Understanding the Impact of Coalitions between EV Charging Stations

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Abstract

The rapid growth of electric vehicles (EVs) is driving the expansion of charging infrastructure globally. This expansion, however, places significant charging demand on the electricity grid, impacting grid operations and electricity pricing. While coordination among *all* charging stations is beneficial, it may not be always feasible. However, a subset of charging stations, which could be jointly operated by a company, could coordinate to decide their charging profile. In this paper we investigate whether such coalitions between charging stations is better than no coordination.

We model EV charging as a non-cooperative aggregative game, where each station's cost is determined by both monetary payments tied to reactive electricity prices on the grid and its sensitivity to deviations from a *nominal* charging profile. We consider a solution concept that we call $\mathcal{C}$-Nash equilibrium, which is tied to a coalition $\mathcal{C}$ of charging stations coordinating to reduce their costs. We provide sufficient conditions, in terms of the demand and sensitivity of charging stations, to determine when independent (uncoordinated) operation of charging stations could result in lower overall costs to charging stations, the coalition, and charging stations outside the coalition. Somewhat counter to intuition, we demonstrate scenarios where allowing charging stations to operate independently is better than coordinating as a coalition. Jointly, these results provide operators of charging stations insights into how to coordinate their charging behavior, and open several research directions.

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1 Introduction

The proliferation of electric vehicles (EVs) has brought major changes to road transportation. EVs could play a significant role in the transition to a sustainable energy-based future and are projected to surpass traditional internal combustion engine-based vehicles in the coming decades(Bloomberg, 2021). The growth in EVs has led to the genesis of a new ecosystem around building faster and more accessible charging infrastructure (Brown et al., 2024). However, there are several challenges associated with the design of charging infrastructure, including the distribution and affordability of public charging and integrating new infrastructure into the current electricity grid (Kampshoff et al., 2022). One such major challenge is that charging stations, especially fast-charging stations, draw a considerable amount of electricity when in operation (America, 2021; EVgo, 2021; Tesla, 2019, 2021) and may adversely impact the grid infrastructure (Alexeenko and Bitar, 2023; Escudero-Garzas and Seco-Granados, 2012; Lee et al., 2014). The ideal solution to this challenge is to coordinate the demands from all charging stations to distribute the load on the grid (Alexeenko and Bitar, 2023; Obeid et al., 2023). However, this is hard to implement in practice due to the high communication and computational costs of such centralized approaches, and concerns about privacy of the information shared by EV charging stations (Brown et al., 2024).

Given the impracticality of achieving *global coordination* among EV charging stations in most cases, we consider coordination between a *subset* of charging stations, forming strategic coalitions. These coalitions can be organically fostered by emerging electric vehicle charging companies (EVCCs) like Tesla, EVgo, and Chargepoint, each of which oversees multiple charging stations (refer Figure 1 for a pictorial depiction). Although it may appear intuitive that any level of coordination is preferable to none at all, we present counter-examples to illustrate that this is not always valid. This solicits further analysis of the regime between no and full coordination of charging stations.

We use an aggregative game theoretic model to study the interaction between charging stations in the energy market. Charging stations can have heterogeneous charging demands, which they require to be fulfilled from the grid over a finite time horizon. We model each charging station to have *nominal* charging profile over this time window. Furthermore, each charging station is strategic and wants to minimize its cost comprising of (i) a monetary payment for power demanded from the grid, and (ii) the deviations from their desired operating charging profile. We allow the charging stations to have heterogeneous sensitivities to deviation from their nominal charging profile. Since charging stations can have a large demand from the grid, we model them as "price-makers", and

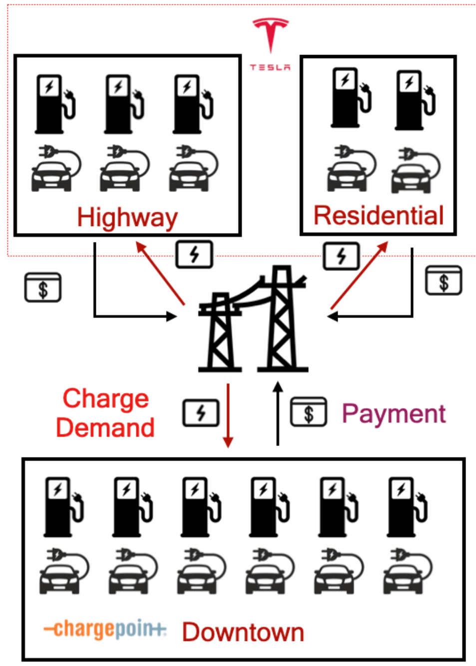


Figure 1: A schematic depiction of charging stations located in different areas (e.g. residential, downtown, highway) and owned by different companies (e.g. Tesla, Chargepoint). Each charging station demands charge from the grid, which determines the prices.

not "price-takers" (Deori et al., 2017; Gan et al., 2012; Ma et al., 2011; Paccagnan et al., 2018). Specifically, we model the unit power price to be contingent upon the aggregate power demanded by all charging stations.

Under this modeling framework, we study the following question in our paper:

> **Q:** *Could coordination between a few charging stations lead to undesirable consequences?*

To answer this question, we compare two separate scenarios: *(a)* when charging stations form a *coalition* (e.g., the charging stations owned by an EVCC), to optimize their combined cost, and *(b)* when each charging station operates independently without any coordination. The two related solution concepts in these settings, $\mathcal{C}-$Nash and Nash equilibrium respectively, are our main objects of study. We characterize them analytically in Theorem 4.1 and Corollary 4.5 respectively. In general, we observe that the equilibrium decomposes into two components: a charging profile uniformly distributed across time and a correction term to account for the coalition between charging stations (Theorem 4.1).

Next, we assess the outcomes under the Nash equilibrium and $\mathcal{C}-$Nash equilibrium from three perspectives: (*i*) the societal perspective, considering the overall cost experienced by all charging stations; (*ii*) the coalition's perspective, focusing on all charging stations operated by an EVCC; and (*iii*) the perspective of charging stations not in the coalition. Under some specific parameter settings, we provide conditions under which the formation of a coalition is better than the independent operation of charging stations in Theorem 4.7 and 4.11. Additionally, we corroborate these theoretical findings by presenting numerical instances when these conditions are satisfied.

Finally, we demonstrate several instances of problem specific parameters like the size of the coalition and composition of the coalition (in terms of demand and sensitivity of stations), which show that coordination may lead to undesirable outcomes to society, coalition and non-coalitional stations. These example highlight instances where the intuition about coordinating charging stations fails.

2 Related Works

In this section, we review several related line of works.

Aggregative games and EV charging. Several works have proposed EV charging as a non-cooperative game (Deori et al., 2017; Gan et al., 2012; Ma et al., 2011; Paccagnan et al., 2018). These works mainly study existence, uniqueness, and computation of Nash equilibria of the EV charging game, and analyze these equilibria via measures such as the price of anarchy (PoA) (Paccagnan et al., 2018). Our key differentiation from this line of literature is to understand the impact of the formation of a coalition of a subset of EV stations.

