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Lecture 20: Non-malleable zero knowledge

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1 Simulation-extractible protocol for small tags

The construction of the family of "small tags" arguments (based on [1]) will be based on techniques similar to Barak's non-blackbox ZK construction. We recall Barak's protocol:

Barak's Protocol

P **V**

$$\xleftarrow{\quad h \quad}$$

$$\xrightarrow{\quad c = Com(0^k, s) \quad}$$

$$\xleftarrow{\quad r \leftarrow_{\$} \{0,1\}^k \quad}$$

. . . . UARG . . . .

Where UARG is a WI universal argument in which P proves to V that either $x \in L$ *or* $\exists(\Pi, s)$ s.t $c = Com(h(\Pi), s) \wedge \Pi(c) = r$.
We will use a modified version of this protocol where the length of r is $l(k)$, and during the UARG P proves that $x \in L$ or there exists (Π, y, s) such that the following conditions all hold:

- $c = Com(h(\Pi), s)$
- $\Pi(y) = r$
- $|y| \leq |r| - k$

Where P has the freedom to choose y in order to try and match r. This is still sound, because of the length difference between y and r - since P commited to C before it got r, it has no more than probability 2^{-k} to "hit" r by choosing y. It is also zero-knowledge if we assume $l(k) \geq 3k$ and the commitment scheme outputs a commitment of size $\leq 2k$. The simulator will use c as y and the length will be suitable to convince V. Also, when these 3 statements hold, we say that $((h, c, r), (\Pi, y, s)) \in R_{sim}$.
We are now ready to describe the protocol for small tags of length $\log n + 1$. Fix a length $l(n) \geq 3n$, and a $tag \in [2n]$:

Protocol $< P_{tag}, V_{tag} >$

Common Input : $An\ \mathbf{in}stance\ x \in \{0,1\}^n$

Stage 0 (**Set up**) :

$V \rightarrow P: \ send\ h \leftarrow \mathcal{H}_n$

Stage 1 (**Slot 1**) :

$P \rightarrow V: \ send\ c_1 = Com(0^n)$

$V \rightarrow P: \ send\ r_1 \leftarrow \{0,1\}^{tag \cdot l(n)}$

Stage 1 (**Slot 2**) :

$P \rightarrow V: \ send\ c_2 = Com(0^n)$

$V \rightarrow P: \ send\ r_2 \leftarrow \{0,1\}^{(2n+1-tag) \cdot l(n)}$

Stage 2 (**UARG**) :

$P\ proves$ **to** $V\ that\ one\ of\ the\ following\ statements\ is$ **true** :

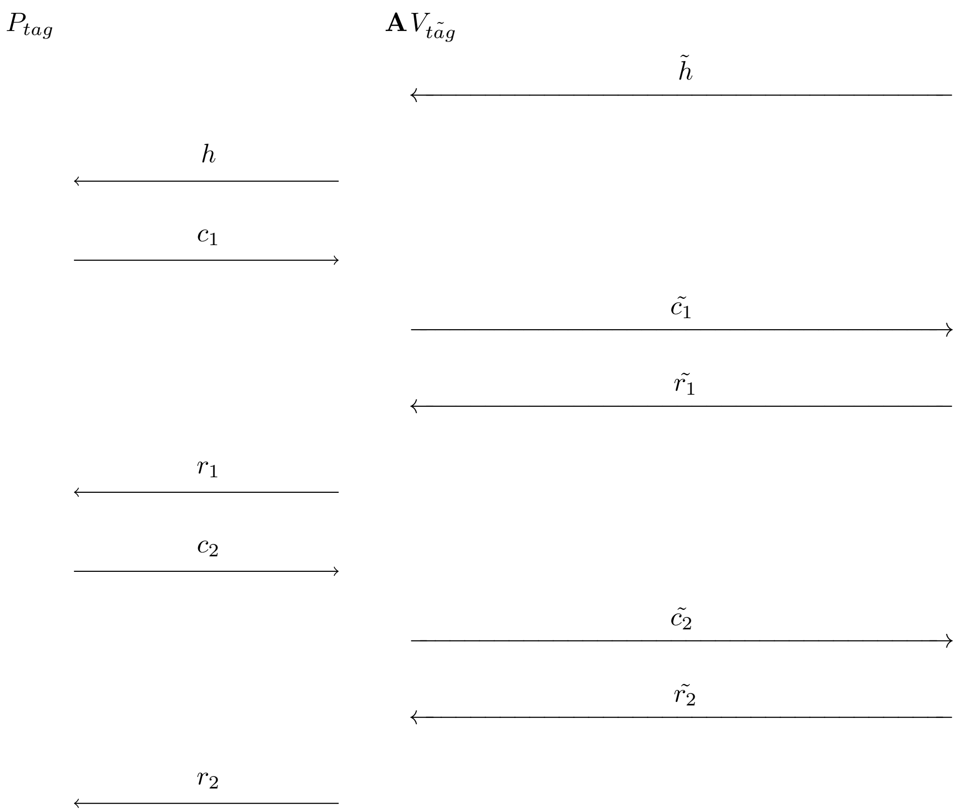
1. $\exists\ w \in \{0,1\}^{poly(|x|)}\ s.t\ (x,w) \in R_L$
2. $\exists\ (\Pi, y, s)\ s.t\ ((h, c_1, r_1),\ (\Pi, y, s)) \in R_{sim}$
3. $\exists\ (\Pi, y, s)\ s.t\ ((h, c_2, r_2),\ (\Pi, y, s)) \in R_{sim}$

