Enhancements for Contractive Receding Horizon Control ⋆

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Abstract: We propose a new stability constraint for contractive receding horizon control (RHC), which was inspired by a non-monotone line search rule for optimization algorithms. Previously proposed stability constraints for contractive RHC can be seen as special cases of our new constraint. The new stability constraint guarantees asymptotic stability, but it does not ensure the monotone decrease of the state to zero. The non-monotone nature of our constraint is less restrictive than previous contractive constraints, and in principle, should result in a larger region of attraction (stability region). We present simulation results which show that this is in fact the case. We also present several enhancements which result in a faster system response and/or a larger stability region.

Keywords: receding horizon control, model predictive control, constrained nonlinear systems, stability constraint, initial feasible set.

1. INTRODUCTION

Receding Horizon Control (RHC) also known as a Moving Horizon or Model Predictive Control (MPC) systems are a form of sample-data feedback system in which the control to be applied over the sampling interval is computed by solving a finite-horizon optimal control problem (FHOCP). Stability is ensured by endowing the FHOCP with set of appropriate properties. RHC is particularly effective for controlling nonlinear systems and/or when the input is bounded. In Mayne et al. (2000) we find an excellent survey of various schemes used to ensure stability of RHC system with input and state-space constraints. In this paper, we are concerned with contractive RHC systems, in which stability is ensured by means of a terminal inequality in the governing optimal control problem.

A contractive test for stability was first introduced in Polak and Mayne (1981). It was subsequently applied to contractive RHC in a series of three papers by Polak and Yang (Polak and Yang (1993a,b); Yang and Polak (1993)), in which stability was ensured by requiring that the system states x_k, at the sampling times t_k, satisfy an inequality of the form $\|x_{k+1}\|^2 \leq \alpha \|x_k\|^2$, with $\alpha \in (0, 1)$. Since one cannot be sure that such an inequality can be satisfied with the horizon fixed in the optimal control problem, Polak and Yang solved a free-time optimal control problem, which resulted in an asynchronous sample-data system with a rather large stability region. They showed that the such an asynchronous sample-data RHC was robustly stable and was able to attenuate L_∞ bounded disturbances.

The use of asynchronous sample-data RHC did not gain favor in industry, where synchronicity was considered important. Because of this, de Oliveira Kothare and Morari (2000) revisited the problem of RHC with a contractive stability constraint, with the dynamics in discretized form. They fixed the horizon in the optimal control problem (in their case a discrete time optimal control problem) and they showed that their algorithm renders the closed-loop system exponentially stable. However, because they coupled the contractive stability constraint with a fixed horizon, the set of initial states for which the governing optimal control problem is smaller than in the case of Yang and Polak (1993), and hence the guaranteed region of attraction of the origin is also smaller. Another variation of contractive RHC was presented in Cheng and Krogh (2001).

In this paper, we propose a new contractive stability constraint for RHC, which was inspired by the non-monotone line search rule proposed in Grippo et al. (1986) for optimization algorithms. It can be seen that the contractive stability constraints in de Oliveira Kothare and Morari (2000) and Cheng and Krogh (2001) are special cases of our new constraint. The new stability constraint guarantees asymptotic stability. However, unlike the constraints in Yang and Polak (1993) and de Oliveira Kothare and Morari (2000), it does not enforce a monotone decrease of the state to zero. The use of a non-monotone constraint is less restrictive than the existing contractive constraint, and in principle, should result in a larger region of attraction (stability region). We present simulation results which show that this is in fact the case. We also present several enhancements which result in a faster system response and/or a larger stability region.

In Section 2 we present our new non-monotone asymptotic stability condition, in Section 3 we state RHC algorithm, in Section 4 we provide asymptotic stability result of our RHC algorithm, in Section 5 we present numerical results, and our concluding remarks are in Section 6.

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2. NON-MONOTONE ASYMPTOTIC STABILITY CONDITION

Before we present the non-monotone asymptotic stability concept, we need the following lemma.

Lemma 1. Suppose that the sequence $\{\gamma_k\}_{k=0}^{\infty}$, $\gamma_k \in \mathbb{R}$, $k \geq 0$, is such that

(i) $\gamma_k \geq 1$ and $\gamma_k \geq \gamma_{k+1}$ for all $k \geq 0$;
(ii) $\prod_{k=0}^{\infty} \gamma_k < \infty$.

