Correlated Failures of Power Systems: Analysis of the Nordic Grid

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***Abstract*—In this work we have analyzed the effects of correlated failures of power lines on the total system load shed. The total system load shed is determined by solving the optimal load shedding problem, which is the system operator's best response to a system failure. We have introduced a Monte Carlo based simulation framework for estimating the statistics of the system load shed as a function of stochastic network parameters, and provide explicit guarantees on the sampling accuracy. This framework has been applied to a 470 bus model of the Nordic power system and a correlated Bernoulli failure model. It has been found that increased correlations between Bernoulli failures of power lines can dramatically increase the expected value as well as the variance of the system load shed.**

I. Introduction

Power systems are among the largest and most complex systems created by mankind. The complexity of power grids keeps increasing as power grids are expanded and new functionalities are added, as with the current developments of the SmartGrid [1]. Since many vital parts of today's society require reliable supply of electricity, the reliable and secure operation of power systems is indisputably essential [2], [3]. We give a brief overview of the research in the two distinctive areas of security against adversarial attacks and reliability of power systems against random failures. We conclude that correlated failures in power systems represent a gap between these two research areas.

In the area of security, research on characterizing optimal attack and defense strategies has gained momentum over the past years. In [4] the optimization problem of maximizing the power outage, for a given number of power transmission lines that an adversary is capable of disconnecting, is considered. The system operator is assumed to take the best action to minimize the damage in form of compulsory load shedding. The problem is game theoretic by nature and gives rise to a maximin optimization problem, where the outer maximization seeks the most disruptive attack for a given budget of the adversary, and the inner minimization solves the optimal load shedding problem which minimizes the consequences of an attack. Because of the non-convexity and the existence of integer variables, the problem is inherently hard to solve for large systems. [5] approximates the nonlinear mixed integer bi-level program by a mixed integer linear program, and derives an upper bound on the severity of adversarial attacks.

Traditionally, the reliability of power systems has often been characterized by deterministic means, such as the widely uses $N-k$ criterion [6], and in almost all cases $k=1$ is used [7]. A power system satisfying the $N-k$ criterion is able to withstand any contingency consisting of k outages. The main drawback of the $N-k$ criterion and other deterministic reliability criteria is that they do not take into account the probabilities of the contingencies. Furthermore, the number of events which have to be considered when evaluating the $N-k$ criterion grows exponentially in k, making the reliability evaluation computationally intractable. More recent research on the reliability of power systems has emphasized that many events governing the reliability of power systems are by nature stochastic, e.g. demands and generation capacities. Various statistical and sampling based methods for evaluating the reliability of power systems have been developed to analyze stochastic phenomena in power systems [8], [9]. In [10] a two state Markov model for the failure of various power system elements is considered, and the statistics of the power system are calculated using Monte Carlo techniques. However, the model does not take correlated failures into account. Other than failure correlations due to cascading failures [11], we found no model which attempts to model correlated failures in power systems.

This work aims at studying the effects of correlations between failures of power system components, and in particular power lines. In contrast to previous papers on the subject, we introduce a failure model explicitly taking into account correlated system failures. We evaluate the impact of correlations of failures by the covariances between the failures. Our research is motivated by the increased deployment of of-the-shelf hardware and software in SCADA systems governing the power systems. When similar or even identical

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software is deployed in several system components, software failures are likely to be correlated between those components. Indeed, current literature suggests that bugs present in one system component are likely to also be present in a similar or identical components [12]. From a cyber security point of view, this work is motivated by the possibility of malicious code exploiting identical software bugs and security flaws. A computer virus spreading in the SCADA network is likely to affect multiple of its target components, and thus causing correlations between failures. The increased interconnection of control and communication systems as well as their connection to the Internet, facilitates the exploitation of software bugs. There are indications that software bugs caused failures leading to a blackout in the northeast blackout of 2003 [13]. We believe that correlated failures provides a good mean of understanding the affects of failures caused by malicious software affecting multiple system components. Correlated failures may also occur due to natural disasters such as earthquakes or hurricanes [14], [15]. Examples of major power system outages caused by natural disasters include the New York city blackout in 1977, where lightning struck a substation and a power line almost simultaneously [16].

We measure the impact of a system failure by the minimal system load shed required to restore the system to a safe state. This formulation gives rise to an optimization problem, which under simplified conditions can be made linear. For different values of the correlations of the failure distribution, we compute the sampled statistics of the total system load shed by Monte Carlo techniques, and provide guarantees on the convergence rate of the sampled statistics. In particular, we use a weighted sum of the mean and variance of the total system load shed as a risk measure of the failure statistics. To obtain statistical data from a realistic power system, we apply our techniques to a 470 bus model of the Nordic power system, acquired from publicly available sources. We have found that increasing correlations between Bernoulli failures of power lines lead to increased expected value and variance of the system load shed under various topological structures of the correlations.

The rest of the paper is organized as follows. In Section II the optimal load shedding problem is presented and the total system load shed is defined. In section III a novel model of the Nordic power system is presented and evaluated . In Section IV a Monte Carlo sampling technique is presented and the effects of correlated failures on the Nordic power system are studied, followed by concluding remarks in Section V.