Note that $< P_{tag}, V_{tag} >$ is a proof of knowledge - if any prover P^* convinces the honest verifier that $x \in L$, we can extract a witness w in expected polynomial time. This will be used later to produce the `EXT` algorithm.

To prove simulation-extractability we need to construct `(SIM,EXT)`. To that end, we construct a simulator S that is able to generate the messages on the left hand interaction of A (the ones from P_{tag}) when the right hand messages are coming from an "external" verifier (S does not have the code of that verifier). We will than construct `SIM` by running internally the honest verifier $V_{\tilde{tag}}$ and forward its messages to S. We will construct `EXT` by using S to construct a stand-alone prover $P_{\tilde{tag}}$ (Emulate A while using S to generate the left interaction and forwarding the right messages to an external honest verifier). We can then extract a witness using the proof of knowledge property.

The main problem is constructing S can be summed up in the scheduling presented in the diagram below. The core of the problem is that S, trying to simulate the left interaction, has to commit to the code of the "verifier" A, but the messages A sends on the left (its output in this context - r_i) are not dependent only on c_i, but also on the "external" messages in the right interaction. The simulation therefor cannot get away by setting $\Pi = A$ and $y = c_i$, because in the scenario in the diagram below the r_i are also dependent on $\tilde{r_i}$, and the simulator will not produce correct views in the universal argument stage.

Problematic scenario in the simulator S



We can get around this technical problem by noting that for at least one of the slots, we will have $|c_i| + |\tilde{r_i}| \leq |r_i| - n$. This allows us to set $y = (c_i, \tilde{r_i})$ (by the UARG stage we have it), which will give the correct answer (because $r_i = A(c_i, \tilde{r_i})$), and still respect the bound in R_{sim}, namely $|y| \leq r - n$.

This works because of how the lengths of the r_i's depend on the tags. If $tag \neq \tilde{tag}$, for at least one of the i's we have $|\tilde{r_i}| \leq |r_i| - l(n)$. And using the facts that $l(n) \geq 3n$ and $|Com(\alpha, s)| \leq 2n$ for α of size n, the required inequality follows.

2 From tags in $[2n]$ to tags in $\{0,1\}^n$

Suppose we have a family of protocols $\{< P_{tag}, V_{tag} >\}_{tag \in [2n]}$ that are simulation-extractable, we can use them to construct a family of protocols $\{< P_{TAG}, V_{TAG} >\}_{TAG \in \{0,1\}^n}$ that are simulation-extractable. The idea is to take the $TAG \in \{0,1\}^n = (TAG_1, \ldots, TAG_n)$, and to generate run the protocol n times in parallel with the small tags generated from the big tag:

Protocol $< P_{TAG}, V_{TAG} >$

Common Input : *An* **in***stance* $x \in \{0,1\}^n$

The protocol :

for $i \in \{1, \dots , n\}$ (**in** *parallel*) :

1. *Set* $tag_i = (i, TAG_i)$
2. *Run* $< P_{tag_i}, V_{tag_i} >$ *with common* **in***put* x *and* **length** $l(n)$

