# Appendices for "Whitespaces after the USA's TV incentive auction: a spectrum reallocation case study"

Kate Harrison, Vidya Muthukumar, Vijay Kamble, Angel Daruna, and Anant Sahai
Wireless Foundations, EECS, UC Berkeley

These appendices to [1] serve to detail our computational methods, describe the datasets we used, provide additional results, and to give the reader more detail on the incentive auction. We also describe some of the problems that arise when working with real-world data and how we overcome them.

## Contents

## APPENDIX A
## METHODS

In this section, we discuss the methods used for the calculations and analysis presented in [1]. We begin by outlining the design and methodology of our repacker. In particular, we describe how we used the FCC's public datasets and a well-known SAT solver to output multiple feasible reallocations for each band plan. We also briefly touch upon some technicalities with respect to the SAT solver and how they impact the output of our repacker.

Next, we discuss the assumptions made while designing the repacker, and explain the reasoning behind these assumptions.

Finally, we describe our method for determining the amount of available whitespace, as well as viewable TV, for a particular reallocation. We also describe in detail how the figures in [1] were created. We make use of the FCC's public datasets and our open-source software, WEST [2], for these computations.

### A. *Details of our repacker*

We used the FCC's domain, interference, and baseline files—released on 20 May 2014 at [3]—as a starting point. We then used PycoSAT, a Python wrapper for the well-known SAT solver library PicoSAT [4], in order to synthesize the constraints and output a feasible repacking. Two other studies on repacking have used PicoSAT [5], [6] and it was also featured in an FCC workshop on the topic of repacking in the incentive auctions [7].

The repacking problem can be naturally cast as a boolean satisfiability problem. The problem is to determine whether a set of stations can be packed into a given set of channels under the various interference and domain constraints. These constraints can be encoded as boolean expressions as follows. Let the set of stations be labelled $i = 1, \cdots, N$ and the set of channels be labelled $j = 1, \cdots, M$. For each station $i$ and channel $j$, define the binary variable $x_{ij}$, which takes value 1 if station $i$ is assigned to channel $j$ and 0 otherwise. We have the following type of constraints on these variables:

- **Domain constraints:** For each station $i$, there is a set of channels $M_i \subseteq \{1, \cdots, M\}$ that it can be feasibly assigned to. These constraints can be expressed as follows:

  $$\neg x_{ij} \text{ for each } i \text{ and each } j \notin M_i. \tag{1}$$

  Further, each station can be assigned to at most one channel in its set of feasible channels. This can be expressed as:

  $$\neg x_{ij} \vee \neg x_{ir} \text{ for each station } i \text{ and every pair of channels } j,\, r \in M_i. \tag{2}$$

  Finally, each station must be assigned to *at least* one channel in its set of feasible channels. This is expressed as

  $$\vee_{j \in M_i} x_{ij}. \tag{3}$$

- **Co-channel interference constraints:** For each channel $j$, there is a list of stations $S_j \subseteq \{1, \cdots, N\}$ such that no two stations in $S_j$ can be assigned to channel $j$ together. These constraints can be expressed as follows:

  $$\neg x_{ij} \vee \neg x_{kj} \text{ for each channel } j \text{ and stations } i,\, k \in S_j. \tag{4}$$

- **Adjacent channel interference constraints:** For each pair of adjacent channels $j$ and $j+1$ where $j \in \{1, \cdots, M-1\}$, there is a set of stations $A_{j,j+1} \subseteq \{1, \cdots, N\}$ such that no two stations in $A_{j,j+1}$ can be assigned to channel $j$ and channel $j+1$ or vice versa. These constraints can be expressed as:

  $$(\neg x_{ij} \vee \neg x_{k(j+1)}) \wedge (\neg x_{i(j+1)} \vee \neg x_{kj}) \text{ for each pair of channels } j,\, j+1 \text{ and stations } i,\, k \in A_{j,j+1}. \tag{5}$$

The boolean expression that needs to be satisfied by the SAT solver is the conjunction of all clauses listed above. Note that this produces a boolean expression in conjunctive normal form (CNF), i.e. a conjunction of a series of disjunctions, which is the standard format in which SAT solvers accept their input.

In [1] we also found the minimum number of stations that must be removed in order to meet a particular spectrum clearing target. This entails asking the following question: can a set of stations be packed into a given set of channels under the interference and domain constraints, assuming that up to a number $K$ $(< N)$ of these stations need not be assigned to any channel? One can start from a large value for $K$ to obtain a "SAT" from the

SAT-solver and subsequently decrease the value of $K$ to find out the minimum value at which a "SAT" output is no longer obtained within a reasonable amount of time. We have borrowed this technique from [5]. The next section discusses why we use the "timeout" method as opposed to obtaining "UNSAT" directly.

This new condition can be encoded as a clause as follows. For each station $i \in \{1, \cdots, N\}$, one defines a binary variable $x_{iH}$, which takes value 1 if the station remains unassigned and 0 otherwise. In order to ensure that a station is not simultaneously assigned to some channel and $x_{iH} = 1$ (i.e. marked unassigned), we need to add the following set of expressions:

$$\neg x_{ij} \vee \neg x_{iH} \text{ for each station } i \text{ and every channel } j \in M_i. \tag{6}$$

Also since each station can now also remain unassigned, the clause (3) gets modified to

$$(\vee_{j \in M_i} x_{ij}) \vee x_{iH}. \tag{7}$$

Further, not more than $K$ of the set of variables $\{x_{iH} : i \in \{1, \cdots N\}\}$ can be assigned a value of 1, i.e.

$$\sum_{i=1}^{N} x_{iH} \leq K. \tag{8}$$

There are many ways of translating this type of a *boolean cardinality constraint* into CNF clauses. Following the precedent set in [5], we use the encoding described in [8]. This encoding essentially encodes a binary-logic adder into CNF clauses by adding intermediate variables. Interested readers should refer to [8] for details.

### B. *Timeout vs. UNSAT*

SAT solvers typically yield an answer of either "SAT" or "UNSAT," indicating whether the problem was satisfiable or not. In the case of a "SAT", an actual assignment meeting the constraints is given as proof of satisfiability. If "UNSAT", the solver has found a proof of unsatisfiability and offers up information related to that proof.

However, the size and complexity of the repacking problem (see Appendix E) is such that both types of solutions may take a long time to obtain. In this case, we "time-out" the SAT solver (i.e. halt its execution early) in the interest of time. This method has precedent: [5] set a time-out of 60 seconds (much shorter than any of ours). Typical run time of different SAT solvers on instances of repacking problems has also been studied in [9]. In our repacker, we set a time-out threshold that varied from 150 to 300 seconds, depending on the difficulty of the repacking problem.

