CS174 Fall 98: Lecture Note 0

Alistair Sinclair, August 1998

Some sample problems

1. Balls and bins/load balancing

   Throw n (labeled) balls "at random" into n (labeled) bins.

   - How many bins do you expect to be empty?
   - How many bins do you expect to contain exactly one ball?
   - How many balls do you expect there to be in the fullest bin?

2. Coupon collecting

   There are n different types of coupon. Each cereal box contains exactly one coupon, with all types equally likely.

   - How many boxes of cereal must you buy to have a "reasonable" chance of getting at least one copy of each coupon?

3. Random graphs

   Start with n (labeled) nodes. Throw in edges "at random" until the graph is connected.

   - How many edges do you expect the graph to have?
   - What is its diameter?

   Pick a "random graph" G on n nodes.

   - How large is a largest clique in G?
   - Can you find one?

4. A randomized algorithm for 2-SAT

   Given a 2-SAT formula ϕ, e.g.,

   $$\phi = (x_1 \vee \neg x_2) \wedge (x_1 \vee \neg x_3) \wedge (\neg x_1 \vee x_3) \wedge (\neg x_1 \vee \neg x_2),$$

   consider the following algorithm:

   start with $x_i = \textsc{True} \quad \forall i$
   while ϕ contains an unsatisfied clause C **do**
   choose a literal in C at random and make it true

   - Does this algorithm eventually find a satisfying assignment of ϕ (if one exists)? If so, after how long?