Accept **if** *and only* **if** *all* n *executions accept.*

Note that this is a constant-round IP (since every $< P_{tag}, V_{tag} >$ is, and we run them in parallel), and that if $TAG \neq \tilde{TAG}$, for at least one $i \in [n]$ we have that $tag_i \neq \tilde{tag}_i$, which assures us of soundness. The problem is being able to simulate the n interactions on the left - the simulator cannot handle messages forwarded from an external $V_{\tilde{TAG}}$, because they are too long, and it is not clear how to construct the stand alone prover $P^*_{\tilde{TAG}}$ (for the EXT procedure). The way around it is to construct a stand-alone prover for a single $< P_{\tilde{tag}}, V_{\tilde{tag}} >$. We first consider a "many to one" simulator, which for a $TAG = (tag_1, \dots, tag_n)$ generates views for the parallel left interactions $< P_{tag_1}, V_{tag_1} >, \dots, < P_{tag_1}, V_{tag_1} >$ on the common input $x \in \{0,1\}^n$, and the single right interaction $< P_{\tilde{tag}}, V_{\tilde{tag}} >$ on common input $\tilde{x} \in \{0,1\}^n$, where $\tilde{tag}$ and $\tilde{x}$ are chosen by A.
S incorporates A as a sub-routine, and handles the right interaction by having A communicate with an "external" honest verifier $V_{\tilde{tag}}$. On the left, messages are are handled by n sub-simulators $S_1, \dots, S_n$, each responsible for generating the messages of the respective sub-protocol. Ignoring the setup and the universal argument steps for simplicity, the interaction (with one specific scheduling) looks like this:

Many-to-one simulation

P			A	V
$\xrightarrow{c_1^1}$	$\dots$	$\xrightarrow{c_1^n}$		
			$\xrightarrow{\tilde{c_1}}$	
			$\xleftarrow{\tilde{r_1}}$	
$\xleftarrow{r_1^1}$	$\dots$	$\xleftarrow{r_1^n}$		
$\xrightarrow{c_2^1}$	$\dots$	$\xrightarrow{c_2^n}$		
			$\xrightarrow{\tilde{c_2}}$	
			$\xleftarrow{\tilde{r_2}}$	
$\xleftarrow{r_2^1}$	$\dots$	$\xleftarrow{r_2^n}$		

This is in fact a rather complicated scheduling, where $\tilde{c_i}, \tilde{r_i}$ are both, respectively, within slot i of $< P_{TAG}, V_{TAG} >$. If we have a simpler scheduling, for example, such that none of the right side

messages are within the first slot, the output of A will depend only on the c_i's (but on all of them together), and S_j can work as follows. Let A_j be an algorithm that is identical to A, but only outputs r_i^j. We let $\Pi_1 = A_j(x, \cdot)$, and the simulator S_j commits to $c_1 = Com(h(\Pi_1); s)$. During the UARG stage, it will use $(\Pi_1, (c_1^1, \ldots, c_1^n), s_1)$ as a witness for (h^j, c_1^j, r_1^j) (and it can commit to anything in the second slot). The case where the right side interaction is inside slot 2 is similar.
For the more complicated scheduling as in the diagram, where don't have a "free slot", we again exploit the length difference trick as following. Let $\Pi_1 = A_j(x, \cdot), \Pi_2 = A_j(x, c_1^1, \ldots, c_1^n, \tilde{r_1}, \cdot)$, and have S_j commit to $c_1 = Com(h(\Pi_1); s)$ and $c_2 = Com(h(\Pi_2); s)$. Now, as before, send witnesses to the suitable slot as follows:

- If $tag_j > \tilde{tag}$, we can set $(\Pi_1, (c_1^1, \ldots, c_1^n, \tilde{r_1}), s_1)$ as witness to (h^j, c_1^j, r_1^j).
- If $tag_j < \tilde{tag}$, we set $(\Pi_2, (c_2^1, \ldots, c_2^n, \tilde{r_2}), s_2)$ as witness for (h^j, c_2^j, r_2^j).

References

[1] Rafael Pass and Alon Rosen. New and improved constructions of non-malleable cryptographic protocols. In *Proceedings of the Thirty-seventh Annual ACM Symposium on Theory of Computing*, STOC '05, pages 533–542, New York, NY, USA, 2005. ACM.