Coalitions and equilibria. Equilibrium characterization under coalitions has been studied via the notion of *strong Nash equilibrium.* Such equilibria are an outcome wherein no coalition of agents can collectively deviate from their strategy to improve their utilities, and was first introduced as a concept in (Aumann, 1959). Since this seminal paper, numerous studies have delved into necessary and sufficient conditions for its existence (Nessah and Tian, 2014) and computational properties (Gatti et al., 2017). Recently, there is an emerging interest in characterizing the performance of such equilibria in terms of *price of anarchy* (Andelman et al., 2009; Bachrach et al., 2014; Chien and Sinclair, 2009; Epstein et al., 2007; Feldman and Friedler, 2015; Ferguson et al., 2023), which quantifies the worst-case welfare loss at strong equilibrium compared to optimal welfare. In contrast to the concept of strong Nash equilibrium, which serves as a refinement of Nash equilibrium, our $\mathcal{C}-$Nash equilibrium (Definition 3.3) represents a relaxation of the traditional Nash equilibrium.

Unlike strong Nash equilibrium, our primary objective is not to ensure robustness against coalitions but rather to explore and comprehend the implications of naturally occurring coalitions within energy markets.

Users as *price-takers*. Several studies have delved into the potential detrimental effects of uncoordinated Electric Vehicle (EV) charging on the power grid (Gruosso, 2016; Quiros-Tortos et al., 2018). In response, efforts such as those outlined in (Alexeenko and Bitar, 2023; Fu et al., 2020; Obeid et al., 2023) have tackled the challenge of devising incentive mechanisms to coordinate small users, often categorized as "price-takers," to shift their charging windows and mitigate grid impacts. However, in contrast to these approaches, our work conceptualizes charging stations, which serve many EVs and thus can aggregate demand, as "price-makers" within electricity markets.

Coalitions in power systems. Coalitions on the production side of energy markets have been studied extensively (Baeyens et al., 2013; Banaei et al., 2016; Bitar and Poolla, 2012; Blumrosen and Mizrahi, 2023; de la Nieta et al., 2014; Nayyar et al., 2013; Pinson, 2013; Sharma et al., 2014; Zhang et al., 2015, 2018; Zhao et al., 2013). The main goal in this line of literature is to study the impact of coalitions on uncertainty reduction, market power and bidding of renewable power producers. However, our goal here is to study coalitions that could emerge on the demand side, specifically between (heterogeneous) charging stations, and understand the impact of size and composition of coalitions on the resulting equilibrium.

3 Model

Notations. For any positive integer m, we define $[m] := \{1, 2, ..., m\}$. Consider two matrices $A = (a_{ij})_{i\in[m],j\in[n]} \in \mathbb{R}^{m\times n}, B \in \mathbb{R}^{p\times q}$. We define the Kronecker product $A \otimes B \in \mathbb{R}^{pm\times qn}$ such that

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \dots & a_{1n}B \\ a_{21}B & a_{22}B & \dots & a_{1n}B \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}B & a_{m2}B & \dots & a_{mn}B \end{bmatrix}.$$

For any set X, such that $|X| < \infty$, we define $\mathbf{1}_X \in \mathbb{R}^{|X|}$ be a vector with all entries to be 1. For any vector $x \in \mathbb{R}^m$ and $\mathcal{C} \subset [m]$, we use the notation $x_{\mathcal{C}} \in R^{|\mathcal{C}|}$ to denote the components of vector x corresponding to $\mathcal{C}$. Additionally, we denote $x_{-\mathcal{C}} \in R^{m-|\mathcal{C}|}$ to be the components of vector x

corresponding to all entries that are not in $\mathcal{C}$. For any positive integer N, we denote $I_N \in \mathbb{R}^{N\times N}$ to be the identity matrix.

Charging stations as strategic entities. Consider a game consisting of N charging stations, each operating as a strategic entity making decisions on their charging levels throughout a charging period spanning T units of time. Each station $i \in [N]$ is characterized by a *nominal charging profile* $(\bar{x}_i^t)_{t\in[T]}$, where the total charge demanded is d_i. Mathematically, this means $\sum_{t\in[T]} \bar{x}_i^t = d_i$, for every $i \in [N]$. The variation of charge demanded between stations reflect differences in the number of vehicles typically utilizing each charging facility.

However, due to the presence of other stations in the grid, the actual charging profile of any station may differ from the nominal charging profile. Formally, let x_i^t be the charge demanded by station i during step t. It must hold that for every $i \in [N]$, $\sum_{t\in[T]} x_i^t = d_i$. Define $X_i := \{x_i \in \mathbb{R}^T : \sum_{t\in[T]} x_i^t = d_i\}$ to be the set of feasible charging profiles for station i [1]. With a slight abuse of notation, we define $x^t \in \mathbb{R}^N$ as the vector of charging profiles of all stations at time step $t \in [T]$. Additionally, we define $x = (x_i^t)_{t\in[T],i\in[N]}$ to be the joint charging profile of all of stations. We define the set of all joint charging profile of all charging stations to be $X := \prod_{i\in[N]} X_i$.

We model the electricity prices to be *reactive*, i.e. the price of electricity depends on the total charge demanded. More formally, the cost of incurred by any station $i \in [N]$ under a joint strategy $x \in X$ is represented as:

$$c_i(x) := \sum_{t\in[T]} p^t(x) x_i^t + \frac{\mu_i}{2}\|x_i - \bar{x}_i\|^2, \tag{1}$$

where (a) $\mu_i > 0$ is the sensitivity parameter of station $i \in [N]$; (b) for every $t \in [T]$, $p^t(x)$ denotes the price per unit of electricity when the joint charging profile of all stations is x. Using the aggregative game framework (Deori et al., 2017; Gan et al., 2012; Ma et al., 2011; Paccagnan et al., 2018), we consider $p^t(x) := a^t + b^t \mathbf{1}_N^\top x^t$ for some $a^t, b^t > 0$. The parameters a^t represent the price fluctuations due to non-EV demand on the grid, and the parameters b^t represent the marginal cost of electricity for additional unit of production. In this paper, we assume that for all time $t \in [T]$, $b^t = b$ for some positive scalar $b \in \mathbb{R}_+$.

[1] We refrain from imposing non-negativity constraints on the charging profile. This reflects the versatile functionality of charging stations, allowing them not only to draw power from the grid but also to inject power into the grid as needed (Khan et al., 2019). It is an interesting direction of future research to impose additional operational constraints on the set X which align more closely with real-world conditions. Our main goal is to initiate the study of the impact of coalitions on the equilibrium outcome of interaction between EV charging stations.

Remark 3.1. *The heterogeneity in sensitivity parameters μ_i between different stations can be ascribed to the geographic location in which they are situated (cf. Figure 1). For instance, a station positioned in downtown might exhibit heightened sensitivity in meeting its demand requirements compared to one situated within a residential neighborhood.*

Nash equilibrium, defined below, is widely used to characterize the interaction in these kinds of charging games (Deori et al., 2017; Gan et al., 2012; Ma et al., 2011; Paccagnan et al., 2018).