If $\{a_k\}_{k=0}^{\infty}$ is a sequence in $\mathbb{R}$, such that $0 \leq a_{k+1} \leq \gamma_k a_k$ for all $k \geq 0$, then there is an $a^* \geq 0$ such that $a_k \to a^*$ as $k \to \infty$.

Proof. First, since for all k we must have that $a_k \leq \prod_{i=0}^{k-1} \gamma_i a_0 \leq \prod_{i=0}^{\infty} \gamma_i a_0 < \infty$. It follows that the sequence $\{a_k\}_{k=0}^{\infty}$ is bounded and hence must have accumulation points. Suppose it has two accumulation points: $a^* < a^{**}$, and that $a^{**} - a^* = 4\delta$, $\delta > 0$. Let $\beta > 0$ be such that

$$\beta(a^{**} - 3\delta) \leq \delta. \tag{1}$$

Since $\prod_{k=0}^{\infty} \gamma_k < \infty$, there exists a k_1 such that for all $k \geq k_1$, $\prod_{i=k}^{\infty} \gamma_i \leq 1 + \beta$. Next, since a^* and a^{**} are accumulation points, there exist indices $k_1 < k_2 < k_3 < k_4$ such that $a_{k_2} \geq a^{**} - \delta$, $a_{k_3} \leq a^* + \delta$, and $a_{k_4} \geq a^{**} - \delta$. Now, we must have that

$$\begin{aligned} a_{k_4} &\leq a_{k_3} \prod_{k=k_3}^{\infty} \gamma^k \\ &\leq a_{k_3}(1+\beta) \\ &\leq (a^* + \delta)(1+\beta) \\ &\leq (a^{**} - 3\delta)(1+\beta) \\ &\leq a^{**} - 2\delta. \end{aligned} \tag{2}$$

But this contradicts the fact that $a_{k_4} \geq a^{**} - \delta$, and hence the sequence $\{a_k\}_{k=0}^{\infty}$ cannot have more than one accumulation point, i.e., it must converge.

The following non-monotone stability condition was inspired by the non-monotone line search rule that was proposed by Grippo et al. (1986).

Theorem 2. Suppose that we have a scalar sequence $\{\gamma_k\}_{k=0}^{\infty}$ such that $\gamma_{k+1} \leq \gamma_k$ $\forall k \geq 0$, $\gamma_k \to 1$ as $k \to \infty$, and $\prod_{k=0}^{\infty} \gamma_k < \infty$, and a vector sequence $\{x_k\}_{k=0}^{\infty}$ in $\mathbb{R}^n$ such that

(i)
$$\|x_{k+1}\|^2 \leq \max_{0 \leq j \leq m(k)} \gamma_{k-j} \|x_{k-j}\|^2 - \alpha \|x_k\|^2, \tag{3}$$
where $0 < \alpha < 1$, $k \geq 0$, $m(0) = 0$, and
$$m(k) \triangleq \min[m(k-1)+1, M], \tag{4}$$
for some fixed $M \in \mathbb{Z}$.

(ii)
$$\|x_{k+1} - x_k\|^2 \leq \beta \|x_k\|^2, \quad \forall k \geq 0, \tag{5}$$
for some $\infty > \beta \geq 1$.

then

(a) x_k remains in a compact set $\{x_k \in \mathbb{R}^n | \; \|x_k\|^2 \leq C\|x_0\|^2\}$ for all $k \geq 0$, where $C \in [1, \infty)$.
(b) $\|x_k\|$ converges to zero as $k \to \infty$

Proof. For each $k \in \mathbb{Z}$, we define $l(k) \in \{k - m(k), \ldots k\}$ to be an index (not necessarily unique) determined by the relation

$$\|x_{l(k)}\|^2 = \max_{0 \leq j \leq m(k)} \|x_{k-j}\|^2. \tag{6}$$

Note that $k - m(k) \leq l(k) \leq k$ by definition. It follows from (4), the fact that $m(k+1) \leq m(k)+1$, and

$$\begin{aligned} \|x_{l(k+1)}\|^2 &= \max_{0 \leq j \leq m(k+1)} \|x_{k+1-j}\|^2 \\ &\leq \max_{0 \leq j \leq m(k)+1} \|x_{k+1-j}\|^2 \\ &= \max[\|x_{l(k)}\|^2, \|x_{k+1}\|^2] \\ &\leq \max[\|x_{l(k)}\|^2, \gamma_{l(k)}\|x_{l(k)}\|^2] \\ &= \gamma_{l(k)}\|x_{l(k)}\|^2. \end{aligned} \tag{7}$$

Therefore,

$$\|x_{l(k+1)}\|^2 \leq \gamma_{l(k)}\|x_{l(k)}\|^2, \tag{8}$$

and

$$\|x_{l(k+1)}\|^2 \leq \prod_{j=0}^{k} \gamma_{l(j)} \|x_0\|^2, \; \forall k \geq 0. \tag{9}$$

It now follows from (8) and Lemma 1, that the sequence $\|x_k\|^2$ converges as $k \to \infty$.