II. Optimal load shedding

Optimal power flow (OPF) problems are in many ways analogous to transportation problems, with the only difference that the power flows obey the Kirchoff voltage law, and are proportional to the relative phase angle difference between the buses [17]. The optimal load shedding problem is a special case of OPF problems, where the load shed is minimized subject to physical constraints [18], [19]. OPF problems have been solved using a variety of optimization techniques, ranging from linear programming [20], Newton methods [21] to interior point methods [22]. While many techniques rely on the linearized power flow equations, there are OPF problems using the nonlinear power flow equations [23], [24]. While showing promising results, these methods suffer from general limitations of non-convex optimization such as convergence and computational complexity.

We will only consider the linearized power flow equations in our work, thereby ensuring robust and fast convergence. Since we will apply Monte Carlo techniques solving the optimal load shedding problem, computation speed is certainly of importance. By assuming that the admittance in the shunt branch of the power lines is negligible, and that the resistance-to-reactance ratio is sufficiently small, reactive power flows can be neglected, and the real power real power flows are described by the DC-model [25]:

$$P^{line} = V^{line} B \sin\left(A\theta\right) \tag{1}$$

where P^{line} is a column vector of active power flows in the transmission lines, $V^{line} = \operatorname{diag}\left(V_i V_j\right)$ where V_i is the voltage of bus i, $B = \operatorname{diag}(b_{ij})$ where b_{ij} is the admittance of the power line connecting bus i with bus j, θ is the vector of bus phase angles and A is the vertex-edge incidence matrix of the graph of the power system, defined as $A_{ij} = 1$ iff $e_i = (v_j, u) \in E$, $A_{ij} = -1$ iff $e_i = (u, v_j) \in E$ and $A_{ij} = 0$ otherwise. Here $\sin(x) = \left[\sin(x_i), \ldots, \sin(x_n)\right]^T$ for a vector x. By only considering sufficiently small phase angle differences, i.e. $\Delta\theta_{max} = \|A\theta\|_\infty$ being sufficiently small, we may linearize (1) around $A\theta = \mathbf{0}$;

$$P^{line} = V^{line} B A \theta \tag{2}$$

By summing the power flows to each bus, we get the linearized equation for the net power flows into the buses;

$$P = A^T V^{line} B A \theta =: L_B \theta$$

where P is a vector of real power injections to the buses. L_B can be interpreted as a weighted Laplacian matrix of the graph associated with the power system, with weights corresponding to the line admittances times the bus voltages. We may assume, wlog, that the buses of the power system are partitioned as $P = \left[{P^g}^T, {P^l}^T\right]^T$, where $P^g > 0$ are generator buses and $P^l \leq 0$ are load buses. We consider the optimization problem of minimizing the total load shed of the system. The optimal load shedding problem can, for the linearized power flow equations, be formulated as a linear Program

(LP);

$$\min_{\theta} \quad c^T\theta \tag{3}$$
$$\text{s.t.} \quad C\theta \preceq d \tag{4}$$

where

$$C = \begin{bmatrix} V^{line}BA \\ -V^{line}BA \\ L_B \\ -L_B \\ A \\ -A \end{bmatrix} \quad d = \begin{bmatrix} P^{line}_{max} \\ P^{line}_{max} \\ P^{g}_{max} \\ 0^{n_l \times 1} \\ 0^{n_g \times 1} \\ -P^l_d \\ \Delta\theta_{max} \cdot 1^{n_p \times 1} \\ \Delta\theta_{max} \cdot 1^{n_p \times 1} \end{bmatrix} \tag{5}$$

and

$$c = \begin{bmatrix} 0^{1 \times n_g} & 1^{1 \times n_l} \end{bmatrix} L_B \tag{6}$$

and where n_g, n_l and n_p denotes the number of generator buses, load buses and power lines respectively, and $0 < \Delta\theta_{max} \leq \frac{\pi}{2}$ is a sufficiently small real number for which the linearized power flow equation (2) is a valid approximation. The matrix inequality in (4) combines line capacity constraints $\left(|P^{line}| \preceq P^{line}_{max}\right)$, power generation constraints $(\mathbf{0} \preceq P^g \preceq P^g_{max})$, power load constraints $\left(P^l_d \preceq P^l \preceq 0\right)$ and phase angle constraints $\left(\|A\theta\|_\infty \leq \theta_{max}\right)$. The objective function $c^T\theta$ is the sum of the power injections to the demand buses, which is a linearly affine function of the total system load shed. It can be easily seen that the minimum total load shed $S^*(C,d)$, which is the difference between the total power demand and total delivered power, is given by

$$S^*(C,d) \quad = \min_{\theta} \left\{ c^T\theta | C\theta \preceq d \right\} - 1^{1 \times n_l} \cdot P^l_d \tag{7}$$

III. Model of the Nordic power system

In this section, we construct a model of the Nordic power system to demonstrate the previously discussed sampling techniques. While IEEE standard power systems offer great transparency for research and can function as benchmark systems, we believe that it is important to demonstrate our concepts on real power systems. We have built a model of the Nordic power system, using only publicly available data. While there have been similar efforts to model other interconnected power systems, such as the main European power grid [26], there are no known complete models of the Nordic power system that are publicly available.

A. Obtaining the network topology

The topology of the power system. i.e. the geographical positions of the main 400, 300, 220 and 132 kV power lines and HVDC links in the Nordic countries were obtained from the respective TSOs websites. The data obtained included the coordinates of the power buses, the connectivity of the buses through power lines, the voltage of the power lines and the number of parallel power lines, if applicable. The complete model has a total of 470 buses and 717 power lines. The power lines and buses of the Nordic power grid are illustrated in figure 1.