We have further observed a well known quirk of SAT solvers: randomizing the order in which the variables $x_{ij}$ are created considerably changes the time taken by the solver to find a satisfying assignment (if one exists). Because of this, for a particular time-out threshold, one attempt may lead to no output, but another attempt after randomization may lead to a "SAT". Hence, while trying to solve an instance, we make multiple randomized attempts (typically 5-10) with a fixed time-out threshold. If the SAT-solver still does not output a solution, we consider this to mean that the instance was *probably* unsatisfiable. Using this approach, the numbers that we obtain for the minimum number of stations to remove to clear spectrum for the different band plans are considerably lower than those reported in [5]. This indicates that the true minimum numbers may in fact be even lower. The only way to ascertain the true minimum numbers is to obtain an "UNSAT." But in all of our experiments, except for specific small instances, we very rarely obtained an "UNSAT" from the SAT solver, even with time-out thresholds as long as 30 minutes.

### C. *Repacking assumptions*

We note the following assumptions made while designing our repacker:

- Stations currently assigned to a channel in UHF, whether the channel is above or below the spectrum clearing target, may be moved to any band; that is, to VHF or UHF channels.
- Stations currently assigned to a VHF channel may remain in the VHF band or be moved to the UHF band.
- Stations, whether above or below the spectrum clearing target, can either be repacked or relinquish their spectrum usage rights.

- All stations, regardless of their current channel assignment, are equally willing to relinquish their spectrum usage rights.[1]

In essence, we assume that all stations fully participate in the auction and we eliminate the distinction between VHF and UHF. Although this may not accurately reflect the incentive auction scenario, it accurately represents the generic spectrum reallocation scenario which is the focus of [1].

If we wish to mimic or study the outcome of the incentive auctions, there is a wide variety of assumptions that can be made. For instance, many opine that during the incentive auction, stations will be reluctant to move to lower bands, in particular from UHF to VHF. This restriction is easily implemented in our repacker. As another example, leaving stations below the spectrum clearing target untouched and attempting to repack only stations above the clearing target would minimize the additional costs required to relocate repacked stations after the auction.

However, we focus on the efficient clearing method with the assumptions listed above because it places minimal restrictions on the repacking process. This repacking flexibility results in the fewest possible stations being taken off the air. This represents a best-case scenario for viewable TV, and a worst-case scenario for available whitespace as stations will now be efficiently packed into the channels below the clearing target, reducing the number of spectrum holes.

We compare the efficient clearing method with a naive clearing method, where stations above the clearing target simply relinquish their spectrum usage rights, and stations below the clearing target are left completely untouched. The naive clearing method results in the most stations being taken off the air because there is no attempt at repacking. This represents the worst-case scenario for viewable TV, and a best-case scenario for available whitespace as the inefficient packing of stations in the channels below the clearing target will create a larger number of spectrum holes.

With these two methods, we have explored the best-case and worst-case scenarios for both whitespace and TV channel availability. Although they may not perfectly mimic what is expected to happen in the actual incentive auction, they prove useful in understanding the general problem of spectrum reallocation. Future work includes modifying our repacker to study the incentive auctions themselves.

### *D. Description of datasets and figures*

In this section, we describe our methodology for computing the amount of available whitespace (in channels or MHz) and the amount of viewable TV (in channels) for a generic allocation of stations in a region. To do this, we have built extensions for our in-house, open-source software, Whitespace Evaluation SofTware [2]. WEST provides general-purpose tools for computing the amount of whitespace and TV for a generic region, and a generic set of protected entities (e.g. TV stations and Private Land Mobile Radio Services (PLMRS)). In this section we provide a brief description of the datasets used for our calculations, the building blocks of our extensions to WEST, and the process by which we generated the figures in [1].

#### **Datasets used**

Our work utilizes a variety of publicly-available datasets. This section describes these datasets, how we used them, and where to find them.

- **FCC's incentive auction baseline file [3]:** This file, released by the FCC, contains basic information about the TV stations which are eligible to participate in the incentive auction.
- **Datasets for PLMRS entities [10]:** We also consider restrictions due to other incumbent services, specifically PLMRS (Private Land Mobile Radio Services). These restrictions primarily affect metropolitan areas.
- **FCC's domain and interference constraint files [3]:** These FCC datasets give us the explicit domain and interference constraints for each TV station as described in Appendix A-A.
- **FCC's TV station service area data [11]:** The FCC has also made available the official service area polygons of TV stations (defined by 360 pairs of latitude and longitude coordinates). From this, we can determine the

[1] In reality, this is not entirely true. Many stations may not even be willing to participate in the incentive auction. It is difficult to predict this behavior, and so we do not consider the possibility here.