After $k = M - 1$ steps, $m(k)$ saturates and assumes the value of M, by definition. Without loss of generality, ignoring the first $M - 1$ steps, we obtain that

$$\gamma_{k-M} \geq \gamma_{l(k)} \tag{10}$$

and

$$\infty > \prod_{k=0}^{\infty} \gamma_k \geq \prod_{k=M}^{\infty} \gamma_{k-M} \geq \prod_{k=M}^{\infty} \gamma_{l(k)}, \tag{11}$$

which implies that $\prod_{k=0}^{\infty} \gamma_{l(k)} < \infty$.

If we set

$$C = \prod_{k=0}^{\infty} \gamma_k, \tag{12}$$

then we conclude from (9) combined with (6), that

$$\|x_{l(k+1)}\|^2 \leq C\|x_0\|^2, \quad \forall k \geq 0. \tag{13}$$

From (3),

$$\|x_{k+1}\|^2 \leq \gamma_{l(k)}\|x_{l(k)}\|^2 - \alpha\|x_k\|^2, \tag{14}$$

and, by design,

$$\|x_{l(k)}\|^2 \leq \gamma_{l(l(k)-1)}\|x_{l(l(k)-1)}\|^2 - \alpha\|x_{l(k)-1}\|^2. \tag{15}$$

Since $\gamma_{l(l(k)-1)} \to 1$ as $k \to \infty$,

$$\begin{aligned} 0 &= \lim_{k \to \infty} (\|x_{l(k)}\|^2 - \gamma_{l(l(k)-1)}\|x_{l(l(k)-1)}\|^2) \\ &\leq -\lim_{k \to \infty} \alpha\|x_{l(k)-1}\|^2, \end{aligned} \tag{16}$$

which implies $\|x_{l(k)-1}\|^2 \to 0$ as $k \to \infty$. It now follows from (5) that

$$\|x_{l(k)} - x_{l(k)-1}\|^2 \leq \beta\|x_{l(k)-1}\|^2, \tag{17}$$

and hence that $\|x_{l(k)}\|^2 \to 0$ as $k \to \infty$.

Finally, since

$$\|x_k\|^2 \leq \|x_{l(k)}\|^2 = \max_{0 \leq j \leq m(k)} \|x_{k-j}\|^2, \tag{18}$$

$\|x_k\|^2 \to 0$ as $k \to \infty$.

Remark 3. Note that the conclusions of Theorem 2 remain valid even if (3) is violated a *finite* number of times.

Remark 4. Note that setting $\gamma_k = 1$ for all k, results in $\|x_1\| \leq \|x_0\|$. Using a monotone decreasing sequence $\{\gamma_k\}_{k=0}^{\infty}$ results in a useful relaxation. In particular, if the

x_k are sampled state vectors of a non-minimum phase linear dynamic system, enforcing the condition (3) with $\gamma_k = 1$ may result in infeasibility due to undershoot.

Note that, if we set $M = 0$ and $\gamma_k = 1$, then (3) becomes

$$\|x_{k+1}\|^2 \leq (1-\alpha)\|x_k\|^2, \tag{19}$$

which is identical with the contractive constraint in Yang and Polak (1993) and de Oliveira Kothare and Morari (2000). In Cheng and Krogh (2001), a similar stability constraint was applied to linear time invariant systems with a RHC control law.