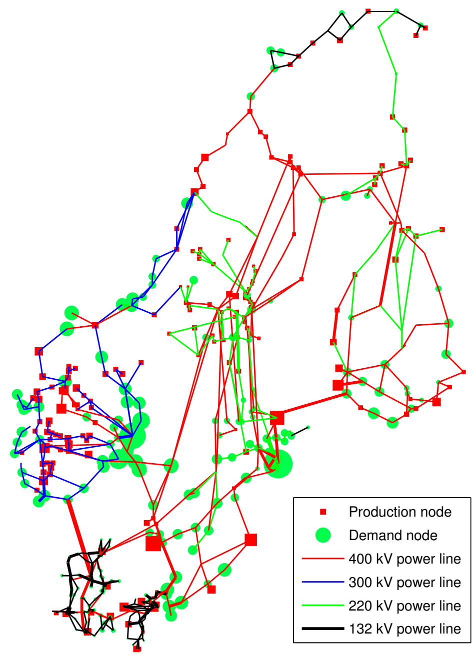

Figure 1. Model of the Nordel high voltage power transmission network. The circle area of the production and demand buses are proportional to the production and demand respectively.

B. Estimating power generation capacities

Information about all power generation facilities with capacities of at least 100 MW were also acquired from public sources [27]. To compensate for the lack of available data of power plants with generation capacity less than 100 MW, we have made assumptions about the remaining power plants. The remaining thermal power plants are assumed to be located in populated areas, and hence the thermal generation capacity is proportional to the demands. As for the remaining wind power capacity, we have assumed that the wind power generation is uniformly distributed over the land surface, and hence over the buses. These assumptions may appear crude, but considering that these assumptions only apply to power plants whose generation capacity is below 100 MW, the net effect of possible errors on the whole model is of minor importance.

C. Estimating power demand data

There is no available electricity demand data, other than cumulative data for the Nordic countries. This data is to rough to be useful for our 470 bus model. Following [26] we have used population census data to estimate the power demand. This methodology relies on the assumptions that household power demand is proportional to the population connected to a substation, as well as industry power demand, since the workforce will settle relatively close to industries. This may however not necessarily be the case for certain energy-intensive industries which are usually co-located with energy sources, nor for certain location-specific industries such as forestry or the oil industry. Population statistics were collected from the Bureau of Statistics of the respective countries. We have collected cumulated population statistics for the major administrative regions of each country, and assumed that the population (and hence the demand) is distributed uniformly over the load buses within each region. The number of administrative regions in each country was between 12 and 21. Using smaller regions would introduce difficulties in assigning the right population to each substation. To estimate the power demands, both the yearly average and yearly maximum of the daily maximum power consumption were used to create two different load situations.

D. Estimating power line parameters

The only known parameters of the power lines obtained from public sources are the line voltages. To solve the optimal load shedding problem, also the line admittances as well as the maximum transmission capacities of each line need to be known. The admittance of power lines can be estimated by the length of the power line. Typically the reactance of high voltage power transmission lines is approximately $0.20\ \Omega/\text{km}$ [28]. The lengths of a power line from a bus with coordinates x to a bus with coordinates y is estimated by the euclidean 2-norm as $l = \operatorname{dist}(x, y) = \|x - y\|_2 = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$ which is always an underestimate of the actual line length. As for estimating transmission capacities, only cross-border transmission line capacity constraints are available from the Nordic TSOs. The transmission capacity of each power line of equal voltage is assumed to be the average transmission capacity of the cross-border lines, which are shown in Table I.

Table I
ESTIMATED TRANSMISSION LINE CAPACITIES.

Voltage	Capacity
400 kV	1030 MW
300 kV	650 MW
220 kV	415 MW
132 kV	143 MW

E. Evaluating the model

The optimal load shedding problem was applied to the previously derived model of the Nordic power transmission grid. By using the YALMIP [29] interface with the GLPK [30] LP solver in MATLAB [31], the optimal load shedding problem was solved. When solving the linear optimal load shedding problem with the yearly maximum loads, the total system load shed was found to be 2 % of the total power demand. When solving the optimal load shedding problem with the yearly average of the daily maximum loads, no system load shed was necessary. This demonstrates that our model is indeed usable for our study.

IV. STATISTICS OF POWER SYSTEM FAILURES

A. Monte Carlo methods

In this paper we will consider stochastic failures of the power system, as in e.g. [8], [9]. To demonstrate the generality of our methods, we will not yet make any assumptions about these failures. Consider the matrices C and d, associated with an arbitrary power system, as random variables endowed with a probability measure $\mu^C \times \mu^d$. Since both the topology and load parameters of the power system are determined by C and d, such a probability measure can represent any type of failures of the power system. It can be shown that for any probability measure $\mu^C \times \mu^d$, the minimum total load shed $S^*(C, d)$ is also a random variable. The total load shed is a commonly used measure of the severeness of a power system outage [32], [33].