Definition 3.2. *A joint charge profile x^* is a* Nash equilibrium *if $c_i(x_i^*, x_{-i}^*) \leq c_i(x_i, x_{-i}^*)$ for every $i \in [N], x_i \in X_i$.*

Coalition between charging stations. Coalitions between charging stations can be easily facilitated by the emerging electric vehicle charging companies (EVCCs) who operate multiple charging stations. We model that any coalition[2] $\mathcal{C} \subset [N]$ that is formed jointly decides the charging profiles of its constituent stations. The goal of the coalition is to choose $x_{\mathcal{C}} \in \prod_{i\in\mathcal{C}} X_i$ that minimizes its cumulative cost function

$$c_{\mathcal{C}}(x) := \sum_{i\in\mathcal{C}} c_i(x), \quad \forall x \in X. \tag{2}$$

While we focus on the formation of a single coalition here, all results presented in this article can be extended to encompass scenarios involving multiple coalitions. Next, we introduce a natural extension of Definition 3.2 to examine the outcomes of interactions in the presence of coalitions.

Definition 3.3. *A joint charge profile $x^\dagger \in X$ is a $\mathcal{C}$-*Nash equilibrium *if*

$$\sum_{i\in\mathcal{C}} c_i(x^\dagger) \leq \sum_{i\in\mathcal{C}} c_i(x'_{\mathcal{C}}, x^\dagger_{-\mathcal{C}}) \quad \forall\ x'_{\mathcal{C}} \in \prod_{j\in\mathcal{C}} X_j,$$
$$c_i(x^\dagger) \leq c_i(x'_i, x^\dagger_{-i}) \quad \forall\ x'_i \in X_i, i \notin \mathcal{C}.$$

When $|\mathcal{C}| = 1$, Definition 3.2 and 3.3 are equivalent.

For the rest of the article, we shall denote $\mathcal{C}-$Nash equilibrium by $x^\dagger$ and Nash equilibrium in the absence of a coalition by x^*.

[2]Without loss of generality, we re-index the numbering of charging stations in the coalition to ensure that stations $\{1, 2, 3, ...|\mathcal{C}|\}$ are in the coalition.

4 Results

In Section 4.1, we undertake an analytical characterization of the $\mathcal{C}-$Nash equilibrium with respect to the game parameters. Subsequently, in Section 4.2, we establish conditions on these parameters, elucidating scenarios where the traditional Nash equilibrium surpasses the $\mathcal{C}-$Nash equilibrium in terms of overall cost experienced by all charging stations, by charging stations within the coalition, and by charging stations outside the coalition. Finally, in Section 4.3, we provide numerical examples that exemplify the conditions delineated in Section 4.2.

4.1 Analytical Characterization of $\mathcal{C}-$Nash equilibrium

Theorem 4.1. *For any arbitrary coalition $\mathcal{C} \subset [N]$, when $b > 0$ and $\mu_i > 0\ \forall i \in N$, the $\mathcal{C}-$Nash equilibrium exists, is unique, and takes the following form*

$$x^{\dagger t} = \frac{d}{T} + \sum_{t' \in [T]} \frac{T\delta^{tt'} - 1}{T} \left(b(\mathbf{1}_N \mathbf{1}_N^\top + \boldsymbol{C}) + \mu \right)^{-1} \cdot \left(\mu \bar{x}^{t'} - a^{t'} \mathbf{1}_N \right),$$

where $\delta^{tt'}$ is the Kronecker delta function ($\delta^{tt'} = 1$ when $t = t'$ and is 0 otherwise), $\mu = \mathsf{diag}([\mu_1, \cdots, \mu_N])$ and

$$\boldsymbol{C} := \begin{pmatrix} \mathbf{1}_{\mathcal{C}} \mathbf{1}_{\mathcal{C}}^\top & 0 \\ 0 & I_{[N]\setminus\mathcal{C}} \end{pmatrix}.$$

Proof. Before presenting the proof, we define some useful notation. Define $A = [a^1, a^2, ..., a^T]^\top$ and the concatenated vector of charging profiles $x^\dagger := [x_1^{1\dagger}, \cdots, x_N^{1\dagger}, \cdots\cdots, x_1^{T\dagger}, \cdots, x_N^{T\dagger}]^\top$.

First, we show that for any coalition $\mathcal{C} \subset [N]$, the resulting game is a strongly monotone game. To ensure this it is sufficient to verify that (i) the strategy set is convex, and (ii) the game Jacobian is positive definite (Facchinei and Pang, 2003). The convexity of strategy sets holds because the strategy sets X_i are simplices. Next, to compute the game Jacobian, we define

$$\mathcal{G}_i(x) = \begin{cases} \frac{\partial c_{\mathcal{C}}(x)}{\partial x_i} & \text{if } i \in \mathcal{C}, \\ \frac{\partial c_i(x)}{\partial x_i} & \text{if } i \notin \mathcal{C}. \end{cases}$$

It can be verified that the game Jacobian, $\nabla \mathcal{G}(x) = \Theta := I_T \otimes \left(b(\mathbf{1}_N \mathbf{1}_N^\top + \mathbf{C}) + \mu \right)$, is a positive definite matrix (cf. Lemma 4.2). Thus, for any $\mathcal{C}$, the game is strongly monotone, and there exists a unique equilibrium.

We now introduce two optimization problems:

$$\begin{aligned}
P_{\mathcal{C}}(x^{\dagger}_{-\mathcal{C}}) : \min_{x_{\mathcal{C}} \in X_{\mathcal{C}}} \; & \sum_{j \in \mathcal{C}} c_j(x_{\mathcal{C}}, x^{\dagger}_{-\mathcal{C}}) \\
\text{s.t.} \; & \sum_{t' \in [T]} x_j^{t'} = d_j \quad \forall j \in \mathcal{C}, \\
\forall\, i \in [N] \backslash \mathcal{C},\; P_i(x^{\dagger}_{-i}) : \min_{x_i \in X_i} \; & c_i(x_i, x^{\dagger}_{-i}) \\
\text{s.t.} \; & \sum_{t' \in [T]} x_i^{t'} = d_i.
\end{aligned}$$

By definition of $\mathcal{C}-$Nash equilibrium, charging stations within $\mathcal{C}$ jointly solve the optimization problem $P_{\mathcal{C}}(x^{\dagger}_{-\mathcal{C}})$ when the charging profiles of other stations $x^{\dagger}_{-\mathcal{C}}$ is known. Similarly, each player $i \in [N] \backslash \mathcal{C}$ solves the optimization problem $P_i(x^{\dagger}_{-i})$ when the charging profile of other players $x^{\dagger}_{-i}$ is known. Note that each of the optimization problems has linear constraints, and hence the Karush–Kuhn–Tucker (KKT) conditions are necessary for optimality. We solve the KKT conditions of all these problems simultaneously to get a unique solution. Since we established that the equilibrium is unique, this unique solution to the KKT conditions is the $\mathcal{C}-$Nash equilibrium.

