analysis: (x, y)

Would like $Pr[x, y \text{ cut by ball} | x \text{ in ball}] \leq \frac{8d(x,y)}{\Delta}$
(Only consider cut by x, factor 2 loss.)

At level Δ

At some point x is in some Δ level ball.
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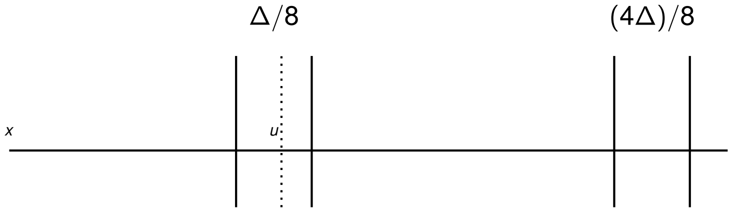
A picture.



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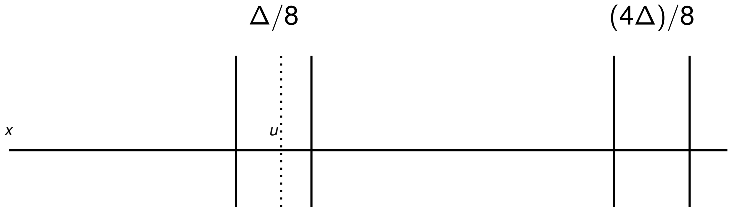

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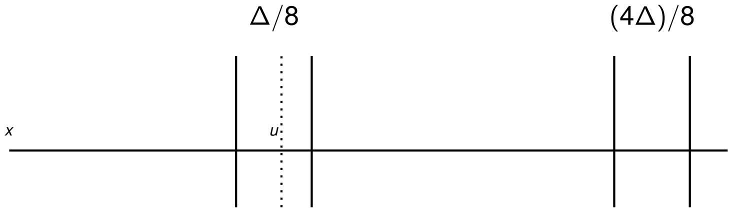

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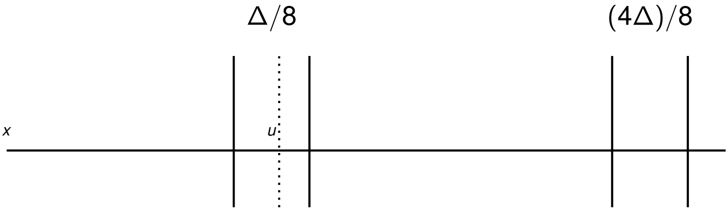

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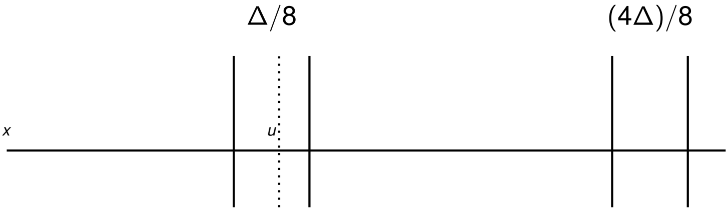

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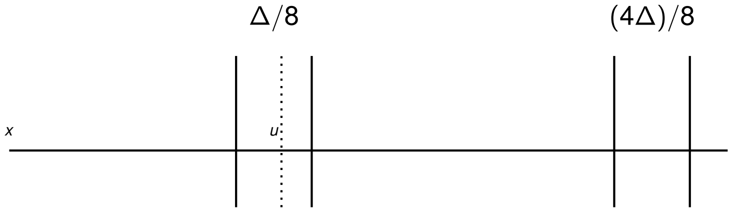

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When Δ was 4Δ the nodes that capture x are separate.

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We gave an algorithm that produces a distribution of trees.

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We gave an algorithm that produces a distribution of trees.

The expected stretch of any pair is $O(\log n)$.

Metric Labelling

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Input: graph $G = (V, E)$ with edge weights, $w(\cdot)$, metric labels (X, d), and costs for mapping vertices to labels $c : V \times X$.

Find an labeling of vertices, $\ell : V \to X$ that minimizes

$\sum_{e=(u,v)} c(e) d(l(u), l(v)) + \sum_v c(v, l(v))$

Idea: find HST for metric (X, d).

Solve the problem on a hierarchically well separated tree metric.

Kleinberg-Tardos: constant factor on uniform metric.

Hierarchically well separated tree, "geometric", constant factor.

$\to O(\log n)$ approximation.