We will consider the sampled probability distribution of the minimum total system load shed $S^*(C, d)$, and in particular we will consider the mean and the variance of the sampled probability distribution. Because $0 \leq S^*(C, d) \leq -\sum P_d^l$, as seen from (4), the mean $\bar{S}^*$ and variance $\sigma_{S^*}^2$ of $S^*(C, d)$ always exist and are finite. We will use $\bar{S}^* + \alpha \cdot \sigma_{S^*}, \alpha \in \mathbb{R}^+$ as a risk measure for the distribution $\mu^C \times \mu^d$. We show that $\bar{S}^* + \alpha \cdot \sigma_{S^*}$ is closely related to the commonly used risk measure value at risk (VaR), which for a random variable S^* is defined as follows [34]:

$$\mathrm{VaR}_\alpha(S^*) = \inf\{l \in \mathbb{R} : \Pr(S^* > l) \leq 1 - \alpha\}$$

The intuition of the expression $\mathrm{VaR}_\alpha(S^*)$ is that the maximum loss, in our case system load shed, is bounded by $\mathrm{VaR}_\alpha(S^*)$ with probability $1-\alpha$. One serious computational drawback with using VaR on sampled probability distributions, is that it requires knowledge of the full probability distribution of the random variable S^*. When dealing with samples of random variables, estimating VaR becomes hard since it requires estimation of the tail of the distribution S^*. The following proposition allows us to obtain an upper bound $\mathrm{VaR}_\alpha(S^*)$ using a linear combination of the mean and the variance of S^*, which can be estimated more robustly:

Proposition 1. *The risk measure value at risk (*VaR_α*) satisfies*

$$\mathrm{VaR}_\alpha(S^*) \leq \bar{S}^* + \frac{1}{\sqrt{\alpha}} \cdot \sigma_{S^*}$$

The proof is given in the appendix. Since obtaining analytical expressions for $\bar{S}^*$ and $\sigma^2_{S^*}$ is in general not possible, we will use Monte Carlo techniques [35] to estimate the mean and the variance of the load shed. By drawing N samples from the distribution $\mu^C \times \mu^d$, we obtain the following approximations of $\bar{S}^*$ and $\sigma^2_{S^*}$;

$$\bar{S}^* \approx \hat{S}^* = \frac{1}{N}\sum_{i=1}^{N} S^*(C_i, d_i) \tag{8}$$

$$\sigma^2_{S^*} \approx \hat{\sigma}^2_{S^*} = \frac{1}{N-1}\sum_{i=1}^{N}\left(S^*(C_i, d_i) - \hat{S}^*\right)^2 \tag{9}$$

Due to $S^*(C, d)$ being bounded, $\hat{S}^*$ and $\hat{\sigma}_{S^*}$ are guaranteed to converge to S^* and σ_{S^*} respectively.

Proposition 2. *Given $\epsilon > 0$, $\delta > 0$, the number of samples N_1 and N_2 which assure that*

$$\Pr\left[\left|\hat{S}^{*,N_1} - \bar{S}^*\right| \geq \epsilon\right] \leq \delta$$
$$\Pr\left[\left|\hat{\sigma}_{S^*,N_2} - \sigma_{S^*}\right| \geq \epsilon\right] \leq \delta$$

are

$$N_1 \geq \left\lceil \frac{\hat{S}^2}{4\delta\epsilon^2} \right\rceil \qquad N_2 \geq \left\lceil \frac{\hat{S}^4}{8\delta\epsilon^4} \right\rceil$$

Proof: Follows by lemma 3 and lemma 5 in the appendix, and the fact that $0 \leq S^*(C, d) \leq -\sum P^l_d$. ∎

With proposition 2 we have guaranteed bounds of the estimation error of both the sampled expected value and the sampled variance of the load shed. The proposition can of course be used in the reverse direction. For given N_1 and N_2, we can obtain bounds on δ and ϵ. With these explicit bounds on the error of the estimated mean and variance of the load shed, the number of samples can be chosen according to given accuracy requirements, and trade-offs between accuracy and the number of samples can be made a priory.

B. Sampling of correlated system failures

In this section we examine the effects of correlated system faults on the statistics of the minimum total system load shed. As discussed in the introduction, the study of correlated faults in power systems is motivated by the increased deployment of of-the-shelf hardware and software in SCADA systems governing the power systems. When identical software is deployed in several system components, software faults between these identical components are likely to experience correlation. In the following empirical study we will consider failures on the form of power line disconnections. We model the disconnection of power line i as a binary random variable $X_i \in \{0, 1\}$ where $X_i = 0$ corresponds to line i being fully functional with all parameters set to default, and $X_i = 1$ corresponds to line i being disconnected. Thus, the failure statistics of the power system are given by

$$P(X_1 = Y_1, \ldots, X_{n_p} = Y_{n_p}) \;\forall\, Y_i \in \{0, 1\}$$

Since parameterizing the full joint Bernoulli distribution would require $n^{n_p} \approx 10^{216}$ variables, we will consider the Bernoulli distribution with the first two central moments given explicitly, i.e.

$$\bar{X}_i = \mathrm{E}[X_i] \quad \forall i \in \{1, \ldots, n_p\}$$
$$\sigma_{ij} = \mathrm{E}\left[(X_i - \bar{X}_i)(X_j - \bar{X}_j)\right] \;\forall\, (i, j) \in \{1, \ldots, n_p\}^2$$

To consider the effects of increasing correlations $\bar{X}_i = 0.02$ is kept constant, while the covariances σ_{ij} are increased. We consider two different scenarios where the correlation between a subset of the power lines is increased.

1) Correlations between incident power lines: We first consider the case where σ_{ij} is increased equally for all incident power lines, from 0 to 0.016 in steps of 0.004. For each step, 1000 Monte Carlo simulations are performed with the Bernoulli sampling algorithm described in [36] to acquire sampled statistics S^*. By proposition 2 the relative error of $\hat{S}^*$ is guaranteed to be less than 7 % with certainty 95 %. In figure 2, the histograms of the sampling processes are shown for different σ_{ij}, together with fitted Weibull distributions. The mean and the variance of the load shed for different correlations are shown in figure 4. Clearly both $\bar{S}^*$ and σ_{X^*} are strongly increasing in σ_{ij}.