3. RECEDING HORIZON CONTROL ALGORITHM

3.1 Preliminaries

We define the set of admissible controls by

$$\mathbf{U} \triangleq \{u \in L_\infty^m[0,\infty) | u(t) \in \mathcal{U},\ \forall t \in [0,\infty)\} \tag{20}$$

with $\mathcal{U} \triangleq \{u \in \mathbb{R}^m | \|u\|_\infty \leq c_u\}$. For any control $u(\cdot) \in \mathcal{U}$, let $x(t; x_k, u) \in \mathbb{R}^n$ be the solution of the following ordinary differential equation

$$\begin{aligned} &\dot{x}(t) = f(x(t), u(t)) \\ &x(t) \in \mathbb{R}^n, \quad u(t) \in \mathbf{U}, t \in [0,\infty) \end{aligned} \tag{21}$$

at time t with the initial condition $x(0; x_k, u) = x_k$. We assume (i) that $f(x, u)$ is continuously differentiable in both arguments, which ensures that the solution $x(t; x_0, u)$ is unique and differentiable in u, and (ii) that $0 = f(0, 0)$. Let $0 < \Delta < \infty$ be the sampling interval of the RHC scheme. We define $t_k = k\Delta$, $k = 0, 1, 2, \ldots$.

3.2 RHC Algorithm

Now we are ready to define the finite-horizon optimal control problem (FHOCP) for our receding horizon control algorithm. We assume that this optimal control problem is defined on a fixed horizon, of length $N\Delta$, where $N \geq 1$ is an integer. This problem has a cost function $J(u)$ whose exact structure is not important at this point, since it does not impact our stability result.

Problem 5. We assume that we are given a set of vectors $x_k, x_{k-1}, \ldots, x_{k-m(k)}$ in $\mathbb{R}^n$. The optimal control problem to be solved is:

$$\min_{u \in \mathbf{U}} J(u), \quad t \in [0, N\Delta] \tag{22}$$

subject to the system dynamics (21) with initial condition $x(0) = x_k$, and the inequality constraints

$$\|x(t_{k+1}; x_k, u)\|^2 \leq \max_{0 \leq j \leq m(k)} \gamma_{k-j}\|x_{k-j}\|^2 - \alpha\|x_k\|^2 \tag{23}$$

$$\|x(t; x_k, u) - x_k\|^2 \leq \beta\|x_k\|^2,\ t \in [t_k, t_{k+1}], \tag{24}$$

where $\alpha \in (0, 1)$, $k \geq 0$, $m(\cdot)$ are as defined in (4), and $\infty > \beta \geq 1$.

Remark 6. There is no objection to including in Problem 5 additional state-space constraints, but we did not do this for the sake of simplicity.

Note that (24) implies (5). Although this functional constraint adds many inequality constraints in the discretized FHOCP, these are rarely active, since β can be set to a large number. The external active-set strategy in Polak et al. (2007) can be used to exclude inactive inequality constraints during optimization so that the computational burden induced by (24) is minimized. In de Oliveira Kothare and Morari (2000), (24) was regarded as an assumption, and not included in the FHOCP formulation for RHC algorithm.

The FHOCP defined above is used to define the RHC algorithm below. Note that this is a "theoretical" version, since it does not take into account computing time or the discrepancies between the model state and the actual state. For a more realistic approach, see Yang and Polak (1993).

Algorithm 1. Receding Horizon Control

Data: x_0, the state of the system (21) at $t = 0$.
Set $k = 0$.
loop
  if $t = t_k$ **then**
    Solve Problem 5, with initial state x_k, and using the preceding states $x_{k-1}, \ldots, x_{k-m(k)}$, for the optimal control $\hat{u}(t)$, $t \in [0, N\Delta]$.
    Apply the control $u(t) = \hat{u}(t - t_k)$ for $t \in [t_k, t_{k+1}]$ to the dynamical system (21).
    Set $x_{k+1} = x(t_{k+1}; x_k, u)$ and set $k = k + 1$.
  end if
end loop

4. NON-MONOTONE ASYMPTOTIC STABILITY

With bounded input constraints, it is not possible to stabilize an unstable system globally. Even for stable systems, when the state is very large, it may not be possible to satisfy the constraint (23). Hence we need the following assumption.

Assumption 7. We assume that there exists a $r \in (0, \infty)$ such that for all $x \in B_r \triangleq \{x \in \mathbb{R}^n |\ \|x\|^2 \leq r^2\}$, Problem 5 has feasible solutions.

We are now ready to state our main result.

Theorem 8. Suppose that Assumption 7 is satisfied and consider the sample-data dynamical system resulting from the use of the RHC scheme defined by Algorithm 1. Let $\Omega_f \triangleq \{x \in \mathbb{R}^n | C\|x\|^2 \leq r^2\}$ with C defined in (12). Then (i) for any $x_0 \in \Omega_f$, the trajectory defined by Algorithm 1 is well defined, and (ii) the resulting RHC feedback system is asymptotically stable on the set Ω_f.