The KKT conditions can be written in a compact form as in (3), where $\lambda = (\lambda_i)_{i \in [N]}$ is the vector of Lagrange multipliers associated with the linear constraint of every charging station.

$$\Theta x^{\dagger} = (I_T \otimes \mu)\bar{x} - (I_T \otimes \mathbf{1}_N)A - (\mathbf{1}_T \otimes I_N)\lambda \tag{3a}$$

$$(\mathbf{1}_T^{\top} \otimes I_N)x^{\dagger} = d, \tag{3b}$$

where $\Gamma := (\mathbf{1}_T^{\top} \otimes I_N)\Theta^{-1}(\mathbf{1}_T \otimes I_N)$. We prove in Lemma 4.2 that Γ and Θ are invertible. Using this fact and solving (3) by eliminating λ, we obtain a closed-form expression for the $\mathcal{C}-$Nash equilibrium in (4).

$$x^{\dagger} = \Psi_1(\Gamma, \Theta)\left((I_T \otimes \mu)\bar{x} - (I_T \otimes \mathbf{1}_N)A\right) + \Psi_2(\Gamma, \Theta)d, \tag{4}$$

with $\Psi_1(\Gamma, \Theta) := \left(I_{NT} - \Theta^{-1}(\mathbf{1}_T \otimes I_N)\Gamma^{-1}(\mathbf{1}_T^{\top} \otimes I_N)\right)\Theta^{-1}$ and $\Psi_2(\Gamma, \Theta) := \Theta^{-1}(\mathbf{1}_T \otimes I_N)\Gamma^{-1}$. Expanding the terms using the time index, it can be checked that equation (4) is equivalent to (3a), completing the proof. □

Next, we present a technical result, used in proof of Theorem 4.1, the proof of which is deferred to Appendix A.1.

Lemma 4.2. *Θ and Γ are positive definite and hence invertible matrices.*

We now make some remarks about Theorem 4.1.

Remark 4.3. *The closed-form equilibrium solution in (3a) is comprised of a fixed term and a correction term. The fixed term represents a charging profile that is uniform across time. The second term is the correction to account for the aggregate effects of the coalition, demand and charging preferences of the charging stations, and exogenous non-EV demand. As expected, when the time-dependent factors $a^{t'}$ and $\bar{x}^{t'}$ are constant across time, the correction term is zero and charging stations charge uniformly across time.*

Remark 4.4. *The result in Theorem 4.1 extends to the setting of multiple (non-overlapping) coalitions $\mathcal{C}_1, \mathcal{C}_2, ...\mathcal{C}_K$ by setting*

$$\boldsymbol{C} = \begin{pmatrix} \mathbf{1}_{\mathcal{C}_1}\mathbf{1}_{\mathcal{C}_1}^\top & 0 & \cdots & 0 \\ 0 & \mathbf{1}_{\mathcal{C}_2}\mathbf{1}_{\mathcal{C}_2}^\top & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \mathbf{1}_{\mathcal{C}_K}\mathbf{1}_{\mathcal{C}_K}^\top & \vdots \\ 0 & 0 & 0 & I_{N \setminus \cup_{i=1}^K \mathcal{C}_i} \end{pmatrix}.$$

In fact, all theoretical results in this article can be directly extended to the multiple coalition case by using the above $\boldsymbol{C}$ matrix. For the sake of clear presentation, we shall always work with $K = 1$ in the following text.

We conclude this section by stating the following specialization of Theorem 4.1 that characterizes Nash equilibrium.

Corollary 4.5. *The Nash equilibrium x^* takes the following form:*

$$x^{*t} = \frac{d}{T} + \sum_{t' \in [T]} \frac{T\delta^{tt'} - 1}{T} \left(b(\mathbf{1}_N \mathbf{1}_N^\top + I_N) + \mu \right)^{-1} \cdot \left(\mu \bar{x}^{t'} - a^{t'} \mathbf{1}_N \right),$$

where $I_N \in \mathbb{R}^{N \times N}$ is the identity matrix.

Proof of Corollary 4.5 follows by setting $\mathcal{C} = \{1\}$ in Theorem 4.1.

4.2 When is $\mathcal{C}-$Nash equilibrium beneficial?

In this subsection, we study conditions under which $\mathcal{C}-$Nash equilibrium will be preferred over Nash equilibrium and vice-versa. For a charging profile x, define the total cost incurred by a group of charging stations in $\mathcal{S} \subseteq [N]$ as

$$c_{\mathcal{S}}(x) := \sum_{i \in \mathcal{S}} c_i(x), \quad \forall\, x \in X. \tag{5}$$

In order to compare the outcome under Nash equilibrium and $\mathcal{C}-$Nash equilibrium, we use the following metrics:

$$\mathsf{M}_{[N]} := \frac{c_{[N]}(x^*)}{c_{[N]}(x^\dagger)}, \quad \mathsf{M}_{\mathcal{C}} := \frac{c_{\mathcal{C}}(x^*)}{c_{\mathcal{C}}(x^\dagger)}, \quad \mathsf{M}_{[N]\backslash\mathcal{C}} := \frac{c_{[N]\backslash\mathcal{C}}(x^*)}{c_{[N]\backslash\mathcal{C}}(x^\dagger)}. \tag{Eval-Metric}$$

For any $\mathcal{S} \in \{[N], \mathcal{C}, [N]\backslash\mathcal{C}\}$, if $\mathsf{M}_{\mathcal{S}} < 1$ then Nash equilibrium is preferred, otherwise $\mathcal{C}-$Nash equilibrium is preferred[3] by the coalition $\mathcal{S}$. Next, we theoretically characterize these metrics in two cases:

4.2.1 Case A: Exogeneous price fluctuations a^t are uniform across time

Here, we assume that there exists some $a \in \mathbb{R}_+$ such that $a^t = a$, for every $t \in [T]$. This can happen when the price is influenced by a constant non-EV demand throughout the day. Under this setting, the equilibria $x^\dagger$ and x^* are represented below.

Proposition 4.6. *Suppose $a^t = a^{t'}$ for all $t, t' \in [T]$. Then for every $t \in [T]$,*

$$x^{*t} = \frac{d}{T} + \sum_{t' \in [T]} \frac{T\delta^{tt'} - 1}{T} \left(b(\mathbf{1}_N \mathbf{1}_N^\top + I_N) + \mu \right)^{-1} \mu \bar{x}^{t'},$$
$$x^{\dagger t} = \frac{d}{T} + \sum_{t' \in [T]} \frac{T\delta^{tt'} - 1}{T} \left(b(\mathbf{1}_N \mathbf{1}_N^\top + C) + \mu \right)^{-1} \mu \bar{x}^{t'}.$$

Further, consider the scenario when all the charging stations prepare for similar peak and low demand hours, and use charging rate recommendations from EV manufacturers to predict their demand requirements. That is, they desire similar demand profiles (up to a constant factor) across the day. This scenario is captured in Assumption 4.1.

Assumption 4.1. *The desired demand of each charging station $i \in [N]$ at time step $t \in [T]$ is*

[3] An implicit assumption here is that cost is always non-negative, which is a natural assumption in real-world.

$\bar{x}_i^t = d_i\alpha^t$, *where* $\sum_{t\in[T]} \alpha^t = 1$ *and* $\alpha^t \geq 0$ *for all* $t \in [T]$.