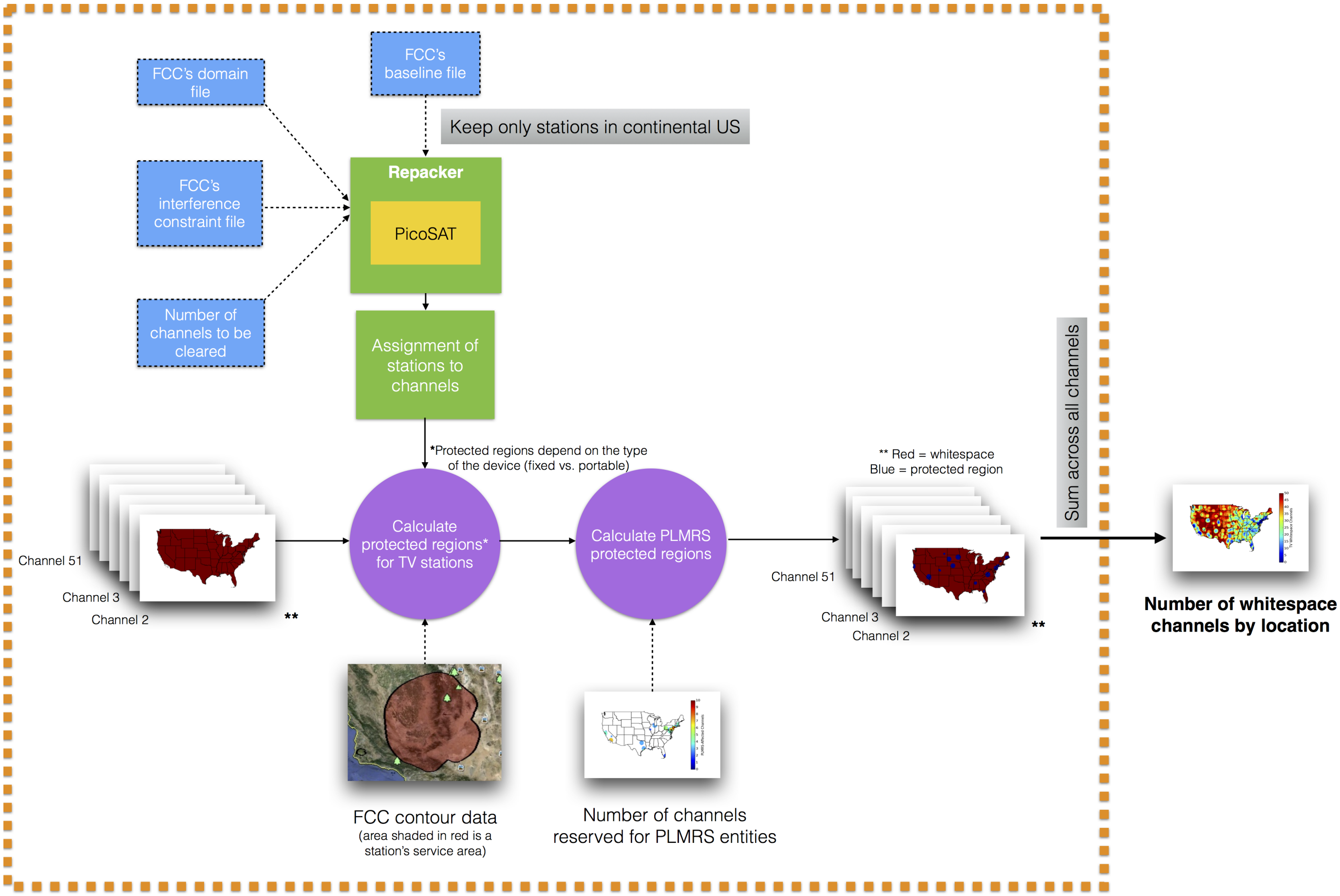



Fig. 1. Data flow diagram showing the process used to evaluate the number of available whitespace channels at every location in the US. Note that this process is executed for a particular band plan and for a particular repacking scenario.

protected contour polygons by adding a buffer of the appropriate size (separation distance) for the whitespace device in question[2].

- **United States 2010 census data [12], [13]:** We use the census data to create a discretized map representing the population of the United States. This population map is used in weighted statistical metrics (for example, mean, median, and CDF) of available whitespace or TV in the United States.

## Basic whitespace- and TV-availability calculations

As mentioned previously, WEST is a free, open-source, and general-purpose software for whitespace studies. It has well-defined modules that are designed to independently support generic geographic regions, protected entities, and rulesets. These modules are easily customized for the US, and for pre- and post-incentive auction scenarios.

Figure 1 outlines our core algorithm for computing whitespace availability. We begin by reading from the FCC's baseline file [3], considering only stations within the continental US. We then provide these stations, along with the FCC's domain and interference constraints [3] to our repacker, a thin wrapper on top of PicoSAT. The repacker outputs a feasible repacking for the given band plan. This process is shown in the top half of Figure 1.

From the re-assignment of TV stations obtained from the repacker, and the service area polygons in [11], we

[2]We were able to make use of these for almost all stations. For about 2% of the stations, there were discrepancies in the data which forced us to fall back to an inferior method of calculating the protected region. We discuss this in more detail in Appendix C.

can determine the protected regions for each TV station, and therefore protected locations on each channel. We also mark off regions that are further protected for PLMRS entities. Finally, we obtain maps that show how much whitespace is available at each location. This process is shown in the bottom half of Figure 1.

The process for evaluating the amount of viewable TV is extremely similar to the above. On every channel, locations within the service area of a TV station are considered "TV-viewable." Therefore, we can similarly use the FCC's service area data to determine TV-viewable locations on every channel, and thereby obtain maps showing how many TV channels are viewable across the US.

Both of these processes are used extensively as the baseline data for the generation of our other figures. **Figure 5 in [1]** shows how much whitespace is available at each location in the continental United States after an efficient reallocation. We can also create "loss maps" that show us where and how much whitespace/TV is lost post-incentive auction, as seen in **Figure 3 of [1]**.

**Figure generation**

Here, we briefly describe the process used to generate Figures 6-10 in [1]. These figures are crucial in understanding the effect of repacking on available whitespace and viewable TV.

- **Figures 6, 7, and 8 in [1]** are 2-D histograms that represent the distribution of people who have a certain number of available whitespace channels pre-auction and a certain number of available whitespace channels post-auction. Figure 2 in this appendix describes the process by which these figures were made. For every possible ordered pair describing the number of whitespace channels pre- and post-auction, $(x, y)$, we determine the number of people that have $x$ available whitespace channels pre-auction and $y$ whitespace channels post-auction. We then color the pixel at position $(x, y)$ according to the number of people. For robustness, we calculate the pixel-wise average of these 2-D histograms over multiple repackings for a given band plan to obtain the final histograms presented in [1].
- **Figures 9 and 10 in [1]** show plots of the amount of viewable TV versus the amount of available whitespace for various spectrum clearing targets. Figure 3 in this appendix describes the process by which these figures were made. We use the whitespace availability map obtained according to Figure 1 along with population data to determine how much whitespace is available to the median person in the US in a particular repacking scenario. Similarly, we use the TV availability map to determine how much TV is viewable to the median person in the US for a particular repacking scenario. We repeat this process for several potential repackings for a given spectrum clearing target, and then for all possible spectrum clearing targets, and plot the amount of viewable TV versus the amount of available whitespace spectrum (in MHz).

## APPENDIX B
## TELEVISION AVAILABILITY AFTER REPACKING

One way of looking at the impact of the incentive auction is to examine which places lose access to TV under various repacking scenarios. Figure 3 in [1] showed which places lose at least one TV channel under the naive clearing method (orange), the efficient clearing method (green), or both (blue) when 14 channels are to be cleared for LTE.

We present here more detailed maps showing how many TV channels were lost in each market. Figure 4 makes the difference between the naive clearing method and the efficient clearing method even more stark. With the efficient clearing method, only San Francisco, Los Angeles, and the most populous portions of the East Coast lose more than four channels. With the naive clearing method, a variety of regions (including Utah, for example) lose six or more TV channels.

Beyond answering the question of how much TV coverage will be lost, these more detailed figures are important because they give insight into how the repacking will work. In particular, we see that the areas that currently have a lot of TV stations (e.g. New York City, Los Angeles) have so many that some must be removed rather than repacked. However, most of the country is not brimming with TV stations (as evidenced by the current amount of whitespace in these regions) and so no stations would need to be removed to meet most clearing targets.