Proof. (i) It follows from Theorem 2(a), by induction, that for every $x_0 \in \Omega_f$, and any $k = 1, 2, \ldots$, x_k, determined by Algorithm 1 is in B_r and hence that the trajectory defined by Algorithm 1 is well defined.

(ii) To prove that our sample-data dynamical system is asymptotically stable on the set Ω_f, we must show that (a) it is stable, and (b) that the trajectory of the closed-loop system converges to the origin for any $x_0 \in \Omega_f$.

(a) Let $\delta > 0$ be given. It follows from the constraint (24) that for all $k \geq 0$ and $t \in [t_k, t_{k+1}]$,

$$\|x(t; x_k, u)\|^2 \leq (1+\beta)\|x_k\|^2 \tag{25}$$

because

$$\|x(t; x_k, u)\|^2 - \|x_k\|^2 \leq \|x(t; x_k, u) - x_k\|^2. \tag{26}$$

Hence, from Theorem 2(i) and the above inequality, we conclude that

$$\|x(t; x_k, u)\|^2 \leq (1+\beta)\|x_k\|^2 \leq (1+\beta)C\|x_0\|^2, \quad (27)$$

for all $k \geq 0$ and $t \in [t_k, t_{k+1}]$.

Now, to show that the closed-loop system is stable, we must to show that for any $\delta > 0$, there exists an ϵ such that $\|x_0\|^2 < \epsilon$ implies that $\|x(t; x_0, u)\|^2 < \delta$ for all $t \geq 0$.

For any given δ, let $\epsilon = \min\{r, \delta/(C(1+\beta))\}$, $\|x_0\|^2 < \epsilon$. Then we must have that

$$\begin{aligned}\|x(t; x_k, u)\|^2 &\leq (1+\beta)C\|x_0\|^2 \\ &< \delta.\end{aligned} \quad (28)$$

(b) By Theorem 2(ii), for any $x_0 \in \Omega_f$, x_k defined by Algorithm 1 for $k = 1, 2, 3, \ldots$, satisfies $\|x_k\| \to 0$ as $k \to \infty$. Therefore, because of the constraint (24), $\|x(t; x_k, u_{[t_k, t]})\| \to 0$ as $t \to \infty$, which completes our proof.

Remark 9. It is possible to consider three generalizations of Problem 5 aimed at either increasing the stability region or the speed of response of the system to disturbances.

(i) In inequality (23), replace the constant value of α by adaptively defined value of α:

$$\alpha(x_k) = \min[\alpha_0, \exp(-\alpha_e\|x_k\|^2)] \quad (29)$$

with $\alpha_0 \in (0, 1)$ and $\alpha_e > 0$, which makes α small when the state is large, making (23) easier to satisfy.

(ii) Incorporate α in the cost function so that (22) becomes replaced with

$$\min_{u \in \mathbb{U}} J(u) - p\alpha, \quad 0 < \alpha < 1, \quad (30)$$

with $p > 0$ large, which should result in a faster system response.

(iii) Fix α and γ_k in FHOCP and resolve FHOCP to find the smallest value of M which result in a feasible solution. Again, this should result in speeded up system response.

5. NUMERICAL EXAMPLE

In order to test our RHC algorithm, we use the nonlinear oscillator example in Mayne and Michalska (1990) with a small modification, which causes the zero-input response to become an ellipse (rather than a circle) in the phase plane:

$$\begin{aligned}\dot{x}_1 &= -4x_2 + u[\mu + (1-\mu)x_1] \\ \dot{x}_2 &= x_1 + u[\mu - 4(1-\mu)x_2].\end{aligned} \quad (31)$$

We define our performance index as

$$J(u) = \frac{1}{2}\int_0^{N\Delta} \|x(\tau; x_k, u)\|^2 + \|u(\tau)\|_2^2 d\tau, \quad (32)$$

with $\mu = 0.5$. We chose the sampling interval $\Delta = 1$, the horizon length also to be Δ ($N = 1$), and the control input is constrained by $|u| \leq 0.5$. We defined γ_k as

$$\gamma_k = \exp(\gamma_0(0.8)^{k-1}). \quad (33)$$

In order to solve Problem 5, the dynamics were discretized using Euler's method using a discretization interval of 1/32, and SNOPT, by Murray et al. (2002), was used as a solver. See Polak (1997) Chapter 4 for a detailed treatment of discrete time optimal control problems.