Theorem 4.7. *Suppose Assumption 4.1 holds and* $a^t = a^{t'}$ *for all* $t, t' \in [T]$. *Any group* $\mathcal{S} \subseteq [N]$ *incurs a lower cost in Nash equilibrium when compared to* $\mathcal{C}-$*Nash equilibrium if and only if*

$$\sum_{i\in\mathcal{S}} f_i^\dagger(\mu, b, d) - f_i^*(\mu, b, d) \geq 0, \tag{6}$$

where for every $i \in [N]$,

$$f_i^\dagger(\mu, b, d) := \Delta^\dagger\Delta_i^\dagger + \frac{\mu_i(\Delta_i^\dagger - d_i)^2}{2b}, \qquad f_i^*(\mu, b, d) := \Delta^*\Delta_i^* + \frac{\mu_i(\Delta_i^* - d_i)^2}{2b},$$

$$\Delta^\dagger := \sum_{i\in[N]} \Delta_i^\dagger \qquad \Delta_i^\dagger := \Big((b(1_N 1_N^\top + \boldsymbol{C}) + \mu)^{-1}\mu d\Big)_i,$$

$$\Delta^* := \sum_{i\in[N]} \Delta_i^* \qquad \Delta_i^* := \Big((b(1_N 1_N^\top + I_N) + \mu)^{-1}\mu d\Big)_i.$$

Proof. For a subset $\mathcal{S} \subseteq [N]$ of charging stations, the total cost under Nash equilibrium is lower than $\mathcal{C}-$Nash equilibrium if and only if

$$\sum_{i\in\mathcal{S}} c_i(x^*) \leq \sum_{i\in\mathcal{S}} c_i(x^\dagger). \tag{7}$$

In the rest of the proof, we calculate these costs in terms of the game parameters. For every $t \in [T]$, define $F^t := T\alpha^t - \sum_{t'\in[T]} \alpha^{t'}$. Since $\sum_{t\in[T]} \alpha^t = 1$ (cf. Assumption 4.1), it holds that $F^t = T\alpha^t - 1$ and $\sum_{t\in[T]} F^t = 0$. Using Assumption 4.1 with Proposition 4.6, we obtain

$$x_i^{\dagger t} = \frac{d_i}{T} + \frac{F^t}{T}\Delta_i^\dagger \tag{8}$$

$$\mathbf{1}_N^\top x^{\dagger t} = \frac{D}{T} + \frac{F^t}{T}\Delta^\dagger \tag{9}$$

$$x_i^{\dagger t} - \bar{x}_i^t = \frac{(1 - T\alpha^t)d_i}{T} + \frac{F^t}{T}\Delta_i^\dagger = \frac{F^t}{T}(\Delta_i^\dagger - d_i), \tag{10}$$

where $D := \sum_{i\in[N]} d_i$. Using (1), (8)-(10) and $\sum_{t\in[T]} F^t = 0$, the cost of station i at $\mathcal{C}-$Nash equilibrium is

$$c_i(x^\dagger) = ad_i + \left(b\sum_{t\in[T]} (1_N^\top x^{t\dagger})x_i^{t\dagger}\right) + \frac{\mu_i}{2}\|x_i^\dagger - \bar{x}_i\|^2$$

$$
\begin{aligned}
&= ad_i + \frac{b}{T^2}\left(TDd_i + \sum_{t\in[T]}(F^t)^2\Delta^\dagger\Delta_i^\dagger\right) \\
&\quad + \frac{\mu_i}{2T^2}\sum_{t\in[T]}(F^t)^2(\Delta_i^* - d_i)^2.
\end{aligned}
$$

Consequently, the total cost of charging stations in $\mathcal{S}$ is

$$
\begin{aligned}
\sum_{i\in\mathcal{S}} c_i(x^\dagger) &= \left(aD_{\mathcal{S}} + \frac{bDD_{\mathcal{S}}}{T}\right) \\
&\quad + \frac{\mathcal{F}}{T^2}\left(\sum_{i\in\mathcal{S}} b\Delta^\dagger\Delta_i^\dagger + \frac{\mu^i}{2}(\Delta_i^\dagger - d_i)^2\right),
\end{aligned}
$$

where $D_{\mathcal{S}} := \sum_{i\in\mathcal{S}} d_i$, $\mathcal{F} := \sum_{t\in[T]}(F^t)^2$. Analogously, we can also compute the total cost at Nash equilibrium. Using these results, we conclude (7) is equivalent to

$$
\begin{aligned}
& aD_{\mathcal{S}} + \frac{bDD_{\mathcal{S}}}{T} + \frac{\mathcal{F}}{T^2}\left(\sum_{i\in\mathcal{S}} b\Delta^\dagger\Delta_i^\dagger + \frac{\mu^i}{2}(\Delta_i^\dagger - d_i)^2\right) \\
& \geq aD_{\mathcal{S}} + \frac{bDD_{\mathcal{S}}}{T} + \frac{\mathcal{F}}{T^2}\left(\sum_{i\in\mathcal{S}} b\Delta^*\Delta_i^* + \frac{\mu^i}{2}(\Delta_i^* - d_i)^2\right) \\
& \iff \sum_{i\in\mathcal{S}} f_i^\dagger(\mu, b, d) - f_i^*(\mu, b, d) \geq 0.
\end{aligned}
$$

□

Remark 4.8. *Note that the condition* (6) *in Theorem 4.7 is independent of* T, α^t, a *and depends only on* μ *and* d.

Corollary 4.9. *When* $c_i(x^\dagger), c_i(x^*) \geq 0\ \forall i \in [N]$, *Theorem 4.7 can be used to analyze (Eval-Metric) as follows. For any* $\mathcal{S} \in \{[N], \mathcal{C}, [\mathcal{N}]\backslash\mathcal{C}\}$, $M_{\mathcal{S}} \leq 1$ *if and only if* $\sum_{i\in\mathcal{S}} f_i^\dagger(\mu, b, d) - f_i^*(\mu, b, d) \geq 0$.

4.2.2 Case B: Desired demand $\bar{x}^t$ is uniform across time

In this subsection, we analyze the setting when $\bar{x}^t$ is uniform in t. This denotes a uniform spread of desired demand by the stations. This resembles the nominal charging profile when the EV adoption will reach a critical mass when each charging station observes a constant flow of EV such that, when averaged over any time window, the nominal charge is uniformly spread.

Proposition 4.10. *Suppose $\bar{x}^t = d/T$ for every $t \in [T]$. Then*

$$x^{*t} = \frac{d}{T} - \sum_{t' \in [T]} \frac{T\delta^{tt'} - 1}{T} \left(b(\mathbf{1}_N \mathbf{1}_N^\top + I_N) + \mu \right)^{-1} \mathbf{1}_N a^{t'},$$

$$x^{\dagger t} = \frac{d}{T} - \sum_{t' \in [T]} \frac{T\delta^{tt'} - 1}{T} \left(b(\mathbf{1}_N \mathbf{1}_N^\top + C) + \mu \right)^{-1} \mathbf{1}_N a^{t'}.$$