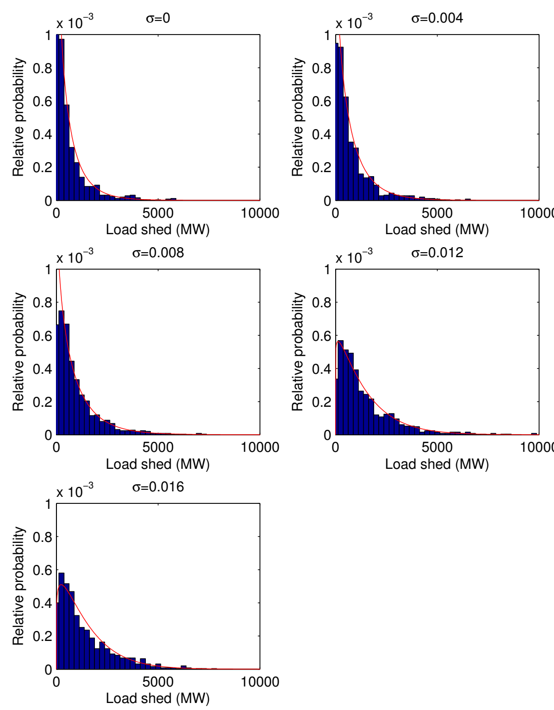

Figure 2. Histograms of the sampled load shed distributions for different σ_{ij} between 0 to 0.016 when the covariance between neighboring lines is increased, with fitted Weibull distributions.

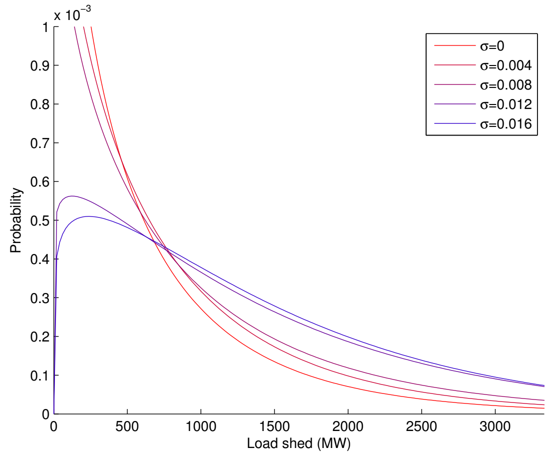

Figure 3. Fitted Weibull distributions of the sampled load shed distribution for σ_{ij} from 0 to 0.016, when the covariance between neighboring lines is increased.

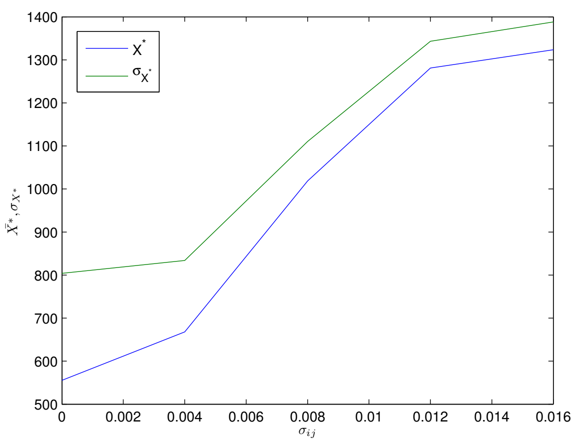

Figure 4. Expected value and standard deviation (MW) of the total load shed for different covariances for power lines connected to the same bus.

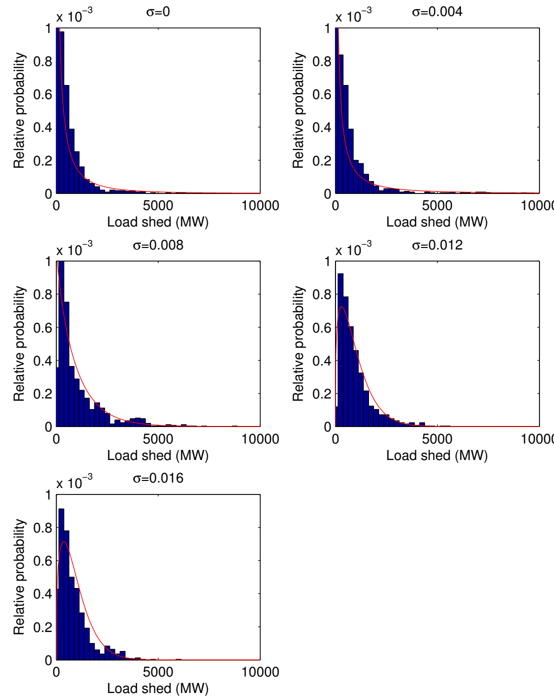

Figure 5. Histograms of the sampled load shed distributions for different σ_{ij} between 0 to 0.016 when the covariance between power lines incident to PMU nodes is increased, with fitted Weibull distributions.