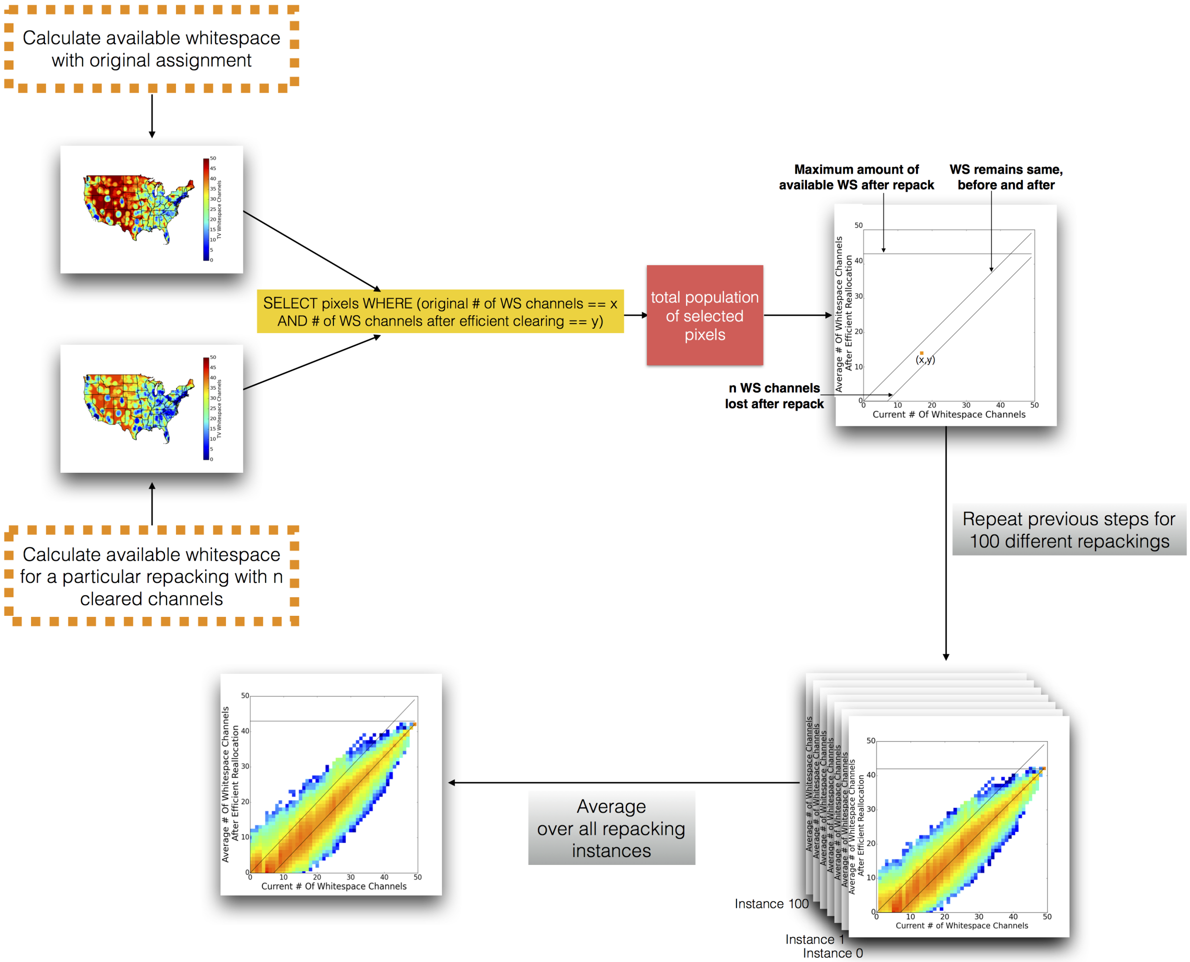



Fig. 2. Data flow diagram showing the process used to generate Figures 6, 7 and 8 in [1]. Note that this process is done for a particular band plan.

## APPENDIX C
## ADDRESSING THE PROBLEMS IN REAL-WORLD DATASETS

In this section, we discuss apparent discrepancies and errors in datasets provided by the FCC. The errors mainly affect our assessment of the service areas (and hence protected contours) of TV stations, and therefore our estimate of the available whitespace and/or viewable TV. We describe techniques we have used to eliminate, or at least minimize, the effects of these errors and discrepancies.

### *A. TV station records with missing HAAT data*

About 18% of the records in our TV station dataset had no height above average terrain (HAAT) data listed. According to an FAQ entry posted on Aug. 23, 2011 at http://www.fcc.gov/encyclopedia/white-space-database-administration-q-page, the HAAT for records missing this value should be calculated from the RCAMSL (Height of Antenna Radiation Center Above Mean Sea Level) in conjunction with the terrain database. Thus we updated the HAAT entries for the relevant records using the FCC's HAAT calculator at [14].

### *B. Stations without service contour data*

The baseline file [3] and the FCC's service contour data points file [11] contain information for slightly different sets of stations. The baseline file contains the list of stations eligible to participate in the incentive auction whereas the contour data points are given for all active records in the FCC's TV Query database. Furthermore, the baseline

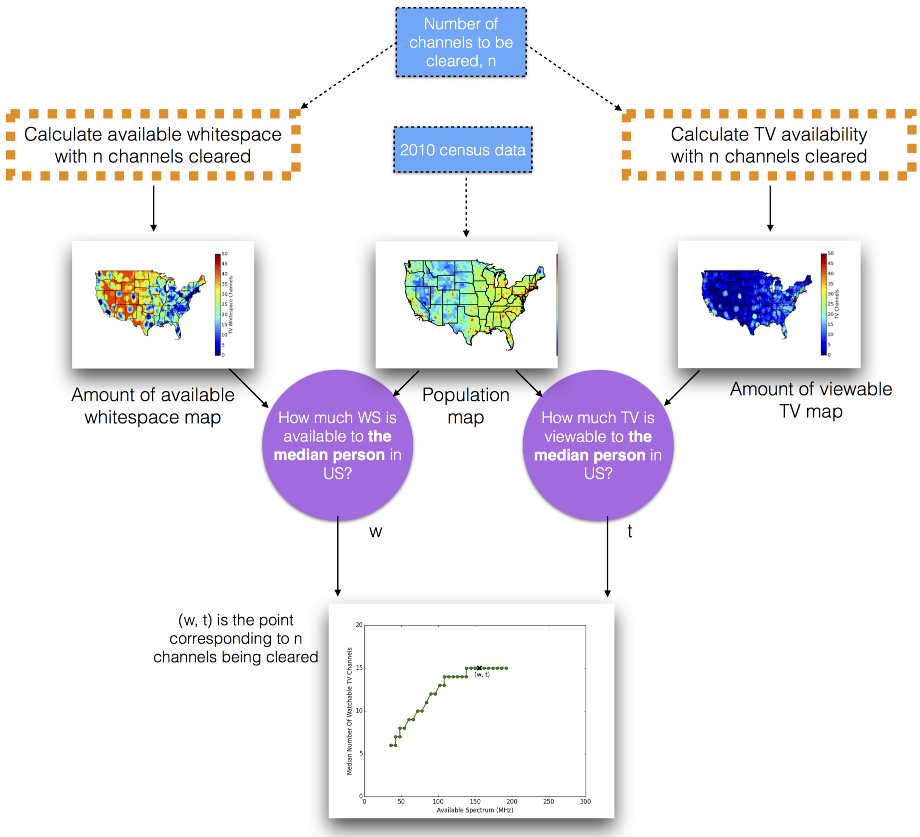
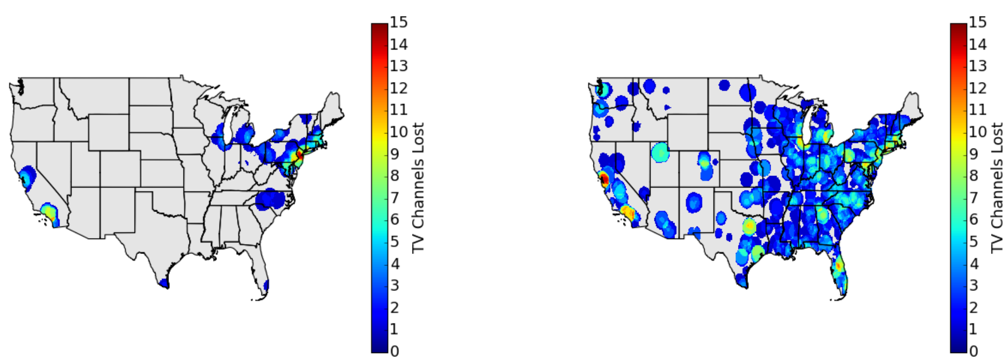