Theorem 4.11. *Suppose $\bar{x}^t = d/T$, for every $t \in [T]$. Any group $\mathcal{S} \subseteq [N]$ incurs a lower cost in Nash equilibrium when compared to $\mathcal{C}-$Nash equilibrium if and only if $g^\dagger_{\mathcal{S}}(\mu, b) - g^*_{\mathcal{S}}(\mu, b) \geq (\Gamma^\dagger - \Gamma^*)h(A)$, where for every $\mathcal{S} \subseteq [N]$,*

$$g^\dagger_{\mathcal{S}}(\mu, b) := \frac{b}{T} \sum_{i \in \mathcal{S}} \Gamma^\dagger \Gamma^\dagger_i + \frac{\mu^i}{2T} (\Gamma^\dagger_i)^2, \qquad g^*_{\mathcal{S}}(\mu, b) := \frac{b}{T} \sum_{i \in \mathcal{S}} \Gamma^* \Gamma^*_i + \frac{\mu^i}{2T} (\Gamma^*_i)^2$$

$$\Gamma^* := \sum_{i \in [N]} \Gamma^*_i, \qquad \Gamma^*_i := \left((b(1_N 1_N^\top + I_N) + \mu)^{-1} \mathbf{1}_N \right)_i,$$

$$\Gamma^\dagger := \sum_{i \in [N]} \Gamma^\dagger_i, \qquad \Gamma^\dagger_i := \left(\left(b(\mathbf{1}_N \mathbf{1}_N^\top + C) + \mu \right)^{-1} \mathbf{1}_N \right)_i,$$

$$h(A) := \frac{\sum_{t,t' \in [T]} a^t a^{t'} \left(T\delta^{tt'} - 1 \right)}{\sum_{t \in [T]} \left(\sum_{t' \in [T]} a^{t'} \left(T\delta^{tt'} - 1 \right) \right)^2}.$$

Proof. The proof is analogous to that of Theorem 4.7 and is deferred to Appendix. □

Remark 4.12. *Note that the condition in Theorem 4.11 is independent of the non-uniformity in the demand of charging stations d.*

Corollary 4.13. *When $c_i(x^\dagger), c_i(x^*) \geq 0 \ \forall i \in [N]$, Theorem 4.11 can be used to analyze (Eval-Metric) as follows.*

$$\mathsf{M}_{\mathcal{S}} \leq 1 \iff g^\dagger_{\mathcal{S}}(\mu, b) - g^*_{\mathcal{S}}(\mu, b) \geq (\Gamma^\dagger - \Gamma^*)h(A)$$
$$\forall \mathcal{S} \in \{[N], \mathcal{C}, [\mathcal{N}] \backslash \mathcal{C}\}.$$

4.3 $\mathcal{C}$-Nash equilibrium may not always be beneficial

In this subsection, using the setup from Section 4.2, we construct instances[4] that may not be conducive to the formation of a coalition. These cases demonstrate that coalitions between a

[4] The code to generate all figures in this section is available at `https://github.com/kkulk/coalition-ev`

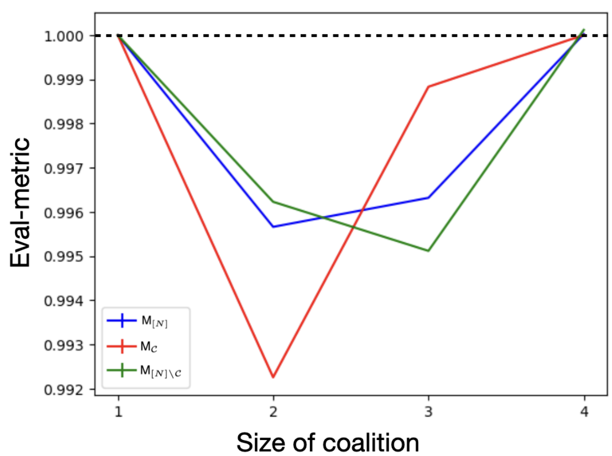


Figure 2: Setting $\eta_1 = 0.4/T, \eta_2 = 1.6/T$ and $\delta = 0$

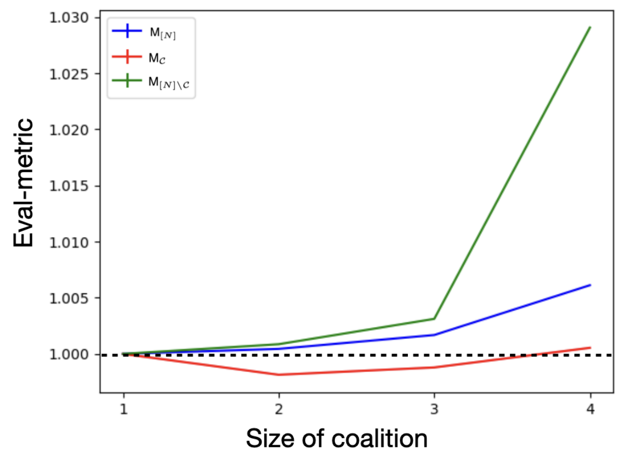


Figure 3: Setting $\eta_1 = 1/T, \eta_2 = 1/T$ and $\delta = 0.4$

subset of charging stations may not always be beneficial, and that the composition of the coalition needs to be considered carefully.

We set $N = 5$, $T = 10$, $b = 0.5$, and consider that each station can be of two types: $\mathsf{Type\ H}$ or $\mathsf{Type\ L}$. For the purpose of this example, we consider that all stations in a coalition will be of $\mathsf{Type\ L}$ and all stations outside the coalition are of $\mathsf{Type\ H}$. A station $i \in [N]$ is said to be $\mathsf{Type\ H}$ if $d_i = 5$ and $\mu_i = 1$, and of $\mathsf{Type\ L}$ if $d_i = 1$ and $\mu_i = 0.1$. Additionally, we set $\alpha^t = \eta_1$ for $t \leq T/2$, and η_2 otherwise. Furthermore, we set $a_t = 0.5 + \delta\nu_t$ where $\nu_t \sim \mathsf{Unif}([0,1])$. We study the impact of the size of the coalition on various metrics presented in (Eval-Metric).

In Figure 2, we study the setting of $\eta_1 = 0.4/T, \eta_2 = 1.6/T$ and $\delta = 0$, which is aligned with the setup considered in Theorem 4.7. Meanwhile, in Figure 3, we study the setting of $\eta_1 = 1/T, \eta_2 = 1/T$ and $\delta = 0.4$, which is aligned with the setup considered in Theorem 4.11. From these figures we conclude that there exist coalitions, where from the coalition's perspective (i.e. $\mathsf{M}_{\mathcal{C}}$) the outcome under Nash equilibrium is more desired. Furthermore, in Figure 2, we find that there exist instances when the $\mathcal{C}-$Nash equilibrium may not be preferred under every evaluation metric. Moreover, in Figure 3, we find that the formation of the coalition not only adversely impacts the coalition but also provides an advantage to stations outside coalitions, which might not be preferred by EVCCs that operates the coalition.