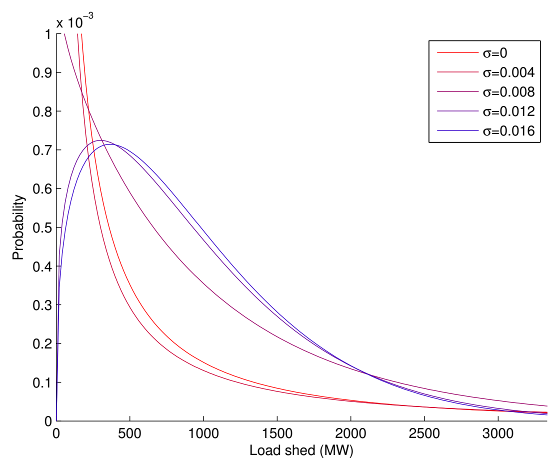

Figure 6. Fitted Weibull distributions of the sampled load shed distribution for σ_{ij} from 0 to 0.016, when the covariance between power lines incident to PMU nodes is increased.

2) Correlations between power lines incident to PMUs: We here consider the scenario of failures being correlated only between lines incident to nodes with phasor measurement units (PMUs). This scenario is motivated by the use of identical software and hardware in PMUs, which could cause correlations between failures of these nodes, and hence the incident lines. In figure 5, the histograms of the sampling processes are shown for different σ_{ij}, together with fitted Weibull distributions. The mean and the variance of the load shed for different covariances are shown in figure 7. While σ_{X^*} is not increasing in σ_{ij}, $\bar{S}^*$ is increasing by a factor 15 with increasing σ_{ij}.

3) Remarks: Although simulations indicate that correlations increase the expected value of the system load shed, it is easy to find counterexamples where increased correlation between power line failures decreases the expected load shed. The simplest possible counterexample is a 3-bus and 2-line power network shown in figure 8. Let the demand bus have demand -1, and the generation bus a capacity $\bar{g} \geq 1$, and the line parameters be such that the demand is satisfied under

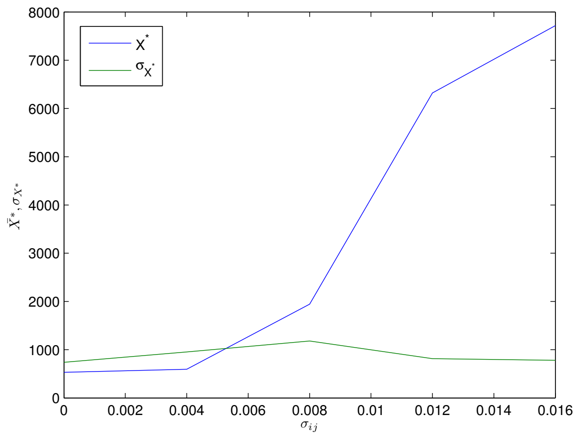

Figure 7. Expected value and standard deviation (MW) of the total load shed for σ_{ij} from 0 to 0.016, when correlations are increased for power lines incident to PMU nodes.

normal operation. It can be shown that the expected load shed of the system is $\mathrm{E}[X_1] + \mathrm{E}[X_2] - \sigma_{12}$, where σ_{12} is the covariance between the failures of power lines l_1 and l_2. The intuition behind this counterexample is that while the probability of both lines failing increases, the probability of each failing individually decreases, with the result that the total probability of any line failing decreases. We here state sufficient conditions

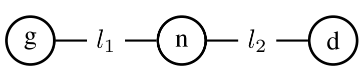

Figure 8. Topology of a 3-bus, 2-line power network where increased correlations between power line failures result in decreased system load shed. g is a generation bus, d a demand bus and n a demand with power demand 0.

under which increased correlations of power line failures imply increased expected load shed.

Proposition 3. *Let the power system satisfy the $n-k$ criterion, i.e. the disconnection of any k power lines does not induce any necessary load shedding. Furthermore, assume that all contingencies with at least $k+1$ line failures induce a total system load shed $\bar{c}$. Assume, wlog, that the moment $\phi_{1,\ldots,k+1} = \mathrm{E}[X_1 \cdot \ldots \cdot X_{k+1}]$ increases by $\Delta\phi$, but all other moments $\mathrm{E}[X_{i_1} \cdot \ldots \cdot X_{i_l}]$ are constant. Then*

1) *The central moment $\sigma_{1,\ldots,k+1} = \mathrm{E}\left[(X_1 - \bar{X}_1) \cdot \ldots \cdot (X_{k+1} - \bar{X}_{k+1})\right]$ also increases by $\Delta\phi$.*
2) *All other central moments of order less than or equal to $k+1$ remain constant.*
3) *The expected load shed $\bar{S}^*$ increases by $\bar{c} \cdot \Delta\phi$.*
4) *If $\mathrm{E}[X_i] < 1/2\ \forall\phi$, the variance of the load shed, σ_{S^*}, increases by $\hat{c}^2 \cdot \Delta\phi$, where $0 < \hat{c} \leq \bar{c}$.*

A proof is given in the appendix. The following corollary follows directly from proposition 3.

Corollary 1. *Assume that all conditions of proposition 3 still hold, except that all contingencies with at least $k+1$ line failures induce a system load shed of* at least *$\bar{c}$. In this case the results of proposition 3 hold instead for the lower bound $\underline{S}^* \leq S^*$ of the system load shed, which is the total system load shed assuming all line failures result in the same system load shed $\bar{c}$.*

V. Conclusions and Future Research

In this work we have demonstrated that increased correlations between power line failures can dramatically increase the expected costs in terms of system load shed, although the expected value of the failure probabilities is kept constant. Furthermore we have demonstrated that increased correlations between power line failures can also increase the variance of the system load shed, thus increasing the risk of large system load sheds. We have demonstrated our results by the sampling of correlated power line failures, using a model of the Nordic power grid. We have furthermore provided sufficient conditions under which the mean and the variance of the total system load shed increase with increasing correlation between line failures.