To obtain a meaningful comparison, we first set $\alpha = 0.3$, $\gamma_k = 1$, and $M = 5$, and computed a value for α to be used with $M = 0$ by least squares curve fitting to ensure that both cases have similar rates of convergence. The resulting value of α was approximately 0.05. Fig. 1 shows the resulting bounds.

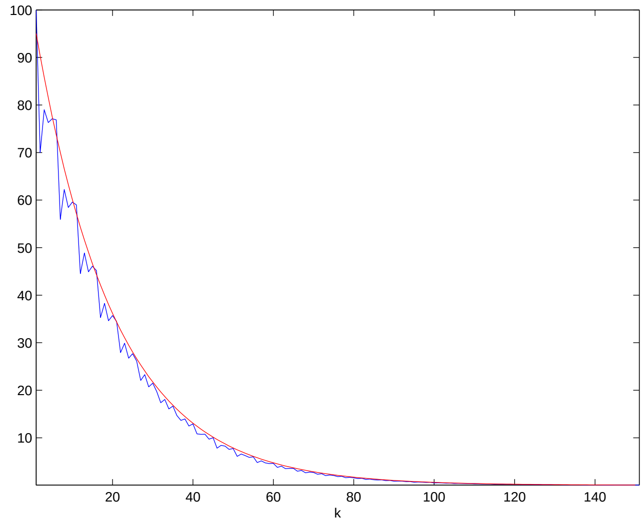

Fig. 1. Comparison of bounds computed by $\|x_{k+1}\|^2 = \max_{0 \leq j \leq m(k)} \|x_{k-j}\|^2 - \alpha\|x_k\|^2$. The jagged curve corresponds to the case where $\alpha = 0.3$ and $M = 5$, and the smooth curve corresponds to the case where $\alpha = 0.05$ and $M = 0$.

Fig. 2 shows the response of the RHC closed-loop system with plant dynamics (31) for two parameter sets in Algorithm 1. We see that they have a similar convergence rate when started from a point where Problem 5 has feasible solutions for both cases.

In order to investigate the effectiveness of our new constraint (23) in enlarging the set of initial states from which our system converges to the origin without violating the feasibility constraint in Problem 5, we performed a series of simulations with initial states in a mesh in the phase plane. An initial state was tagged as 'feasible' if the resulting trajectory satisfied $\|x_k\|^2 \leq 10^{-1}$ for some k without producing an infeasibility error from solver. Otherwise, it was tagged as 'infeasible'. Fig. 3 shows the result with $\alpha = 0.05$, $\gamma_0 = 0$, and $M = 0$, and Fig. 4 with $\alpha = 0.3$, $\gamma_0 = 2$, and $M = 5$. Note that, in Fig. 5, a denser grid was used for identifying more accurately the feasible, i.e., stability, region for the case of $M = 0$.

The results show that our new constraint enlarges the stability region dramatically. The stability region shown Fig. 5 is highly non-convex, and the ball satisfying Assumption 7, which was also used in de Oliveira Kothare and Morari (2000), is very small. Therefore, enforcing the contractive constraint at the end of the fixed horizon, without any relaxation, makes the RHC algorithm very conservative, and thus impractical.

Fig. 7 shows the behavior of our algorithm with the enhancements described in Remark 9. $\alpha_0 = 0.5$ and $\alpha_e = 0.5$ are used for the scheme (29), and $p = \|x_k\|^2$ is used for the scheme (30). As expected, the "optimal" selection of α given by (30) results in the fastest system, without adding to the burden of solving Problem 5. The "adaptive" definition of α (29) is effective, but requires

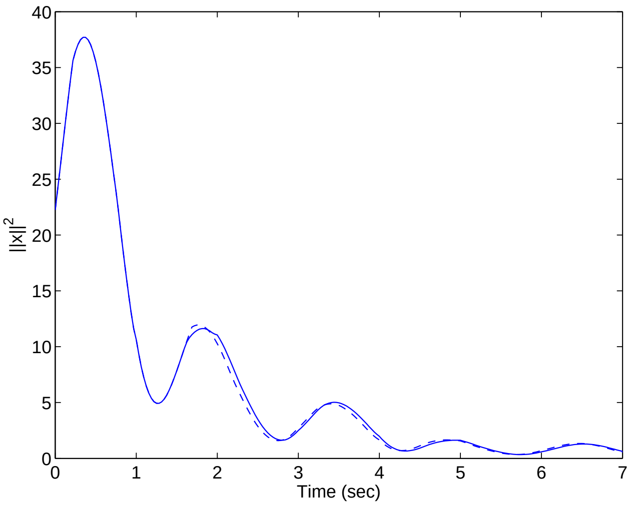

Fig. 2. Convergence rate comparison of closed-loop system under RHC control. The solid line corresponds to the case $\alpha = 0.3$ and $M = 5$, and the dashed line corresponds to the case $\alpha = 0.05$ and $M = 0$.