Fig. 3. Data flow diagram showing the process used to create Figures 9 and 10 in [1].




(a) Efficient clearing method

(b) Naive clearing method

Fig. 4. Map showing how much TV coverage is likely to be lost across the US after the incentive auctions if 14 TV channels are repurposed using the efficient clearing method. Gray denotes areas whose TV coverage is not affected.

file is a snapshot from 20 May 2014 while the service contour data is updated daily and our copy is from 17 June 2014. About 2% of the stations in the baseline file updated their licenses between these two dates (e.g. some stations have changed physical location or transmit power), meaning that the service contours may not correspond to the station parameters given in the baseline file and used when generating the repacking constraint files. Thus it is not possible to obtain official FCC service contours for all auction-eligible stations.

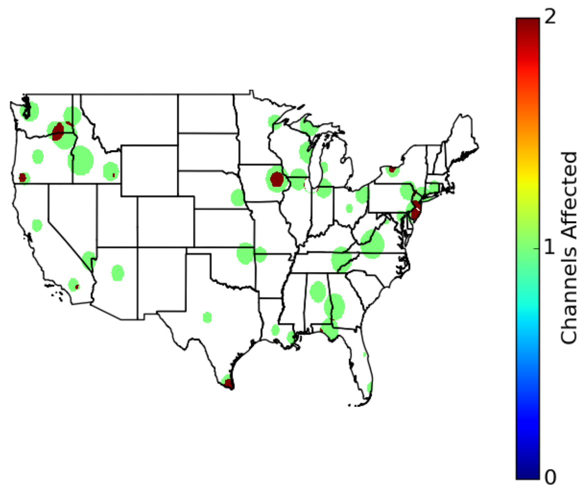
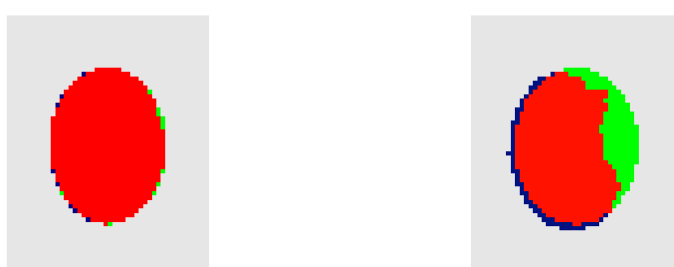



Fig. 5. Map indicating areas in the United States which are serviced on one or more channels by stations where we use circular contours.

For these stations without official FCC service contours, we create approximate contours. Although we use the FCC's F(50,90) propagation curves [15] to create these approximations, we do not incorporate the terrain data which is necessary for reproducing the official contours[3]. Instead, we assume flat terrain which results in circular service contours. In relatively flat areas, our approximate contours will closely match the official FCC contours. In areas with rough or mountainous terrain, the official contours are generally not circular and thus our approximations will have greater error. The difference is shown for two example stations *with* official contours in Figure 6. Light green and dark blue indicate disagreement between the approximate and official contours whereas gray and red indicate agreement.

Figure 5 indicates the locations which are serviced by these stations *according to the approximate (circular) contours*. We see that about 29% of the population is serviced by these circular contours.



(a) Flat terrain

(b) Rough/mountainous terrain

Fig. 6. Comparison of protected contours provided by the FCC and approximate circular contours for different types of terrain.

[3]Although WEST supports the use of terrain data, we have not obtained and integrated terrain data into our code yet.

### C. Overlap in stations' service contours

While calculating protected regions for stations across the US, we observed that a few service contours overlapped for stations on the same channel. We believe this is because the FCC model for television reception allows the receiver to point his antenna towards the station he wishes to receive. This allows for the theoretical possibility of locations where two or more stations can be received on the same channel by pointing the receiver's antenna in different directions. At such locations, there are two or more viewable TV stations on the same channel. Figure 7 highlights these locations in blue and makes it clear that the "overlap effect" occurs in a small portion of the United States.

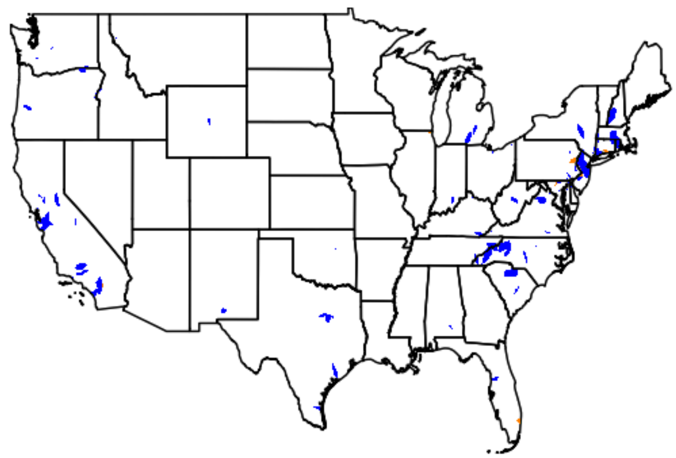


Fig. 7. Map indicating locations in the United States where there are two or more effectively viewable TV stations on any channel. Blue represents overlap between stations' contours given by the FCC, and orange represents an overlap between at least one circular contour.

When evaluating the amount of viewable TV at each location in the US, we take the effect of this overlap into account and count all visible stations at that location. The effect also shows up in locations where there was an overlap in the current allocation but no overlap after repacking. These locations will now "lose" an extra whitespace channel, even in cases where *no* channels are cleared. Conversely, locations where there was originally no overlap on any channel but at least one overlap present after repacking will "gain" at least one extra whitespace channel. These overlap effects explain why a few places lose more than $n$ channels of available whitespace when $n$ channels are cleared for LTE, and why a few places gain whitespace channels. These can be seen in the points that lie outside of the diagonal lines in Figures 7 and 8 of [1].