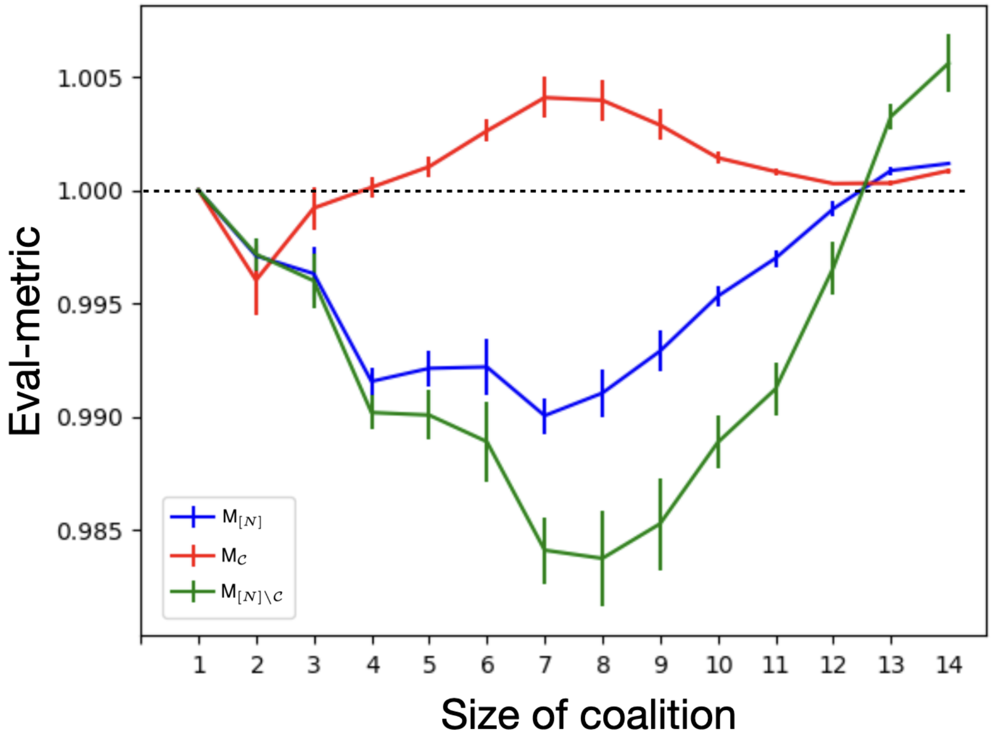


Figure 4: Comparison between Nash equilibrium and $\mathcal{C}-$Nash equilibrium with respect to (Eval-Metric) under different size of coalition.

5 Discussion

In this section we expand on the experimental insights obtained in Section 4.3 by discussing the effects of the following two kinds of variations in the parameters[5]:

Impact of non-uniform variations in both a^t and α^t. In Figure 4, we study the impact of the size of the coalition in terms of (Eval-Metric). In contrast to Section 4.3, we consider both a^t and α^t to be non-uniform and randomly assign a station to be Type H with probability 0.2 if they are within the coalition. All stations outside of coalition are assigned to be Type L. We find that if the size of coalition is small then Nash equilibrium is favorable along all metrics. However, as the size of coalition increases, it may be beneficial to form a coalition. This example demonstrates that whether a coalition is beneficial or not may depend on the *size* of the coalition being formed. Essentially, in this example, the coalition is able to profit off of 'economies of scale' that were not present when the coalition size is small (which is controlled by the probability of a Type H station being in the coalition).

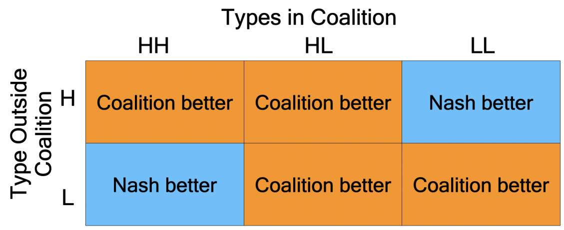
[5]The code to generate all figures in this section is available at `https://github.com/kkulk/coalition-ev`

Impact of the composition of the coalition. Here, we show that whether or not a coalition is beneficial not only depends on the size, but also the composition of the coalition. To illustrate this point, we examine an example with $N = 3$, $T = 10$, $b = 1$ and $a^t = 10$ for all $t \in [T]$. We posit a scenario where the first two stations form a coalition. Each station is one of two types, namely $\mathsf{Type\ H}$ or $\mathsf{Type\ L}$. We call a station $i \in [N]$ to be of $\mathsf{Type\ H}$ if $d_i = 5$ and $\mu_i = 5$, and of $\mathsf{Type\ L}$ if $d_i = 1$ and $\mu_i = 1$. In Figure 5, we present a comparative analysis, evaluating how different coalitions perform in terms of metric presented in (Eval-Metric). We find under some circumstances $\mathsf{M}_{\mathcal{S}} > 1$ for all $\mathcal{S} \in \{[N], \mathcal{C}, [N]\backslash[\mathcal{C}]\}$ indicating that such coalitions would not form. For instance, this is the case when the coalition is comprised of atleast one $\mathsf{Type\ H}$ station and the station outside coalition is of $\mathsf{Type\ H}$. On the contrary, perhaps surprisingly, there are also instances when $\mathsf{M}_{\mathcal{S}} < 1$ for all $\mathcal{S} \in \{[N], \mathcal{C}, [N]\backslash[\mathcal{C}]\}$. For instance, this is the case when the stations within a coalition are of $\mathsf{Type\ H}$ and the station outside the coalition is of $\mathsf{Type\ L}$. This indicates that the heterogeneity between the types of stations that constitute the coalition plays a significant role in whether or not the coalition is beneficial, and opens avenues for further research.

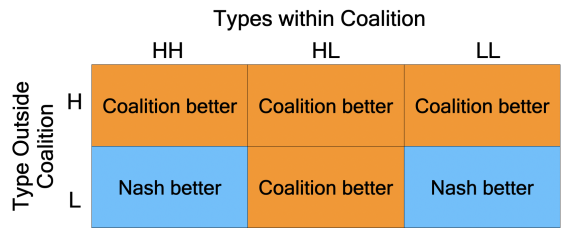
6 Conclusion

In this work, we initiate a study to understand the impact of coalitions that could emerge between EV charging stations as charging infrastructure continues to grow in coming years. As charging stations draw substantial amount of electricity from the grid, they can become "price-makers" and influence the grid operation and electricity prices. More often than not, multiple charging stations are operated by same electric vehicle charging company (EVCC), which could facilitate coordination between charging stations. In this work, we analytically characterize the equilibrium outcome in the presence of coalitions by modeling the interaction as a non-cooperative game. Our analysis hints at potential losses encountered by EVCCs if they coordinate all charging stations owned by them; sometimes it may be beneficial to instead allow all charging stations to decide their demand profile autonomously.

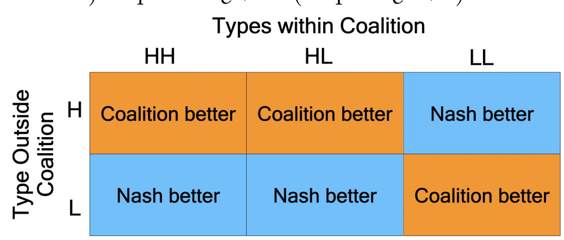
There are several interesting questions for future research in understanding the impact of coalition between charging stations. In the current model we assume that the overall demand of a charging station is fixed. More realistically, this demand could be decided via feedback from electricity prices (Escudero-Garzas and Seco-Granados, 2012; Lee et al., 2014; Yuan et al., 2015). Furthermore, there are additional operational constraints such as the limited capacity of the energy



(a) The phrase "Coalition better" (resp. "Nash better") implies $\mathsf{M}_{[N]} > 1$ (resp. $\mathsf{M}_{[N]} < 1$).