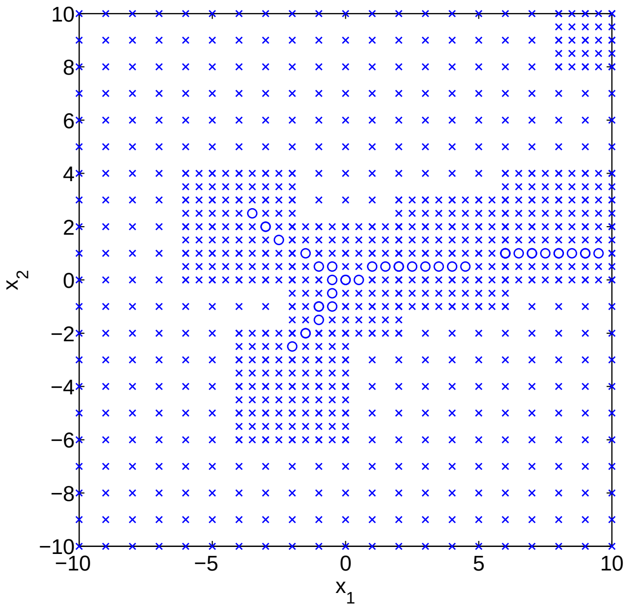

Fig. 3. The case $\alpha = 0.05$, $M = 0$, and $\gamma_0 = 0$. Initial states resulting in feasible trajectories for Problem 5 are marked with ‘o’, those resulting in infeasible trajectories for Problem 5 are marked with ‘x’.

parameter tuning, and hence is less desirable than the “optimal” selection of α .

6. CONCLUSION

We have introduced a new stability constraint for contractive receding horizon control, as well as some enhancements. Our numerical results show that the replacement of the “classical” contractive stability constraint by the new ones results in a very considerable enlargement of the stability region.

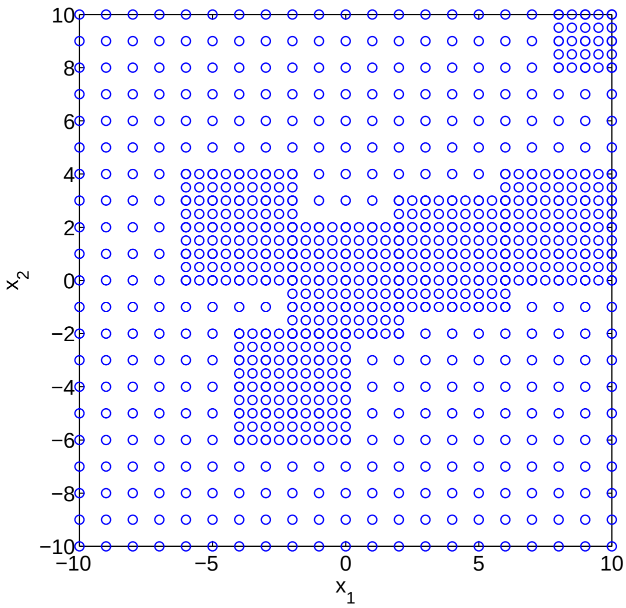

Fig. 4. The case $\alpha = 0.3$, $M = 5$, and $\gamma_0 = 2$. Initial states resulting in feasible trajectories for Problem 5 are marked with ‘o’, those resulting in infeasible trajectories for Problem 5 are marked with ‘x’.