## APPENDIX D
## OVERVIEW OF THE INCENTIVE AUCTION

We provide a brief overview of the incentive auction background and procedures as a convenience to the interested reader. Although our work in [1] heavily uses the data released in connection with the incentive auction, it does not study the auction itself. Studies of the incentive auction are intentionally separated from [1], which is a study of the more general problem of spectrum reallocation.

### A. Background

Until the 1990s, spectrum allocations in the United States were assigned by lottery. When demand began to exceed supply and buyers demanded greater certainty about allocation outcomes, the Federal Communications Commission (FCC) switched to an auction-based system. In this system, spectrum leases were auctioned off to the highest bidder. When the lease expired, the spectrum went back on the market.

However, we have recently seen that the turnover rate in this system is too slow. Rather than wait for new spectrum to come to auction, companies are instead purchasing smaller companies purely for their spectrum holdings. When spectrum *is* available, it commands a hefty price as we saw in the AWS-3 auction that concluded in early 2015, in which carriers collectively paid almost $45 billion for 65 MHz of nationwide spectrum [16].

This, along with the oft-touted wireless boom, demonstrates the great need for new LTE-friendly spectrum. However, there are no upcoming bands where older licenses are expiring, especially at lower frequencies, so the FCC had to get a little creative. In particular, they turned their eye to over-the-air television broadcasting spectrum which is licensed in a fundamentally different way from most spectrum (briefly: broadcasters did not pay for their licenses because they are serving the public good). This left the door open to repurpose their spectrum with minimal cooperation from the broadcasters.

However, the FCC cannot simply break the broadcasters' leases. Instead, an act of Congress [17] passed in February 2012 gave them the authority to conduct a novel double-auction. In November 2012, the FCC put out a Notice of Proposed Rulemaking (NPRM) [18] for these so-called "incentive auctions" and they have since received over 2000 comments. A Report and Order [19] with further auction details was published in June 2014. Although the R&O established the overall framework for the auction, the smallest yet most important details were left for Public Notices (PNs). The first of these PNs was published in December 2014 [20] and the second is due in the first quarter of 2015. In the FCC's own words:

> Well in advance of the auction, also by Public Notice, the Commission will resolve these implementation issues, and provide detailed explanations and instructions for potential auction participants ("Incentive Auction Procedures PN" or "Procedures PN"). [19, ¶15]

This section briefly explains what is currently known about the auction process. The auction procedure is presented as an algorithm in Algorithm 1.

### B. Reverse auction

The first half of the incentive auction is the "reverse auction" in which TV broadcasters indicate the price at which they would be willing to relinquish their spectrum usage rights[4]. The auction will have a descending clock format [19, ¶447], meaning that bidders (broadcasters) will be offered successively lower bids for their spectrum[5]. If the price drops too low for a broadcaster, it can "drop out" of the auction, meaning that it will receive no payment but will keep its spectrum usage rights. If the broadcaster's bid is eventually accepted by the FCC, it will be paid and will be required to cease transmission within three months.

Broadcasters that "drop out" of the auction will be treated equivalently to broadcasters which choose not to participate in the auction. In both cases, the FCC has the authority to "repack" (i.e. relocate in frequency) these stations, compensating broadcasters only for the cost of the move itself. To minimize the negative impact on broadcasters and their viewers, a station will only be moved within its original band (e.g. UHF) and will have its transmission parameters adjusted so that its coverage area remains approximately the same.

### C. Forward auction

The second half of the incentive auctions, called the "forward auction," is in many ways a standard spectrum auction; that is, it will have an ascending-clock format. At each round, bidders will indicate how many $2{\times}5$ MHz bands (i.e. LTE uplink + downlink) of spectrum they would buy and at what price. Expected bidders include AT&T, Verizon, T-Mobile, Sprint, and possibly Dish Network.

Again, we have slightly simplified our description of this stage to improve readability. For example, the FCC will be using two types of generic spectrum blocks ("impaired" and "unimpaired") and will use a post-auction auction to determine which blocks go to which winning bidders. Interested readers should read the source documents for full details [18]–[20].

[4]This is not precisely true. Broadcasters are actually given several choices [19, ¶365] but here we simplify to the most important choice.

[5]In a reverse auction, multiple potential sellers compete to *sell* their goods or services to a single buyer, as opposed to the better-known case of the forward auction, in which multiple buyers compete to *buy* goods or services from a single seller. The descending-clock reverse auction is just the reverse auction counterpart of the well known ascending clock forward auction, also known as the English auction, in which bidding according to true preferences is a weakly dominant strategy. See [21] for more details.

**Algorithm 1** Incentive auction procedure

**Require:** $M_{excess}$: the amount of money needed in addition to the cost for the reverse auction (mandatory expenses)
**Require:** $M_{MHz-pop}$: reserve MHz-pop price ( [20, ¶47] suggests $1.25/MHz-pop)

▷ INITIALIZATION

Broadcasters given initial bids from the FCC
Broadcasters submit applications to participate in the auction
Set $S_{\text{clearing target}}$ (in number of paired LTE channels) based on broadcaster interest

▷ REVERSE AUCTION

**function** ReverseAuction($S_{\text{clearing target}}$)
    Offer broadcasters large amounts of money
    **while** clearing target met **do**
        Offer each broadcaster slightly less money (customized offers)
        Receive bids from broadcasters
        Determine which bids to provisionally accept
    **end while**
    Accept the last bids for which the clearing target was met
    **return** ReverseMoneyNeeded
**end function**

▷ FORWARD AUCTION

**function** ForwardAuction($S_{\text{clearing target}}$)
    **while** demand $>$ supply **do**
        Increase prices from previous round
        Each bidder indicates the number of generic LTE bands it wants in each market at that price
    **end while**
    **return** ForwardMoneyRaised
**end function**

▷ FINAL CONDITION FOR AUCTION SUCCESS

**function** FinalConditionMet(ReverseMoneyNeeded, ForwardMoneyRaised, $S_{\text{clearing target}}$)
    **if** (ReverseMoneyNeeded + $M_{excess} \leq$ ForwardMoneyRaised) AND (MHz/pop($S_{\text{clearing target}}$) $> M_{MHz-pop}$) **then**
        **return** True
    **else**
        **return** False
    **end if**
**end function**

▷ COMBINING THE REVERSE AND FORWARD AUCTIONS

AuctionSucceeded $\leftarrow$ False
**while** $S_{\text{clearing target}} \geq 2$ LTE channels AND not AuctionSucceeded **do**
    ReverseMoneyNeeded $\leftarrow$ ReverseAuction($S_{\text{clearing target}}$)
    ForwardMoneyRaised $\leftarrow$ ForwardAuction($S_{\text{clearing target}}$)
    **if** FinalConditionMet(ReverseMoneyNeeded, ForwardMoneyRaised, $S_{\text{clearing target}}$) **then**
        AuctionSucceeded $\leftarrow$ True
    **else**
        $S_{\text{clearing target}} \leftarrow S_{\text{clearing target}} - 1$
    **end if**
**end while**