The framework presented in this paper should be seen as a general framework for reliability evaluation of power systems where correlations are known a priory, either by empirical data or by improved failure models of power systems.

It should be clarified that the failures we consider do not correspond to cascading failures, but that they could represent potential causes of cascading failures. In many situations, cascading failures further aggravate the state of a partially failing power system, leading to even larger losses in terms of system load sheds. In future work, it would be of interest to consider the impact of correlated failures under power system on cascading failures. Also, it would be interesting to study if the conditions under which the expected value and the variance of the load shed are increasing in the correlations, can be relaxed.

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APPENDIX

Proof: (of proposition 1) Note that

$$\mathrm{VaR}_\alpha(X) \le \bar{X} + \frac{1}{\sqrt{\alpha}}\sigma \quad \Leftrightarrow$$
$$\Pr\left(X < \bar{X} + \frac{1}{\sqrt{\alpha}}\cdot\sigma\right) \ge 1-\alpha$$

which is easily shown using Chebyshev's inequality

$$\Pr\left(X < \bar{X} + \frac{1}{\sqrt{\alpha}}\cdot\sigma\right) \ge$$
$$\Pr\left(\left|X - \bar{X}\right| < \frac{1}{\sqrt{\alpha}}\cdot\sigma\right) =$$
$$1 - \Pr\left(\left|X - \bar{X}\right| \ge \frac{1}{\sqrt{\alpha}}\cdot\sigma\right) \ge 1-\alpha$$

■

Lemma 2. *The variance of a random variable X with compact support $[a,b]$ is bounded by:*

$$\mathrm{Var}[X] \le \frac{(b-a)^2}{4}$$

Proof: Consider the random variable defined by $Y = X - \frac{a+b}{2}$. By basic probability theory we have

$$\mathrm{Var}[X] = \mathrm{Var}[Y] = \mathrm{E}[Y^2] - \big(\mathrm{E}[Y]\big)^2$$
$$\le \left(\frac{b-a}{2}\right)^2 = \frac{(b-a)^2}{4}$$

■

Lemma 3. *Let $\hat{X}_N = \frac{1}{N}\sum_{i=1}^{N} X_i$ be the sampled mean of the random variable X with N samples. Let X have compact support on $[a,b]$, then for any given $\epsilon > 0,\ \delta > 0$*

$$\Pr\left[\left|\hat{X}_N - \bar{X}\right| \ge \epsilon\right] \le \delta$$

for

$$N \ge \left\lceil \frac{(b-a)^2}{4\delta\epsilon^2} \right\rceil$$

Proof: Since the samples are iid, we have

$$\mathrm{Var}[\hat{X}_N] = \mathrm{Var}\left[\frac{1}{N}\sum_{i=1}^{N} X_i\right] = \frac{1}{N^2}\,\mathrm{Var}\left[\sum_{i=1}^{N} X_i\right]$$
$$= \frac{N\sigma^2(X)}{N^2} \le \frac{(b-a)^2}{4N}$$

By Chebyshev's inequality we have

$$\Pr\left\{\left|\hat{X_N}-\bar{X}\right|\geq\epsilon\right\}\leq\frac{\mathrm{Var}[\hat{X}_N]}{\epsilon^2}\leq\frac{(b-a)^2}{4N\epsilon^2}\leq\delta$$

■

Lemma 4. *Let* $\hat{\sigma}_{X_N}=\frac{1}{N}\sum_{i=1}^N\left(X_i-\hat{X}_N\right)^2$ *be the sampled variance of the random variable* X *with* N *samples. Let* X *have compact support on* $[a,b]$*, then for any given* $\epsilon>0,\ \delta>0$

$$\Pr\left[\left|\hat{\sigma}^2_{X_N}-\sigma^2_X\right|\geq\epsilon\right]\leq\delta$$

for

$$N\geq\left\lceil\frac{(b-a)^4}{8\delta\epsilon^2}\right\rceil$$

Proof: By [37], the variance of the sampled variance is given by

$$\mathrm{Var}\left[\hat{\sigma}^2_{X_N}\right]=\frac{2\sigma^4_X}{N}$$

By lemma 2, the variance $\mathrm{Var}[X]=\sigma^2_X$ is bounded by

$$\sigma^2_X\leq\frac{(b-a)^2}{4}$$

Thus, by Chebyshev's inequality

$$\Pr\left[\left|\hat{\sigma}^2_{X_N}-\sigma^2_X\right|\geq\epsilon\right]\leq\frac{\mathrm{Var}[\sigma^2_{X_N}]}{\epsilon^2}\leq\frac{(b-a)^4}{8N\epsilon^2}\leq\delta$$