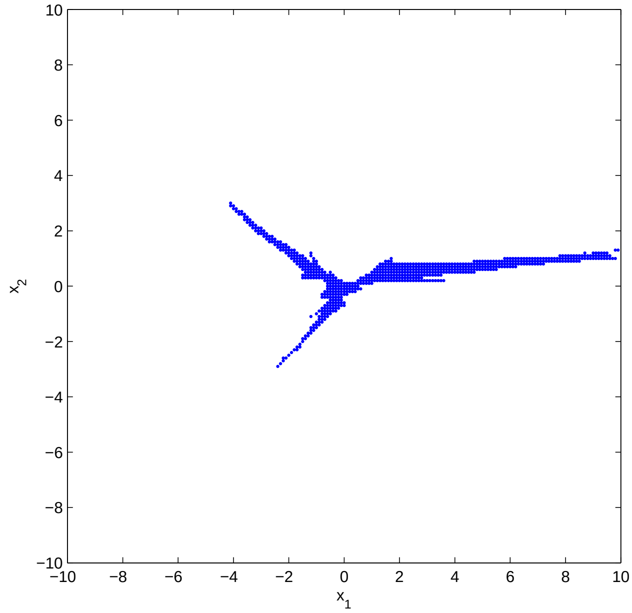

Fig. 5. The case $\alpha = 0.05$, $M = 0$, and $\gamma_0 = 0$ on a finer mesh (201×201). For better visibility, initial states resulting in infeasible trajectories for Problem 5 are not marked here.

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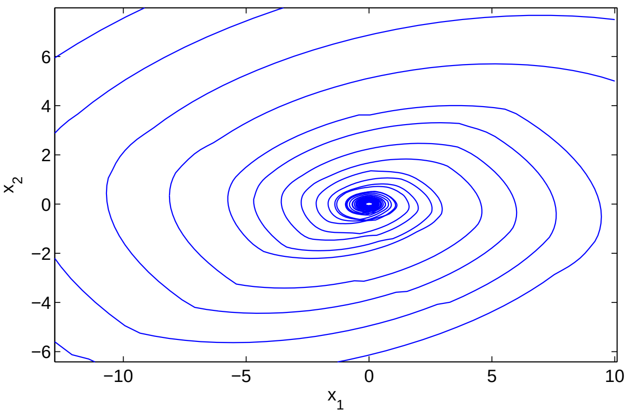

Fig. 6. Phase portrait of the closed-loop system under RHC with $\alpha = 0.3$, $M = 5$, and $\gamma_0 = 2$.

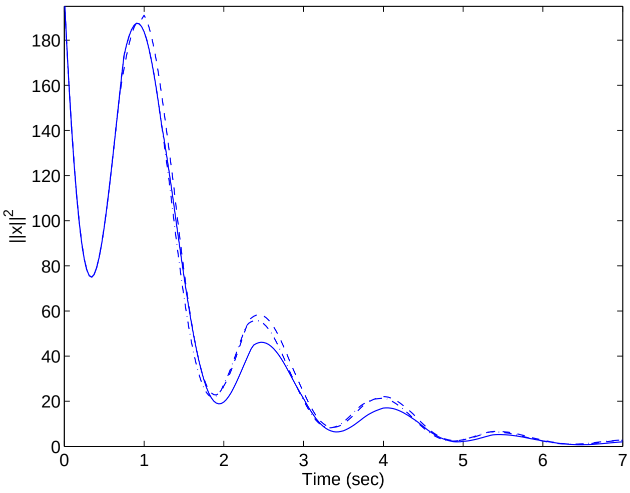

Fig. 7. Convergence rate comparison with enhancements in Remark 9. The solid line corresponds to α determined as in the (30), the dashed line corresponds to the adaptive α, as in (29), and the dotted-dashed line corresponds to the case where $\alpha = 0.3$ and $M = 5$.

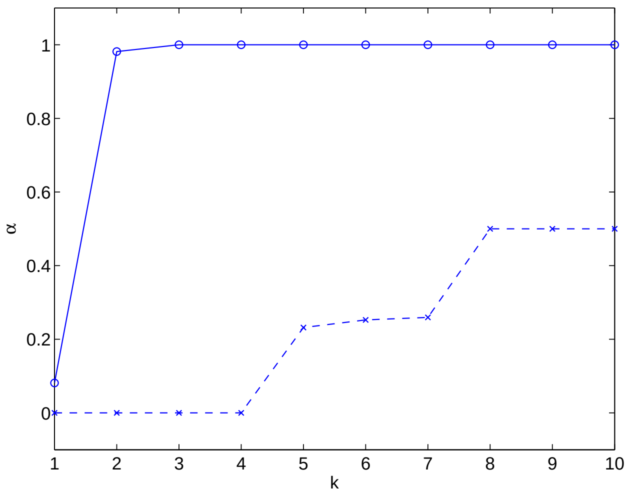

Fig. 8. Plot of α vs. k with the optimal scheme (30) (solid line) and with the adaptive scheme (29) (dashed)

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