▷ POST-AUCTION

**if** AuctionSucceeded **then**
    Forward auction winners bid for specific spectrum blocks
    Repacking optimized
**else**
    Apply egg directly to face
**end if**

### D. Combining the reverse and forward auctions

Since Congress did not authorize the FCC to make any payments itself, the reverse and forward auctions must be sufficiently intertwined to ensure that the proceeds from the forward auction can pay for the reverse auction[6]. In particular, the FCC will set a spectrum clearing target (i.e. number of TV channels to be repurposed) based on initial interest indicated by broadcasters. They will then conduct a reverse auction, provisionally "buying out" enough broadcasters to meet the clearing target. A forward auction is then held with the goal of auctioning the provisionally-cleared spectrum for enough money to actually fund the reverse auction. If the forward auction successfully raises the funds to pay the provisional reverse auction bids, all bids are officially accepted and the auction concludes. If not, the FCC reduces the spectrum clearing target and begins again, starting with the reverse auction.

### E. Remaining details

*1) Specific band plans:* The FCC has created specific spectrum clearing targets, each called a "band plan," as can be seen in Figure 8, which is Figure 23 in [19]. Each row represents a unique band plan, with the lowest clearing target at the top and the highest at the bottom. The paired LTE channels are shown in blue with the higher frequencies being designated for uplink and the lower frequencies for downlink. Remaining TV channels are shown in white (channels 2-20 are not shown but will remain allocated for TV). Band plans include spectrum for the duplex gap (11 MHz between the LTE up- and downlink bands), a guard band between the LTE downlink and remaining TV channels (7-11 MHz, depending on the plan), and a 3-MHz buffer around channel 37 (used by sensitive radioastronomy equipment as well as some medical devices deployed in hospitals) as needed.

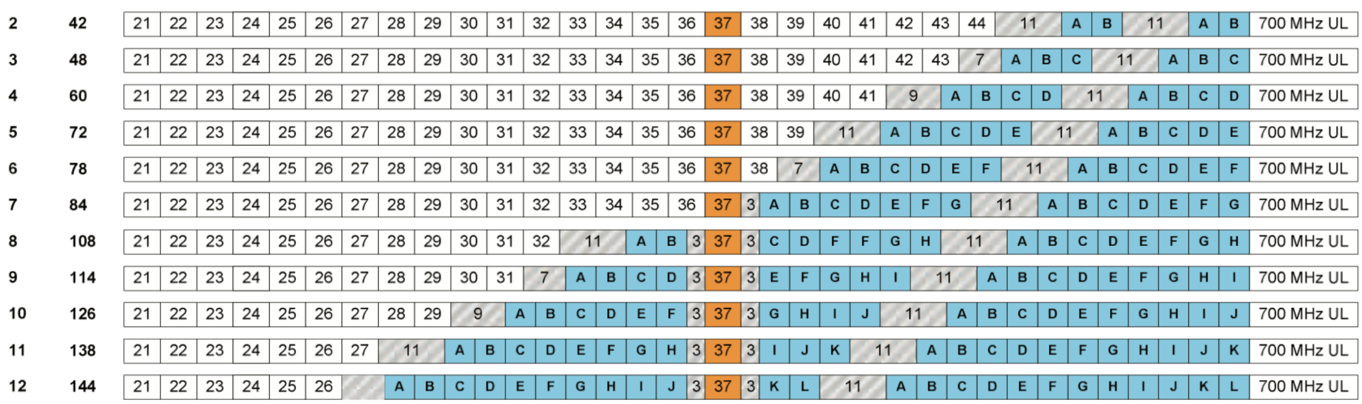



Fig. 8. FCC band plans from [19]. Each row represents a specific band plan. The left column of numbers represent the number of 2×5 LTE channels created. The second column lists the number of repurposed MHz. TV spectrum is shown in white and LTE spectrum in blue.

*2) "Repacking" TV stations:* A major component of the reverse auction that is at first hidden from the reader is that it is essential to be able to relocate TV stations to another channel (within reason) without paying them more than relocation costs. For example, consider a TV station on channel 51, the highest-frequency TV channel. Removing this TV station from channel 51 (either buying its spectrum usage rights or relocating it to another channel) is absolutely necessary for *any* of the band plans to be feasible. If given the option to name a price, the broadcaster could essentially hold the entire auction hostage. This same reasoning applies to any broadcaster in a channel that is included in the spectrum clearing target.

As such, the Spectrum Act also gives the FCC the authority to relocate TV stations within their "home band" (e.g. if a station is currently allocated a channel in the UHF band, it must either be paid or be relocated within the UHF band) [17, §6403(b)(1)(B)(i)]. Although it sounds simple, the repacking is actually critical to the auction.

Repacking also provides a great opportunity for researchers. In order to be able to conduct the auction in real time, the FCC preprocessed a large amount of data about the potential for co- or adjacent-channel interference between TV stations. They also analyzed which stations have "domain constraints." For example, a station near the Canadian border cannot be assigned to a channel being used by a nearby Canadian station (non-US stations may not

[6]In addition, the proceeds must cover reallocation costs and costs of running the auction [19, ¶341], amounting to $2 billion.

participate in the auctions). It is precisely this data which we repurposed in [1], since it completely enumerates the constraints on the efficient repacking of TV stations into existing spectrum. The following appendix gives further details about the data.

## APPENDIX E
## EXPLORING INCENTIVE AUCTION DATA

While working closely with the FCC's incentive auction data, we noticed several interesting phenomena. Although our work in [1] is intended to be somewhat separate from the incentive auction itself, these phenomena will affect our results. Furthermore, we feel strongly that any interesting patterns or potential errors in the incentive auction data should be made public.

### *A. Domain constraints*

A station's domain is the set of channels to which it *may* be assigned during the repacking. For many stations, the domain will be the entire set of TV channels. However, some stations, particularly those along the borders, will have constrained domains:

> In order to protect LM base stations, LMW base stations, Mexican allotments, Canadian allotments and Channel 37, the FCC staff had to consider these as fixed constraints, which will limit the channels on which any U.S. television station can be assigned or reassigned in the incentive auction repacking process. [22, §3.3]

The FCC has published the domains for all stations that may participate in the auction at [3]. Note that the domain data is independent of the spectrum clearing target since it only identifies channels on which a station could be placed *in the context of protecting non-TV incumbents and non-US TV stations*.

Figure 9 shows the number of stations with domain of size $D$. The majority of stations (approximately 57%) have no domain restrictions. All but about 12.5% have a domain size greater than 40.