(b) The phrase "Coalition better" (resp. "Nash better") implies $\mathsf{M}_{\mathcal{C}} > 1$ (resp. $\mathsf{M}_{\mathcal{C}} < 1$).



(c) The phrase "Coalition better" (resp. "Nash better") implies $\mathsf{M}_{[N]\setminus\mathcal{C}} > 1$ (resp. $\mathsf{M}_{[N]\setminus\mathcal{C}} < 1$).

Figure 5: Comparison of societal (overall) cost, coalitional cost, and the cost of the station outside the coalition in the three station game.

infrastructure and bounds on charging rates of EVs that could be accounted for in the equilibrium computation. Jointly, answers to these questions may better illuminate the effects of coordination between charging stations.

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A Excluded proofs from main text

A.1 Proof of Lemma 4.2

Note that $\Theta = I_T \otimes \Big(b(\mathbf{1}_N \mathbf{1}_N^\top + \mathbf{C}) + \mu\Big)$ is positive definite if and only if each of the terms in the Kronecker product is positive definite (Schacke, 2004). Hence, it is sufficient to show that $Q := b(\mathbf{1}_N \mathbf{1}_N^\top + \mathbf{C}) + \mu$ is a positive definite matrix. First, note that Q is a symmetric matrix. Additionally, for any $z \in \mathbb{R}^N$, it holds that

$$z^\top Q z = b(\mathbf{1}_N^\top z)^2 + b(\sum_{i\in\mathcal{C}} z_i)^2 + \sum_{i\in\mathcal{C}} \mu_i z_i^2 + \sum_{i\in N\setminus\mathcal{C}} (b+\mu_i) z_i^2,$$

which is strictly positive for all $z \neq 0$.

Next, we show that Γ is invertible by calculating its inverse. Indeed,

$$\begin{aligned}\Gamma^{-1} &= \Big((\mathbf{1}_T^\top \otimes I_N)\Theta^{-1}(\mathbf{1}_T \otimes I_N)\Big)^{-1} \\ &= \left(\mathbf{1}_T^\top \mathbf{1}_T \otimes \Big(b(\mathbf{1}_N \mathbf{1}_N^\top + C) + \mu\Big)^{-1}\right)^{-1} \\ &= \frac{1}{T}\Big(b(\mathbf{1}_N \mathbf{1}_N^\top + C) + \mu\Big) = \frac{1}{T} Q.\end{aligned}$$

This completes the proof.

A.2 Proof of Theorem 4.11

For every $t \in [T]$, define $G^t := \sum_{t'\in[T]} a^{t'} \Big(T\delta^{tt'} - 1\Big)$. Using this, the expression for $x^{\dagger t}$ from Proposition 4.10 can be re-written as follows

$$x^{\dagger t} = \frac{d}{T} - \frac{1}{T}\Big(b(\mathbf{1}_N \mathbf{1}_N^\top + C) + \mu\Big)^{-1} \mathbf{1}_N G^t. \tag{11}$$

Consequently, it holds that

$$\begin{aligned} x_i^{\dagger t} &= \frac{d_i}{T} - \frac{G^t}{T}\Gamma_i^\dagger \\ \mathbf{1}_N^\top x^{\dagger t} &= \frac{D}{T} - \frac{G^t}{T}\Gamma^\dagger \\ x_i^{\dagger t} - \bar{x}_i^t &= -\frac{1}{T}\Gamma_i^\dagger G^t. \end{aligned} \tag{12}$$

Consequently, combining previous equations with (1), for every $i \in [N]$,

$$
\begin{aligned}
c_i(x^\dagger) &= \sum_{t\in[T]} a^t x_i^{\dagger t} + \frac{b}{T^2} \sum_{t\in[T]} \left(D - G^t \Gamma^\dagger\right) \left(d_i - G^t \Gamma_i^\dagger\right) \\
&\quad + \frac{\mu_i (\Gamma_i^\dagger)^2}{2T^2} \sum_{t\in[T]} (G^t)^2 \\
&= \sum_{t\in[T]} a^t \left(\frac{d_i}{T} - \frac{G^t}{T}\Gamma_i^\dagger\right) + \frac{b}{T^2}\left(TDd_i + \Gamma^\dagger \Gamma_i^\dagger \sum_{t\in[T]} (G^t)^2\right) \\
&\quad + \frac{\mu_i (\Gamma_i^\dagger)^2}{2T^2} \sum_{t\in[T]} (G^t)^2, \qquad (13)
\end{aligned}
$$

where the last equality is because $\sum_{t\in[T]} G^t = \sum_{t,t'\in[T]} a^{t'} \left(T\delta^{tt'} - 1\right) = 0$. Additionally, summing (13) for all $i \in \mathcal{S}$, we obtain

$$
\begin{aligned}
\sum_{i\in\mathcal{S}} c_i(x^\dagger) &= \sum_{t\in[T]} a^t \left(\frac{D_\mathcal{S}}{T} - \frac{G^t}{T}\sum_{i\in\mathcal{S}} \Gamma_i^\dagger\right) + \frac{bDD_\mathcal{S}}{T} \\
&\quad + \frac{b\Gamma^\dagger \sum_{i\in\mathcal{S}} \Gamma_i^\dagger \sum_{t\in[T]} (G^t)^2}{T^2} + \frac{b}{2T^2}\sum_{t\in[T]} (G^t)^2 \sum_{i\in\mathcal{S}} \frac{\mu^i}{b} (\Gamma_i^\dagger)^2 \\
&= \left(\frac{AD_\mathcal{S} - \sum_{i\in\mathcal{S}} \Gamma_i^\dagger \sum_{t\in[T]} a^t G^t}{T} + \frac{bDD_\mathcal{S}}{T}\right) \\
&\qquad + \frac{b\sum_{t\in[T]}(G^t)^2}{T^2}\left(\Gamma^\dagger \sum_{i\in\mathcal{S}} \Gamma_i^\dagger + \sum_{i\in\mathcal{S}} \frac{\mu^i}{2b} (\Gamma_i^\dagger)^2\right),
\end{aligned}
$$

where $A := \sum_{t\in[T]} a^t$. Analogously, we obtain the cost at Nash equilibrium as follows

$$
\begin{aligned}
\sum_{i\in\mathcal{S}} c_i(x^*) &= \left(\frac{AD_\mathcal{S} - \sum_{i\in\mathcal{S}} \Gamma_i^* \sum_{t\in[T]} a^t G^t}{T} + \frac{bDD_\mathcal{S}}{T}\right) \\
&\qquad + \frac{b\sum_{t\in[T]}(G^t)^2}{T^2}\left(\Gamma^* \sum_{i\in\mathcal{S}} \Gamma_i^* + \sum_{i\in\mathcal{S}} \frac{\mu^i}{2b} (\Gamma_i^*)^2\right).
\end{aligned}
$$

Using the above equations, we obtain that $\sum_{i\in\mathcal{S}} c_i(x^\dagger) > \sum_{i\in\mathcal{S}} c_i(x^*)$ if and only if $g_\mathcal{S}^\dagger(\mu, b) - g_\mathcal{S}^*(\mu, b) \geq (\Gamma^\dagger - \Gamma^*) h(A)$. This completes the proof.