■

Lemma 5. *Given* $\epsilon>0,\ \delta>0$*, we have for a random variable* X *with compact support on* $[a,b]$

$$\Pr\left[\left|\hat{\sigma}_{X_N}-\sigma_X\right|\geq\epsilon\right]\leq\delta$$

for

$$N\geq\left\lceil\frac{(b-a)^4}{8\delta\epsilon^4}\right\rceil$$

Proof: By concavity of $\sqrt{\cdot}$, Chebyshev's inequality and lemma 4

$$\Pr\left[\left|\hat{\sigma}_{X_N}-\sigma_X\right|\geq\epsilon\right]\leq\Pr\left[\left|\hat{\sigma}^2_{X_N}-\sigma^2_X\right|\geq\epsilon^2\right]$$
$$\leq\frac{\mathrm{Var}[\sigma^2_{X_N}]}{\epsilon^4}=\frac{2\sigma^4_{X_N}}{N\epsilon^4}\leq\frac{(b-a)^4}{8N\epsilon^4}\leq\delta$$

■

Proof: (of proposition 3) The first part follows directly, since:

$$\sigma_{1,\ldots,k+1}=\mathrm{E}\left[(X_1-\bar{X}_1)\cdot\ldots\cdot(X_{k+1}-\bar{X}_{k+1})\right]=$$
$$\mathrm{E}\left[X_1\cdot\ldots\cdot X_{k+1}\right]-\bar{X}_1\cdot\mathrm{E}\left[X_2\cdot\ldots\cdot X_{k+1}\right]+$$
$$\ldots+(-1)^{k+1}\cdot\bar{X}_1\cdot\ldots\cdot\bar{X}_{k+1}$$

The second part also follows directly by calculation. Wlog, consider

$$\sigma_{1,\ldots,l}=\mathrm{E}\left[(X_1-\bar{X}_1)\cdot\ldots\cdot(X_l-\bar{X}_l)\right]=$$
$$\mathrm{E}\left[X_1\cdot\ldots\cdot X_l\right]-\bar{X}_1\cdot\mathrm{E}\left[X_2\cdot\ldots\cdot X_l\right]+$$
$$\ldots+(-1)^l\cdot\bar{X}_1\cdot\ldots\cdot\bar{X}_l$$

which is constant for all $l\leq k$ by the assumption that all other moments $\mathrm{E}[X_{i_1}\cdot\ldots\cdot X_{i_l}]$ are constant.

As for the third part, let $k=1$ and consider a network with 3 power lines. The expected value of the system load shed is:

$$\bar{c}\cdot(\Pr[X_1=1,X_2=1]+\Pr[X_1=1,X_3=1]+$$
$$\Pr[X_2=1,X_3=1]-\Pr[X_1=1,X_2=1,X_3=1])=$$
$$\bar{c}\cdot(\mathrm{E}[X_1X_2]+\mathrm{E}[X_1X_3]+\Pr[X_2X_3]-\Pr[X_1X_2X_3])$$

which increases by $\bar{c}\cdot\Delta\phi$ as $\mathrm{E}[X_1X_2]$ increases by $\Delta\phi$. One can generalize this and show that for arbitrary k and network size, the expected system load shed will still be proportional to $\bar{c}\cdot\mathrm{E}[X_1\cdot\ldots\cdot X_{k+1}]$. By the assumption that all other moments $\mathrm{E}[X_{i_1}\cdot\ldots\cdot X_{i_l}]$ are constant.

To prove the last claim, we denote $p_s=\Pr[S^*=\bar{c}]$, and $p_n=1-p_s=\Pr[S^*=0]$. By assumption $p_s<\frac{1}{2}$. For this case, the variance of the load shed is simply:

$$\sigma_{S^*}=p_n(0-\bar{S}^*)^2+p_s(\bar{c}-\bar{S}^*)^2=p_s\bar{c}^2(1-p_s)$$

The latter expression is increasing since

$$\frac{\partial}{\partial p_s}\,p_s\bar{c}^2(1-p_s)=\bar{c}^2(1-2p_s)>0$$

for $p_s<\frac{1}{2}$. Since $0<\frac{\partial\sigma_{S^*}}{\partial p_s}\leq\bar{c}^2$, $\bar{S}^*=\bar{c}p_s$, $\bar{S}^*=constant+\bar{c}\Delta\phi$ and

$$\begin{aligned}\Delta\sigma_{S^*}&=\int_{\phi_0}^{\phi_0+\Delta\Phi}\frac{\mathrm{d}\sigma_{S^*}}{\mathrm{d}\phi_{1,\ldots,k+1}}\ \mathrm{d}\phi_{1,\ldots,k+1}\\&=\int_{\phi_0}^{\phi_0+\Delta\Phi}\frac{\partial\sigma_{S^*}}{\partial p_s}\frac{\partial p_s}{\partial\phi_{1,\ldots,k+1}}\ \mathrm{d}\phi_{1,\ldots,k+1}\\&=\int_{\phi_0}^{\phi_0+\Delta\Phi}\frac{\partial\sigma_{S^*}}{\partial p_s}\ \mathrm{d}\phi_{1,\ldots,k+1}\end{aligned}$$

it holds that

$$\Delta\sigma_{S^*}=\int_{\phi_0}^{\phi_0+\Delta\Phi}\frac{\partial\sigma_{S^*}}{\partial p_s}\ \mathrm{d}\phi_{1,\ldots,k+1}>0$$

$$\begin{aligned}\Delta\sigma_{S^*}&=\int_{\phi_0}^{\phi_0+\Delta\Phi}\frac{\partial\sigma_{S^*}}{\partial p_s}\ \mathrm{d}\phi_{1,\ldots,k+1}\\&\leq\int_{\phi_0}^{\phi_0+\Delta\Phi}\bar{c}^2\ \mathrm{d}\phi_{1,\ldots,k+1}=\bar{c}^2\Delta\phi\end{aligned}$$

■