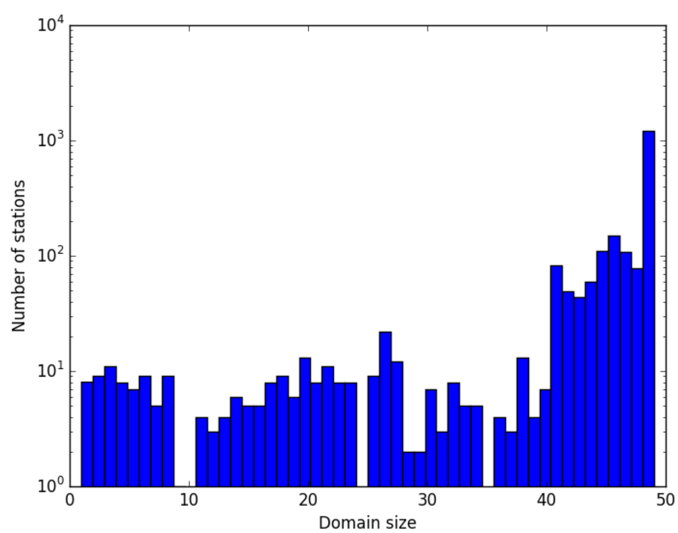



Fig. 9. Frequency distribution of domain sizes of stations that may participate in the incentive auction.

Figure 10 illustrates the domain size as a function of location. Each marker represents a station. White stations are completely unconstrained and may be placed on any TV channel. Blue stations are nearly unconstrained whereas red stations are extremely constrained.

We see that the majority of the constraints are coming from the border protections with Canada and Mexico. For example, Los Angeles, southern Texas, and upstate New York are particularly affected.

In other places such as San Francisco and New York City, most if not all domain constraints come from the LM and LMW protections. Data detailing these protections can be downloaded from [3]. These affect only channels

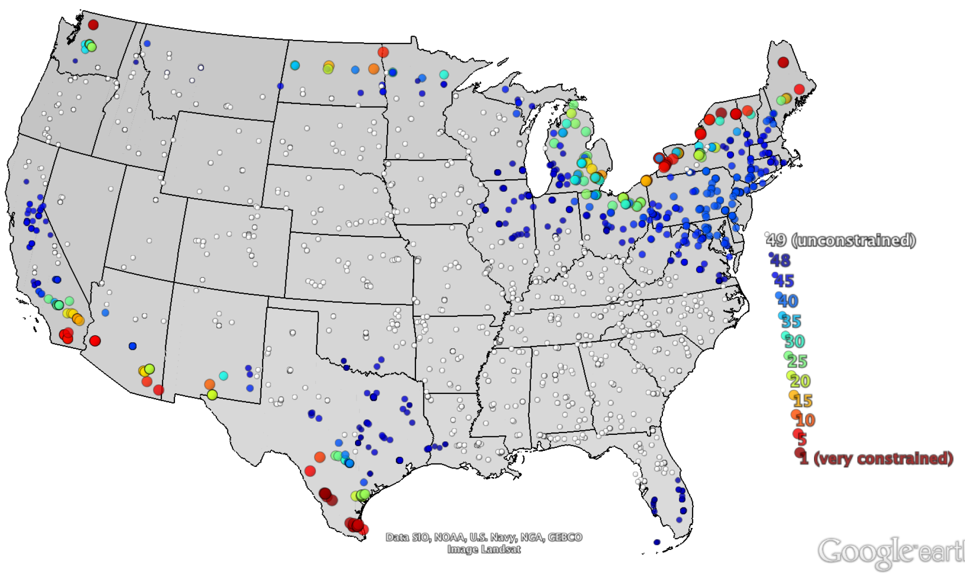



Fig. 10. Color-coded sizes of domains of stations plotted with Google Earth. The source KML can be found at http://www.eecs.berkeley.edu/~sahai/icc2015-incentive-auction-domain-constraints.kml

14-20. This can be seen most clearly in Figure 11. While the protected stations in Canada and Mexico have no particular affinity for one channel over another, there is an obvious decrease in the number of stations that have channels 14-20 in their domain.

Note that channel 37 is not in the domain for any TV station since it is used for radioastronomy, which involves receivers that are too sensitive to coexist with TV stations.

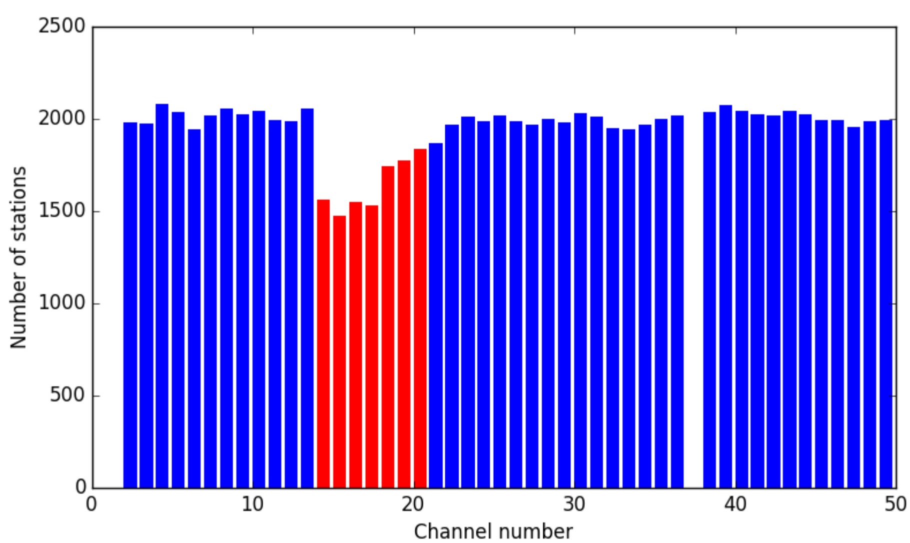



Fig. 11. Number of stations with a particular channel $C$ in their domain. Channels 14-20, which are affected by the LM and LMW protections, are shown in red.

### B. *Possible errors in the domain constraints*

Figure 10 displays some data that appears to be inconsistent with the protection criteria. For example, there are several white (i.e. completely unconstrained in terms of domain) stations in New York, situated among many rather constrained stations. One of the white stations (on the NY-Pennsylvania border) is even located on the same tower as a constrained station, although the difference could be attributed to the relatively low transmit power of the unconstrained station. There are similar apparent anomalies in the Upper Peninsula of Michigan, the North Dakota-Minnesota border, northern Idaho, and near Austin, Texas. Differences in transmit power cannot explain all of these apparent anomalies: for example, the transmit power of the unconstrained station in the Upper Peninsula of Michigan (facility ID 81448) is over three times that of the nearest constrained station (facility ID 59281) and it is approximately the same distance from the border.

## ACKNOWLEDGEMENTS

We thank the U.S. NSF (CNS-1343398, CNS-1321155, AST-1444078, and a Graduate Research Fellowship). The work of the third author was supported in part by the U.S. Air Force Office of Scientific Research (AFOSR) under grant number MURI FA9550-10-1-0573.

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